Joint hazard of earthquake shaking at two or more locations

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ABSTRACT: The continuation of a system, activity or lifeline after an earthquake may depend on one or all of several critical facilities at different sites remaining operational. Hence the joint hazard of earthquake shaking at two or more locations in the same earthquake is of much interest. The joint hazard from either a single earthquake source or multiple earthquake sources can be estimated using a random effects attenuation model, which distinguishes the within-earthquake and between earthquake components of variability. The ratio between the joint and individual hazards can vary widely, and this calls into question the common "scenario" approach adopted in some lifeline studies. To illustrate this point, estimates of joint hazard are compared with the results of a lifelines study in Auckland in which the scenario approach was adopted.

1 INTRODUCTION

In seismic hazard analysis, the probability of earthquake shaking is typically estimated individually at one or many sites. But it is the joint distribution of earthquake hazard over two or more sites that is important for many engineering applications. For example, lifeline systems may have vulnerable elements at two or more points of a network, and failure of the network as a whole may depend on the simultaneous failure of certain combinations of elements. Then there is interest in knowing whether the vulnerable elements are likely to be simultaneously affected by shaking of sufficient strength to disable them. Or there may be several critical points, with failure at any one of them causing failure of the system. In that case there is interest in knowing how often the shaking is likely to be strong enough at any one of the critical points to cause failure.

When facilities at different sites are considered together, the probability of failure at all sites simultaneously can be no greater than the smallest of the individual site probabilities of failure, and the probability of failure at any one site can be no less than the largest. Just how close these probabilities are to their upper and lower bounds, respectively, depends largely on how much within-earthquake variability there is in the levels of motion that could be experienced at equivalent sites in a given earthquake. If the within-earthquake component of variability is large, the return period for damaging motions in the same event at two or more sites could be much greater than the individual return periods, making the common "scenario earthquake" approach to assessing lifeline networks with redundancy highly conservative. Conversely, the return period for damaging motion at any of the sites could be much smaller than the individual return periods, making the scenario approach to assessing lifeline networks without redundancy highly non-conservative.

A rigorous methodology for assessing the joint hazard at a number of sites has been presented by Rhoades & McVerry (2001), taking into account the different components of uncertainty in a strong motion attenuation relation. Here we briefly review that methodology and apply it to an example recently considered as part of a lifelines study in Auckland in which the standard scenario approach was adopted.

2 JOINT HAZARD ANALYSIS

In the "random effects" attenuation model (Brillinger and Preisler 1985; Abrahamson and Youngs 1992), the maximum or spectral acceleration at some site remote from an earthquake source is of the form

$$\log A = h(e,s) + \delta + \varepsilon \tag{1}$$

where e denotes variables which are properties of the earthquake source (e.g. the earthquake magnitude, tectonic setting and focal mechanism), s denotes variables which are properties of the remote site (e.g. its distance from the source, the stiffness of the ground at the site, and path characteristics), and h is a suitable function. The error random variables δ and ε represent the between-earthquake and within earthquake components of variability and are assumed to be independent and normally distributed with variances σ_δ^2 and σ_ε^2, respectively. Both δ and ε are considered to be components of "aleatory" uncertainty in the sense of Toro et al.(1997) or Anderson and Brune (1999), i.e. they represent variability of the data from the fitted model. The other important component of uncertainty is the so-called "epistemic" uncertainty, represented by the uncertainty in the values of the parameters in the model. Taking account of this epistemic uncertainty, the acceleration $A_1,\cdots, A_n$ at n different locations $L_1,\cdots, L_n$ with site-variables $s_1,\cdots, s_n$ in an earthquake of known source properties e is represented by

$$\log A_i = h(e,s_i) + \gamma_i + \delta + \varepsilon_i\,, \qquad i = 1,\cdots,n \tag{2}$$

where γ_i is the error in estimating the parameters of the function h, assumed to be normally distributed with variance σ_γ^2. The value of γ_i is an unknown outcome of the fitting of equation (1), but its value varies with the values of the earthquake and site terms, e and s. The value of δ is common to all sites, it being a random variable determined by the earthquake. On the other hand, the value of ε_i is particular to the earthquake and the site.

2.1 *Single earthquake source analysis*

Rhoades and McVerry (2001) showed that for a single earthquake source, if epistemic uncertainty is ignored, the joint probability of given levels of acceleration being exceeded at a number of sites is given by

$$P(A_i \ge a_i, i=1,\cdots,n \mid \gamma, e) = \frac{1}{\sigma_\delta}\int_{-\infty}^{\infty}\prod_{i=1}^{n}\left\{1-\Phi\left[\frac{\log a_i - h(e,s_i) - \gamma_i - \delta}{\sigma_\varepsilon}\right]\right\}\phi\left(\frac{\delta}{\sigma_\delta}\right)d\delta \tag{3}$$

where ϕ and Φ are the standard normal probability density function and cumulative distribution function, respectively, i.e., $\phi(u) = (2\pi)^{-1/2}\exp(-u^2/2)$, and $\Phi(z) = \int_{-\infty}^{z}\phi(u)du$.

In many studies, the epistemic error is treated as negligible and ignored, i.e., it is implicitly assumed that $\gamma_i = 0$, $i = 1,\cdots,n$. To take account of it requires a further integration:

$$P(A_i \ge a_i, i=1,\cdots,n \mid e) = \int_{\gamma_1}\cdots\int_{\gamma_n} P(A_i \ge a_i, i=1,\cdots,n \mid \gamma, e) f(\gamma_1,\cdots,\gamma_n)d\gamma_1\cdots d\gamma_n \tag{4}$$

where $f(\gamma_1,\cdots,\gamma_n)$ is the joint probability density of $\gamma_1,\cdots,\gamma_n$ and depends on the variance-covariance structure of the estimated parameters. If the earthquake characteristics source characteristics are themselves uncertain, a further integration may be required, viz.,

$$P(A_i \ge a_i, i=1,\cdots,n) = \int_{e} P(A_i \ge a_i, i=1,\cdots,n \mid e)de. \tag{5}$$

In practice, the latter two integrations, (4) and (5), may be most readily accomplished by averaging over many Monte Carlo simulations of the parameter values and earthquake source characteristics, as in the Auckland example below. Rhoades and McVerry (2001) showed in detail how to simulate the parameter uncertainties in the case where the model (2) is linear.

2.2 *Multiple earthquake source analysis*

Let us now consider the joint hazard rate $\lambda(a_1,\cdots, a_n)$, i.e. the average rate of occurrence of events in which the level of shaking exceeds $a_1,\cdots, a_n$ simultaneously at $L_1,\cdots, L_n$ when there are multiple earthquake sources. The contribution $\lambda_e(a_1,\cdots, a_n)$ that an individual earthquake source e makes to this rate is given by

$$\lambda_e(a_1,\cdots,a_n) = \frac{1}{T_e} P(A_i \geq a_i, i = 1,\cdots,n \mid e) \tag{6}$$

where T_e is the average recurrence interval for an event with source e, and $P(A_i \geq a_i, i = 1,\cdots,n \mid e)$ is given by equation (6). If all possible earthquake sources and their average recurrence times were known, the joint hazard rate could be computed as

$$\lambda(a_1,\cdots,a_n) = \sum_e \lambda_e(a_1,\cdots,a_n)\,. \tag{7}$$

However, usually only a few sources are known, and the hazard rate is estimated by some combination of characteristic earthquakes at known sources and an actual or randomly simulated catalogue of events distributed over other possible sources. One approach, which is adopted in the Auckland example discussed below, is to use a historical or simulated catalogue for the distributed seismicity. Then, for the distributed seismicity, T_e is approximated by T_c, the length of the actual or simulated catalogue, and for the characteristic earthquakes we should substitute $P(A_i \geq a_i, i = 1,\cdots,n)$ for $P(A_i \geq a_i, i = 1,\cdots,n \mid e)$ on the right hand side of equation (6), if uncertainties in the source characteristics are being allowed for. Another approach, when the epistemic error is ignored, is to evaluate equation (5) by numerical integration or series approximation with $\gamma_i = 0$. The probability that at least one instance of simultaneous exceedance will occur in a time period of length T is given by

$$P_T(A_i \geq a_i, i = 1,\cdots,n) = 1 - \exp[-\lambda(a_1,\cdots,a_n)T]\,. \tag{8}$$

The ratio of the joint exceedance rate to the individual exceedance rates of given levels of motion at two sites depends on two factors. The first factor is the extent to which both sites are affected by the same mix of earthquakes. As sites become further apart, they are less likely to be affected simultaneously in the same event, and their joint exceedance rates can be very low even when the individual exceedance rates are quite high. The second factor is the degree of variability in motions within a given earthquake. If the within-event variability is small, the joint probability of exceedance for any event approaches the smaller of the individual probabilities of exceedance, while when this variability is high the joint probability approaches the product of the individual exceedances.

The scenario approach that is used commonly to tackle joint hazard situations, such as for reliability analysis of extended lifelines systems or for loss estimates for insurance purposes, assumes that the sites are affected by a similar mix of earthquakes, with the selected scenario dominating. Most scenario approaches choose a particular percentile level of motion for the individual sites and assume that the joint probability of exceedance is the same as the individual probabilities of exceedance. This assumption is correct where there is no within-event variability, but can considerably over-estimate joint probabilities of exceedance.

3 **EXAMPLES**

Before moving on to the more general situation of the effects of all possible earthquake sources, an example is given for a situation where a scenario approach is usually assumed to be appropriate, namely where the hazard at the sites of interest is dominated by a single event.

3.1 *Joint hazard when a single source is dominant*

Consider sites in Upper Hutt and Wellington, both at distances of less than 3 km from the Wellington-Hutt Valley segment of the Wellington fault. The scenario uses the fault parameters and attenuation model from a recently published New Zealand National Seismic Hazard Model (NZNSHM) (Stirling et al. 2000). In the NZNSHM, this segment of the Wellington fault is modelled as producing magnitude 7.3 earthquakes at an average recurrence interval of 600 years. The 50-, 84- and 90-percentile peak ground accelerations estimated for these sites are about 0.6g, 0.95g and 1.1g for Wellington, and 0.65g,1.0g and 1.15g for Upper Hutt. The hazard at both of these sites is dominated by this segment of the Wellington Fault for return periods of about 500 years and greater, especially beyond about 1000 years. The hazard curves for the two sites are very similar, for both the Wellington-Hutt Valley fault event, and when all events are taken into account. The NZNSHM hazard curves incorporating the contributions of all events (not shown), show that the Wellington-Hutt Valley fault 50-, 84- and 90-percentile acceleration values correspond to individual site return periods of about 700 years, 2500 years and 4000 to 5000 years respectively.

Normalised versions of the joint hazard curves for Wellington and Upper Hutt are presented in Figure 1. The normalisation is in terms of the single site exceedance rates at Wellington, which are essentially the same as those for Upper Hutt. The normalised joint hazrd curves are shown for a selection of values of ρ, the ratio of the inter-event to total aleatory variance, covering the entire possible range from 0 to 1. In the NZNSHM, ρ is magnitude dependent, ranging from 0.16 for magnitudes of 5 and less to 0.36 for magnitudes of 7 and greater. These values are similar to those found in other attenuation models. The curve for ρ=0.36 appropriate for the Wellington-Hutt Valley fault segment is shown in the figure.

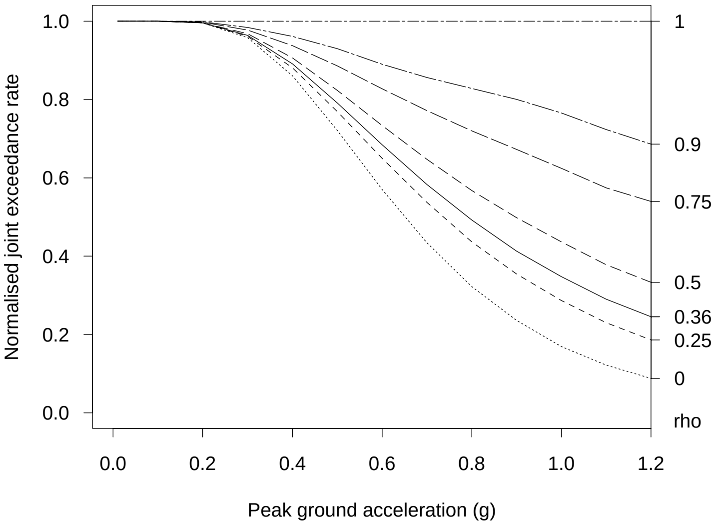

Figure 1. Joint exceedance rate normalised by Wellington single site exceedance rate for PGA in Wellington-Hutt Valley fault event. Each curve is for a different value of ρ (ratio of inter-event variance to total aleatory variance) shown at the right hand side of the plot.

For no intra-event variability (ρ=1), the normalised joint exceedance rate is 1.0, the usual assumption in scenario approaches. When all the variability is within events (ρ=0), the normalised joint exceedance rate falls well below 1.0, with the ratio decreasing as the acceleration level increases. As the joint probabilities of exceedance for this case are the product of the individual probabilities, the normalised joint exceedance rate is the same as the individual exceedance probability for a given acceleration level, i.e. 0.5 at the 50-percentile level, 0.16 at the 84-percentile level and 0.1 at the 90-percentile level. For the actual ρ-value of the attenuation model, the normalised exceedance rates are 0.62, 0.35 and 0.30 at the 50-, 84- and 90-percentile levels, respectively.

These results have important implications for the standard scenario approach. For critical lifelines facilities, 84-percentile or occasionally 90-percentile scenarios may be considered. The 84-percentile acceleration value of 0.95g-1g resulting from the Wellington-Hutt Valley scenario corresponds to individual site return periods of about 2500 years according to the NZNSHM estimates, a return period at about the top end of the range that is considered for most lifelines systems. However, for the ρ-value associated with the attenuation model, this lengthens to about 7500 years for the joint return period, or about 15,000 years if the ρ=0 case applied. The 90-percentile levels show an even more exaggerated effect. These return periods are well beyond what is appreciated in most uses of the scenario approach.

The variation of joint hazard with ρ is more pronounced for individual scenarios than when multiple events are considered. Both theoretical expressions that we have derived, and examples carried out including all sources in the NZNSHM, for example, show that the curves for a range of ρ values become more compressed when multiple events are considered

3.2 *Joint hazard of strong shaking at three Auckland sites*

Let us now consider an example involving multiple sources. Figure 2 shows three sites in the Auckland area at varying distances from the source of an earthquake on the northern segment of the Kerepehi fault and its extension into the Hauraki Gulf. The Kerepehi fault is the largest known surface fault in the Auckland area, and is closest to Auckland city on its northward extension. We distinguish two seismicity regions A and B, as shown in Figure 2. The eastern region B has historically had a higher level of seismicity than the western region A, which includes the Auckland urban area. The seismicity parameters assumed for each region are given in Table 1, where a_4 is the expected number of earthquakes of magnitude 4 or greater per 1000 km^2 per year, b is the Gutenberg-Richter b-value, and $M_{\max}$ is the maximum magnitude. The rate of occurrence of earthquakes exceeding magnitude M is given in events/1000km^2/yr by

$$\lambda(M) = a_4[10^{-b(M-4)} - 10^{-b(M_{\max}-4)}]. \tag{9}$$

The position of the Kerepehi fault source and the seismicity regions and their parameters are taken from a lifelines study (McVerry et al.1997; Berryman et al. 1995).

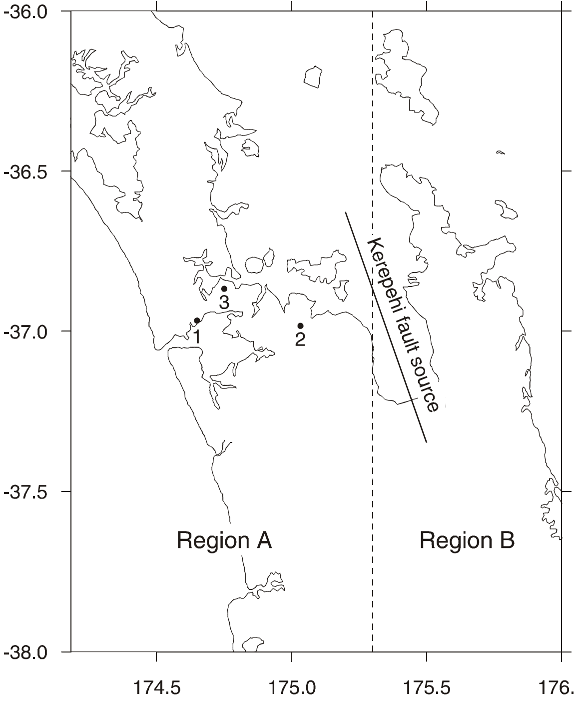

Figure 2. Map showing locations of three sites (1-3), the Kerepehi fault source and the seismicity regions A and B.

The three sites could represent, for example, three critical nodes of a lifelines network. We seek to evaluate the joint probability of earthquake shaking exceeding a given peak horizontal ground acceleration (PGA) at any one of the sites or simultaneously at all three sites over a 1000 year period using the method of Monte Carlo simulations. This involves generating many random catalogues composed of both characteristic earthquakes on the Kerepehi fault (M6.9, return period 5000 yr) and seismicity distributed in regions A and B (parameters in Table 1).

Table 1. Seismicity parameters for Regions A and B

Region	a_4	b	$M_{\max}$
A	0.01	0.89	7.0
B	0.103	1.27	7.0

Here we adopt the simple attenuation model estimated by Rhoades (1997), from the Joyner and Boore (1981) data. The model is a special case of equation (2), given by

$$\log_{10}(A_i R_i) = \alpha + \beta M + \gamma R_i + \delta + \varepsilon_i \tag{10}$$

where M is earthquake magnitude, A_i is the peak horizontal acceleration at site i, and

$$R_i = (D_i^2 + h^2)^{1/2} \tag{11}$$

where D_i is the horizontal distance in kilometres from the source to the site and h is a depth parameter common to all earthquakes. The parameter estimates and standard errors given by Rhoades (1997) for this model are listed in Table 2. Note that σ_ε is about three times larger than σ_δ, showing that most of the variability occurs in the within-earthquake term in this case.

Table 2. Parameters of attenuation model (after Rhoades, 1997).

Parameter	Estimate	Standard error
α	-1.24	0.25
β	0.28	0.04
γ	-0.0022	0.0004
h	6.57	0.55
σ_δ	0.08	0.02
σ_ε	0.23	0.01

A different set of regression parameters, generated from the distribution of parameter uncertainties, was applied to each of 200 simulated 1000-year catalogues of earthquakes with $M \geq 5.0$. For each earthquake a value of δ was simulated and for each site and earthquake a value of ε_i. From each earthquake the acceleration at each of the three sites was noted and also the maximum and minimum acceleration across the two sites. The probability of any given level of shaking being exceeded was estimated from the resulting statistics. The results are shown in Figure 3, in which the probability curves plotted are logistic local regression fits to the simulation estimates.

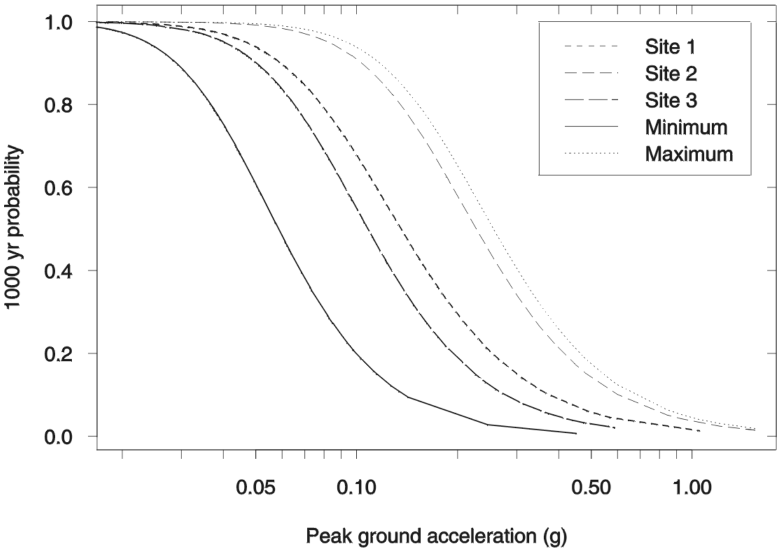

Figure 3. 1000 year probabilities of exceedance for PGA at the three Auckland sites (Fig. 2) and for the maximum and minimum PGA across the three sites.

The probability that a given threshold of acceleration is exceeded simultaneously at all three sites is the probability that the minimum of the accelerations at the three sites exceeds the threshold. Similarly, the probability that a given threshold is exceeded at at least one site is the probability that the maximum of the three sites exceeds the threshold.

3.3 *Comparison with scenario approach for Auckland*

In the scenario approach, one seeks to identify a single earthquake source and return period that epitomize the earthquake risk across a region. This is easy to do only when the earthquake risk is dominated by a single large source. In the Auckland example, it is difficult to do because the distributed seismicity, although low compared with other New Zealand areas, dominates the risk contributed by the Kerepehi fault source. McVerry et al. (1997) developed four different scenarios producing PGAs up to 0.45g for parts of the Auckland urban area. One particular scenario, of an earthquake centred offshore east of the Waitemata Harbour entrance, was chosen because it produced shaking intensities within the main urban area close to those estimated from a uniform hazard model.

There are clear disadvantages associated with the scenario approach when compared to the present method:

- It may not be possible to find a single scenario that adequately represents the hazard.
- The scenario PGAs cannot be relied on for all locations in the region.
- There is no formal accounting for uncertainties, whether epistemic or aleatory.
- The scenario is only designed around a single return period.

3.4 *Effect of epistemic error*

The epistemic (parameter) uncertainties contribute to the joint hazard estimates in the example above, but their effect on the hazard curves is not evident in Figure 3. To clarify the important role that parameter uncertainties play, the calculations of the previous section were repeated, with the attenuation parameters fixed at their estimated values for all simulated catalogues. The effect on the joint hazard estimates is shown in Figure 4.

The parameter uncertainties are seen to have quite a large effect on the results. Ignoring parameter uncertainties would lead to underestimation of the joint probabilities for any PGA greater than about 0.05g., as shown by a comparison of the “Fixed, Minimum” and “Uncertain, Minimum” curves. At 0.1g, these curves differ by about a factor of 4. Similarly at 0.5g, the “Fixed, Maximum” and “Uncertain, Maximum” curves differ by about a factor of 4, so that ignoring parameter uncertainties would also result in underestimation of the probability of a given PGA being exceeded at at least one of the three sites for high values of PGA.

It can thus be seen from this example that it is in general unwise to ignore parameter uncertainties.

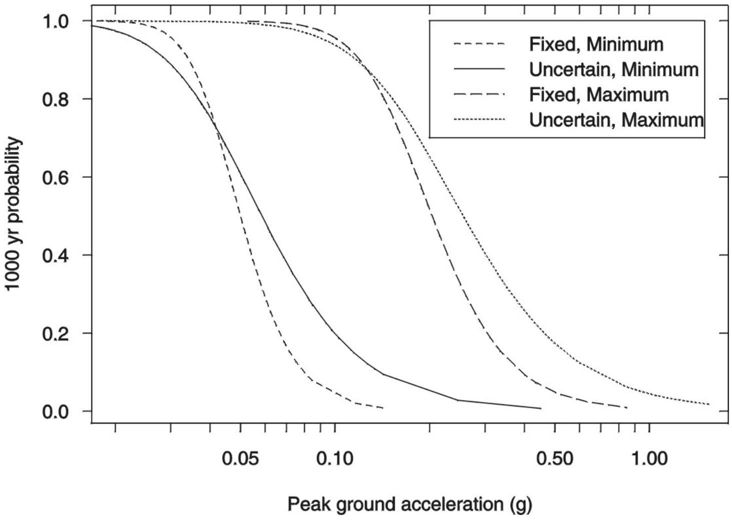

Figure 4. Comparison of 1000 yr probabilities of maximum and minimum PGA across the three Auckland sites (Fig. 2) assuming fixed and uncertain attenuation parameters.

4 CONCLUSION

Estimating the joint hazard of given thresholds of strong shaking being exceeded at two or more sites in the same earthquake is useful where the continuation of a system, activity or lifeline depends on at least one of several critical facilities at different sites remaining operational. The examples presented here are simple, but the method can be applied with no greater difficulty to situations of much greater complexity, e.g. complex networks involving any number of sites, with variable thresholds of shaking at different sites, and attenuation models that account for more site and source effects.

The joint hazard method has distinct advantages over scenario approach commonly applied in lifelines studies. Unlike the scenario approach, it can adequately represent the hazard at all locations in the region, and for all return periods, and can formally account for uncertainties.

Both the epistemic and aleatory components of uncertainty in the attenuation model affect the outcome of a joint hazard analysis. Ignoring the epistemic uncertainty can result in underestimation of joint (and individual) hazards. The partitioning of the aleatory uncertainty into within-earthquake and between-earthquake components also has important effects.

Further current developments of this work are extensions to the theory for joint probabilities including equivalent scenarios for multiple events to allow joint probabilities to be mapped in a manner analogous to uniform-hazard maps for single sites. Code to estimate joint probabilities for pairs of sites has been incorporated recently in the NZNSHM software

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6 RETURN TO INDEX