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PREDICTION OF FUNDAMENTAL PERIOD OF REGULAR FRAME BUILDINGS

Pavan K. Aninthaneni[1] and Rajesh P. Dhakal[2]

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ABSTRACT

The most important structural parameter in the estimation of the seismic demand on a building is the natural period of the building's fundamental/first mode of vibration. There are several existing empirical, analytical, and experimental methods which can be used to estimate the fundamental period of a building. The empirical equations prescribed in the building codes are simple, but they do not consider actual building properties, and are very approximate. On the other hand, analytical methods like Eigenvalue analysis and Rayleigh method are able to consider most of the structural parameters that are known to affect the period of a building. Nevertheless, the analytical methods require considerable effort and expertise; often requiring structural analysis software's to estimate the fundamental period of a building.

In this paper, a generic method is developed to estimate the fundamental period of regular frame buildings and a simple yet reliable equation is proposed. The equation is derived using the basic concept of MacLeod's method for estimation of roof/top deflection of a frame building, which is modified to more accurately predict the lateral stiffness of moment resisting frames under triangular lateral force distribution typically used in seismic design and analysis of frame buildings. To verify the reliability and versatility of the developed equation, the fundamental periods predicted are compared with the periods obtained from Eigenvalue analysis for a large number of low to medium rise RC frame buildings. The fundamental period predicted using the proposed equation is also verified using the period obtained using the Rayleigh method and measured in experimental tests. Since the proposed equation was found to closely predict the fundamental period, the results are used to study the limitations of the empirical equations prescribed in building codes. The applicability of the proposed equation to predict the fundamental period of low to medium rise frame buildings with minor irregularity is also investigated, and it was found that the proposed equation can be used for slightly irregular frame buildings without inducing any additional error. The proposed equation is simple enough to be implemented into building design codes and can be readily used by practicing engineers in design of new buildings as well as assessment of existing buildings.

INTRODUCTION

The most important parameter in the seismic design of a building is its period of fundamental mode of vibration, which controls the seismic demand on the building and subsequently its structural element sizes. The fundamental period of a building depends on the lateral stiffness and seismic mass and it cannot be precisely calculated for a building yet to be designed. In reality, it is very difficult to predict the actual period of vibration of a building under real earthquake shaking because of many uncertain parameters (i.e. actual material properties, seismic mass of a building during earthquakes, soil condition, contribution of secondary elements to the lateral stiffness of a building, etc.). Therefore, it is common practice to use approximate empirical, analytical and experimental methods to estimate the fundamental period for the design of a new building as well as assessment of an existing building.

The empirical equations prescribed in most of the building codes to predict the fundamental period are developed using an actual database of recorded periods of a real building in earthquakes [1, 2]. Most of the empirical equations relate the fundamental period of a building to the height of the building *H*; usually in the form $T_a = C_t H^x$[3]. In the Japanese building code [4], the fundamental period is a linear function of the building height which is given as $T_a = (0.02 + 0.01\varphi)H$ [5], where the value of φ for concrete and steel buildings are 0 and 1 respectively. The full compilation of existing empirical equations and seismic demand prediction using equivalent static procedure in different building codes can be found in literature [6-9]. The empirical equations prescribed in the building codes are simple, but they are very approximate. In most cases, they underestimate the fundamental period, which is conservative (hence acceptable) in design of a new building, but are unsuitable for assessment of an existing building as they under estimate seismic displacement [10].

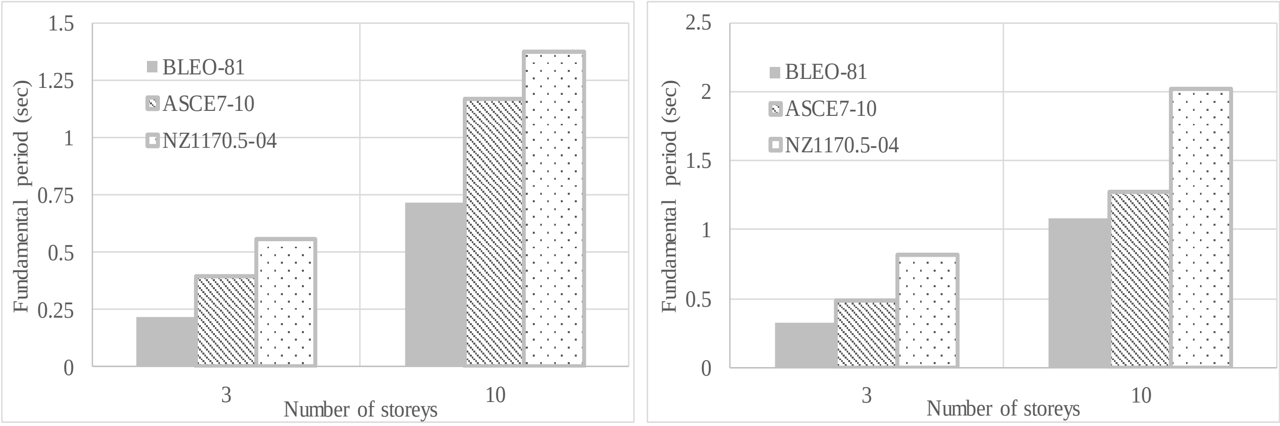
Building codes in different countries prescribe different empirical equations to estimate the fundamental period at ultimate and serviceability limit states. The values of the coefficients to be used in those empirical equations (i.e.$T_a = C_t H^x$) in the different building codes are shown in Table 1. Obviously, the different values of the coefficients result in different fundamental periods, and hence different seismic demands for the same building. It is important to note that some building codes (e.g. New Zealand) prescribe the periods at different limit states, whereas most of the other building codes define the period at only one limit state, and then the seismic demand for other limit state is scaled up or down [11]. The fundamental period of 3 and 10 storey RC and steel frame buildings calculated using empirical equations given in the Japanese, American and New Zealand building seismic design standards [4, 12, 13] are shown in Figure 1. It is clear that there is a lot of variation in the estimated fundamental periods from one building code to another.

[1] *PhD Student, University of Canterbury, Christchurch, New Zealand, pavan.aninthaneni@pg.canterbury.ac.nz (Member)*
[2] *Professor, Corresponding Author, University of Canterbury, Christchurch, New Zealand, rajesh.dhakal@canterbury.ac.nz (Member)*

Table 1: Values to be used in the empirical equation $T_a = C_t H^x$ in different building codes.

Country	Limit state	Concrete buildings		Steel buildings		Building code
		C_t	x	C_t	x	
United states/Chile	ULS	0.0466	0.9	0.0724	0.8	ASCE7-10 & NCh433-11
New Zealand	ULS & (SLS=ULS/1.25)	0.0937	0.75	0.1375	0.75	NZS1170.5-04
Europe/India	SLS	0.075	0.75	0.085	0.75	EN1998-1-04 & IS1893-02
Japan*	ULS	0.02	1	0.03	1	BLEO-1981

** Though the empirical equation in Japanese code is in a different form, the coefficient values given in the table are derived after converting the Japanese expression into the form $T_a = C_t H^x$, thereby resulting in the same period.*



(a) RC moment resisting frame building ***(b) Steel moment resisting frame building***

Figure 1: Fundamental period calculated using empirical equations in different standards.

The building codes allow to use any method of analysis (i.e. Eigenvalue, Rayleigh method, etc.) to predict the fundamental period, but some codes limit the analytical predicted period to 20% to 25% more than the period calculated using empirical equations prescribed in the codes [12, 13]. This upper bound limit is intended to avoid underestimating design base shear force due to unreasonable assumptions in analytical modelling. Some building codes specify the upper bound limit on the predicted period obtained from analytical methods only for capacity design (i.e. sizing the structural members), but not for calculating lateral displacements at serviceability limit state [12]. It is also important to note that some building codes do not impose any limit on period calculated from analytical methods in strength and displacement checks [11]. Also, building codes differ from each other in the calculation/use of the gross or effective section properties (i.e. moment of inertia) in the estimation of the fundamental period using analytical methods [14, 15].

There are some hand calculation equations available for estimation of the fundamental period of frame buildings [16, 17]. In these methods, a building frame is idealized as an equivalent cantilever column with separate flexural and shear modes of deformation. The periods of shear and flexural modes of vibration are then combined using Dunkerley's combination rule to obtain the period of the frame [17].

The existing analytical methods like Eigenvalue analysis and Rayleigh method take into account all parameters that affect the period of a building. These methods are able to predict the fundamental period of frame buildings with reasonable accuracy, but require considerable effort and expertise in using structural analysis software. In most cases, the periods predicted using analytical methods (e.g. Eigenvalue analysis and the proposed equation) using effective section properties will be higher than those predicted using empirical equations, which will be proved in later sections. There is no simple and reliable analytical equation which accounts for all structural parameters that are likely to affect the period and can be used both in design of new building as well as in the assessment of an existing building. The main objective of the paper is to theoretically derive a generic expression to predict the fundamental period of regular frame buildings with constant storey height and uniform mass along the building height which takes into account most structural parameters that are known to affect the period. The other objective of the paper is to modify and improve the accuracy of the original Macleod's equation in predicting the lateral deflection and the lateral stiffness of a moment resisting frame building under lateral seismic forces. Although the proposed equation can be used for any type of frame buildings, only reinforced concrete (RC) frame buildings are analysed in this paper for validation. The equation developed in this paper to predict the fundamental period of frame buildings, which has been simplified for design engineers' use in day to day projects, can be used for the following purposes:

1. To quickly (and much more accurately than the available empirical formulae) estimate the seismic demand in design of regular frame buildings.
2. To predict seismic demand and to estimate the seismic displacement for assessment of an existing frame building.
3. To verify stiffness and mass properties of finite element models by comparing the time periods obtained from finite element analysis and the proposed equation.
4. Handy tool for practicing structural engineers to understand change in seismic demand with change in cross sectional properties.

Firstly, the basis for the proposed equation and brief details of the original Macleod's model to estimate roof/top deflection of a frame building is discussed. Thereafter, the lateral stiffness of frames are obtained with modified Macleod's equation are compared with initial lateral stiffness obtained from pushover analysis performed using the structural analysis software SAP 2000 [18]. The error in predicting the lateral stiffness with original and modified Macleod's equation is compared. Later, the fundamental periods predicted by the proposed equation are compared with those obtained from Eigenvalue analysis. The accuracy and reliability of the proposed equations are thoroughly verified by comparing predicted results with those obtained from computer analysis for a wide range of low-medium rise RC frame buildings covering a wide range of bay widths, seismic mass and beam to column relative stiffness. Also, the fundamental period predicted using the proposed equation is compared with the period obtained using Rayleigh method and measured from experimental tests [19, 20]. The paper also investigates the reliability of the empirical equations in predicting fundamental period of frame buildings for design or assessment purpose. The results obtained using the proposed equation and Eigenvalue analysis are used to scrutinize the limitations of the empirical equations; for example their inability to capture the effects of geometric configuration, seismic mass, and effective section properties. Finally, the ability of the proposed equation to predict fundamental period of slightly irregular buildings with varying bay lengths, storey heights and storey seismic mass is also investigated.

ANALYTICAL MODEL

The proposed analytical model/equation to predict the fundamental period is developed by modifying and extending original Macleod's method [21] which was originally developed to estimate top deflection of a frame building. The basic philosophy of the Macleod's method is to condense the multiple bays into an equivalent single bay frame. Because of symmetry, only a half of the frame is considered. Firstly, the lateral deflection of the beam-column subassembly in a single storey of the half bay frame is calculated using any standard analysis methods. To estimate the top deflection of a frame, thus calculated inter-storey deflection is then integrated over the total building height assuming linear variation of deflection over storey height. Key features of the original Macleod's method are discussed again in the paper; and interested readers can obtain its full details from the literature [21].

It is important to note that the original Macleod's model is developed to predict top deflection only, not to predict the fundamental period of a frame building. Two main assumptions in the original method are: (i) cross sections are un-cracked; and (ii) the point of contra-flexure is at the mid-height of a storey. Because of these assumptions, the original model cannot be directly used in estimating the lateral deflection and the lateral stiffness of a RC frame building in seismic regions. For these reasons the original model is modified and extended herein by incorporating factors to account for the above mentioned issues.

The modified model is generic and can predict the top deflection, the lateral stiffness under different lateral load profiles; such as a point load at the top of a frame, a uniformly distributed load over the whole building height to represent wind load or a triangular load which can be used to represent the lateral seismic force profile in frame buildings at different limit states. The modified expression for lateral deflection is further extended to predict the lateral stiffness and the fundamental period of frame buildings. In Figure 2, I_b and I_c represent the gross moment of inertia of the beams and columns respectively. The figure also shows the process of condensing a multi-bay frame into an equivalent single bay frame by taking into account the effective sectional properties of beams and columns through factors α_b and α_c, respectively. Note that the factors α_b and α_c are calculated as the ratio of gross to effective moments of inertia at corresponding limit states, hence the values of the factors are always greater than or equal to 1. These values are generally prescribed in the corresponding building codes or can be calculated using Table 2 which are taken from the New Zealand concrete structures standard [15].

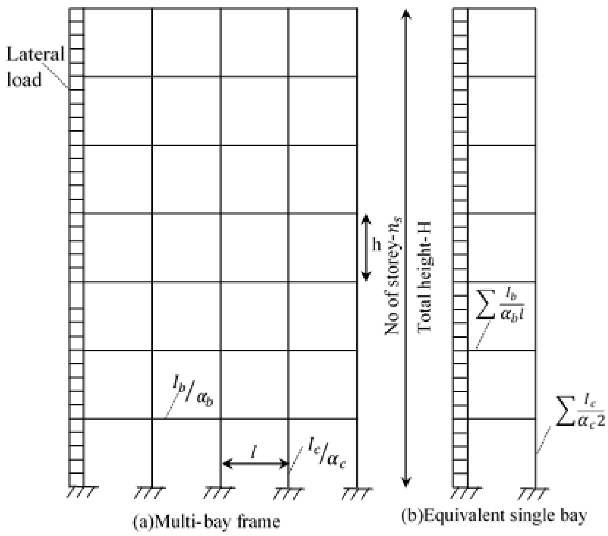


Figure 2: Condensation of a multi bay frame into an equivalent single bay frame.

Figure 3 shows the overall modified Macleod's model. Idealization of a single bay frame into a half bent is shown in Figure 3a; and the factors α and β in Figure 3b define the location of point of contra-flexure in the columns. It is assumed that the point of contra-flexure in beams is at mid length, which is a reasonable assumption. The linear variation of the sectional properties with height is kept the same as in the original Macleod's model, which is shown in Figure 3c. Most of the notations and symbols are kept the same as in the original Macleod's model, so that the readers can easily identify the modifications and understand the extension of the original model from the estimation of top deflection to the prediction of the fundamental period.

Table 2: Effective moment of inertia of RC sections at different limit states [15].

Type of member	Ultimate limit state		Serviceability limit state		
	f_y = 300 MPa	f_y = 500 MPa	μ = 1.25	μ = 3	μ = 6
1 Beams					
(a) Rectangular ¶	0.40 I_g (use with E_{40}) §	0.32 I_g (use with E_{40}) §	I_g	0.7 I_g	0.40 I_g (use with E_{40})§
(b) T and L beams ¶	0.35 I_g (use with E_{40}) §	0.27 I_g (use with E_{40}) §	I_g	0.6 I_g	0.35 I_g (use with E_{40})§
2 Columns					
(a) $N^*/A_g f'_c > 0.5$	0.80 I_g (1.0 I_g) ‡	0.80 I_g (1.0 I_g) ‡	I_g	1.0 I_g	As for the ultimate limit state values in brackets
(b) $N^*/A_g f'_c = 0.2$	0.55 I_g (0.66 I_g) ‡	0.50 I_g (0.66 I_g) ‡	I_g	0.8 I_g	
(c) $N^*/A_g f'_c = 0.0$	0.40 I_g (0.45 I_g) ‡	0.30 I_g (0.35 I_g) ‡	I_g	0.7 I_g	

NOTES –
(§) With these values the *E* value should be the elastic modulus for concrete with a strength of 40 MPa regardless of the actual concrete strength.
(‡) The values in brackets apply to columns which have a high level of protection against plastic hinge formation in the ultimate limit state.
(¶) For additional flexibility, within joint zones and for conventionally reinforced coupling beams refer to the text.

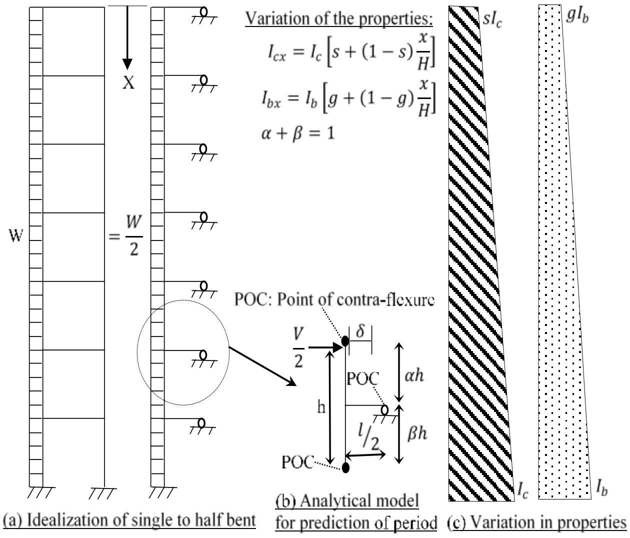


Figure 3: Modified Macleod's model to predict the fundamental period of a frame building.

The top deflection of a single storey half bent shown in Figure 3b can be calculated using any standard analysis methods (e.g. moment area theorem); which is given by:

$$\delta = \frac{\alpha_c V h^3[\alpha^3+\beta^3]}{6E_c I_c} + \frac{\alpha_b V h^2 l}{12 E_b I_b} \tag{1}$$

For the equivalent single frame, $I_c = \sum \frac{I_c}{2}$ and $\frac{I_b}{l} = \sum \frac{I_b}{l}$.

By substituting these, Equation 1 turns into:

$$\delta = \frac{V h^3}{12 \sum E_c I_c}\left(4\alpha_c[\alpha^3+\beta^3] + \alpha_b \frac{\sum \frac{E_c I_c}{h}}{\sum \frac{E_b I_b}{l}}\right) \tag{2}$$

$$\delta = \frac{V h^3}{12 \sum E_c I_{cef}}(4\,[\alpha^3+\beta^3] + \lambda\,) \tag{3}$$

Where $\lambda = \frac{\sum \frac{E_c I_c}{h\alpha_c}}{\sum \frac{E_b I_b}{l\alpha_b}}$ is calculated as a ratio of the sum of column effective stiffness to sum of beam effective stiffness at the first storey level, n_b is number of bays, I_{cef} is the effective moment of inertia of the column, and V is the storey shear at the level of sub-assembly under consideration (as shown in Figure 3b). For the integration to be performed later, the deflection of the single storey half bent is converted to the differential form for the height dX (with the assumption of linear variation over storey height, which is reasonable for low-medium rise buildings).

$$d\delta = \frac{V h^2}{12 \sum E_c I_{cefx}}(4[\alpha^3+\beta^3] + \lambda\,)dx \tag{4}$$

Equation 4 is further modified by incorporating factors to account for variation of section properties, and finite size of the beam-column joint:

$$d\delta = \frac{V h^2}{12 \sum E_c I_{cef}}\left(\frac{4\,[\alpha^3+\beta^3](1-\beta_d)^3}{s+(1-s)\frac{x}{H}} + \frac{\lambda\,(1-\beta_c)^3}{g+(1-g)\frac{x}{H}}\right)dx \tag{5}$$

Where s and g are the ratio of the moment of inertias of column and beam at the top and bottom of the frame respectively. The factors $\beta_d = \frac{D}{h}$ (h to be measured between top of successive floors) and $\beta_c = \frac{C}{l}$ (l to be measured between centrelines of columns) are to account for the finite size of beam-column joint as mentioned before; D is beam depth and C is column depth. Note that the factor β_d should be calculated differently for low rise frames if h is measured to the centre of the beams. As frames do not have any beam at the base, for one storey frame the factor becomes $\beta_d = \frac{D}{2h}$.

The term $[\alpha^3 + \beta^3]$ in Equation 5 is replaced with a single factor $\frac{\gamma}{4}$, where the value of γ depends on the location of point of contra-flexure in the columns. The factor γ is evaluated by calculating the actual deflection of the sub-frame shown in Figure 3b by varying the parameters α and β (i.e. location of point of contra-flexure in columns) as defined. Thereafter, the calculated deflection is normalized with the deflection corresponding to point of contra-flexure at mid-height, which give the γ values shown in Table 3. Equation 5 is now integrated over the whole building height $H = n_s \times h$ for the different load profiles to predict the corresponding top deflection Δ, which is given by Equation 6. Here, the additional top deflection contribution due to axial deformation of columns is neglected; this is a fair assumption for low to medium rise buildings.

$$\Delta = \frac{V h^3 n_s}{12 \sum E_c I_{cef}}\left(\gamma F_s\,(1-\beta_d)^3 + \lambda F_g\,(1-\beta_c)^3\right) \tag{6}$$

Table 3: Values of γ with variation of location of point of contra-flexure in columns.

Location of POC in the columns	α	β	γ
At 0.5h (i.e. mid height)	0.5	0.5	1
At 0.4h	0.4	0.6	1.12
At 0.35h	0.35	0.65	1.27
At 0.3h	0.3	0.7	1.48

The factors F_s and F_g in Equation 6 depend on the type of loading profile, and variation of section properties with the height of the building, these factors can be calculated using the equations given in Table 4.

Table 4: F_s and F_g values with variation of sectional properties and different load profiles [21].

Type of loading	F_s (m=s) and F_g (m=g)	s=g=1
Point load (F_{sp}, F_{gp})	$\frac{log_e m}{(m-1)}$	1
UDL (F_{su}, F_{gu})	$\frac{1}{(1-m)} + \frac{m log_e m}{(1-m)^2}$	0.5
Triangular load (F_{st}, F_{gt})	$\frac{log_e m}{(m-1)} + \frac{(-1.5+2m-0.5m^2-log_e m)}{(m-1)^2}$	0.67

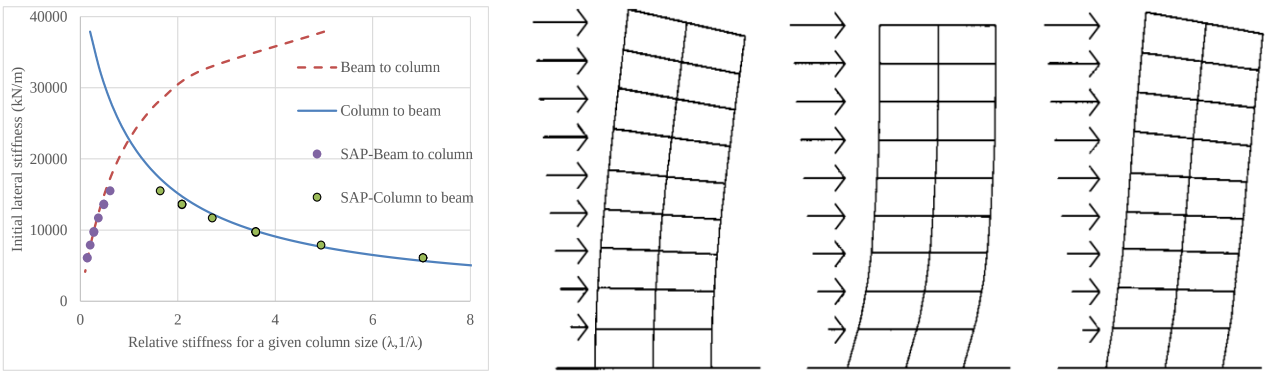
Thereafter, the lateral stiffness of the frame is calculated as the ratio of total load (i.e. base shear) and top deflection as shown below:

$$K_{l1} = \frac{12 \sum E_c I_{cef}}{h^3 n_s(\gamma F_s\,(1-\beta_d)^3 + \lambda F_g\,(1-\beta_c)^3)} \tag{7}$$

As indicated by the Equation 7, the lateral stiffness of a frame predominantly depends on the relative flexural stiffness between its beams and columns. The relative stiffness is a function of moment of inertia of the beams and the columns, span length and storey height. To understand better the factors which affect the relative stiffness, here it is assumed that the storey height and the span length are constant. Then, the relative stiffness of beam to column can be increased either for a given column size by increasing the beam size or for a given beam size by decreasing the column size. The lateral stiffness of a frame can be increased by increasing the relative stiffness of beam to column (represented by 1/λ) or by decreasing the column to beam relative stiffness (represented by λ) for a given column size and vice versa.

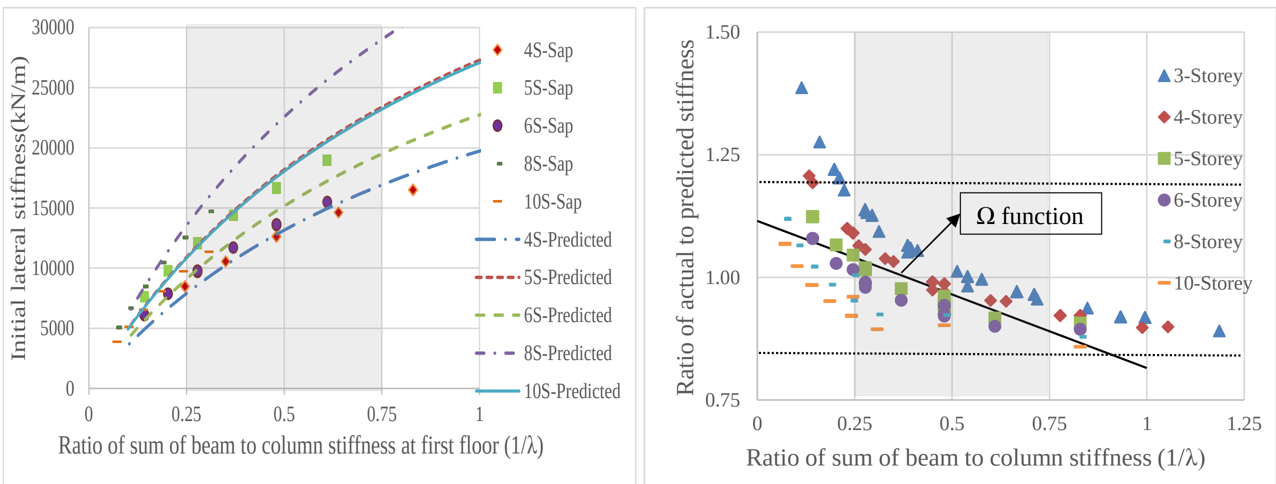
To clarify the difference between the two different ways of interpretation of the relative stiffness, Figure 4a plots the initial lateral stiffness of the frame as a function of beam-to-column and column-to-beam stiffness ratios. In this figure, for both cases the lateral stiffness is calculated using Equation 7 but plotted as a function of beam-to-column and column-to-beam stiffness ratio respectively, and "SAP" represents the lateral stiffness values obtained from pushover analysis results performed using SAP 2000. For a given column size, very small beam to column relative stiffness 1/λ means that the overall frame behaviour is close to cantilever deformation mode (i.e. flexural response with single curvature throughout the building height) without any point of contra-flexure in the columns throughout the building height, which is illustrated in Figure 4b. As the relative stiffness 1/λ approaches a very large value, the building deforms in shear mode (i.e. columns within the floors deform in double curvature with point of contra-flexure at mid-height), which is shown in Figure 4c. For practical range of relative stiffness 1/λ, the building frame deforms with a combination of flexural and shear deformation modes with no definite pattern of column curvature profiles along the building height (i.e. points of contra-flexure in the columns of different storeys are not at the same location), which is shown in Figure 4d. Generally, lower storeys of a frame deform in flexure dominated mode whereas upper storeys deform in shear dominated mode.

To find the error in prediction of the lateral stiffness with original Macleod's Equation, the lateral stiffness of a wide range of frames predicted by using Equation 7 with the assumption of point of contra-flexure at mid height of the columns (i.e. γ=1) is compared with initial lateral stiffness obtained from the pushover analysis in Figure 5. Pushover analysis of the chosen frames is performed with triangular load pattern as per New Zealand Seismic Standard [12, 13]. In Figure 5a, "*n*S-Sap" represents a *n* storey frame whose lateral stiffness is calculated using SAP 2000, and "*n*S-predicted" represent the same frame but the lateral stiffness is calculated using Equation 7. It is observed from the plot that there is a discrepancy of ±25% between the lateral stiffness predicted by Equation 7 and the initial lateral stiffness obtained from pushover analysis. This error is due to the fact that the point of contra-flexure is never at mid height in columns and not at the same location in the columns of all storeys. Moreover, Equation 7 is developed based on the assumption that the bending moment in a beam is equal to the sum of column moments at the top and bottom of the beam, which is not true for low beam-to-column relative stiffness ratio. Such frames deform in cantilever mode, which adds further to the error in the stiffness predicted using Equation 7 with γ=1. Although the ratio of beam to column stiffness is varied up to 1.5 for verification purpose, in modern frame buildings designed to capacity design principles and confirming to weak beam-strong column hierarchy the beam to column stiffness ratio ranges between 0.25 and 0.75, which is indicated in the figures as the shaded region. The horizontal lines in Figure 5b (and later in Figures 6b and 7b as well) define the upper and lower bound values used in the regression analysis to get a correction factor as described in the next paragraph.



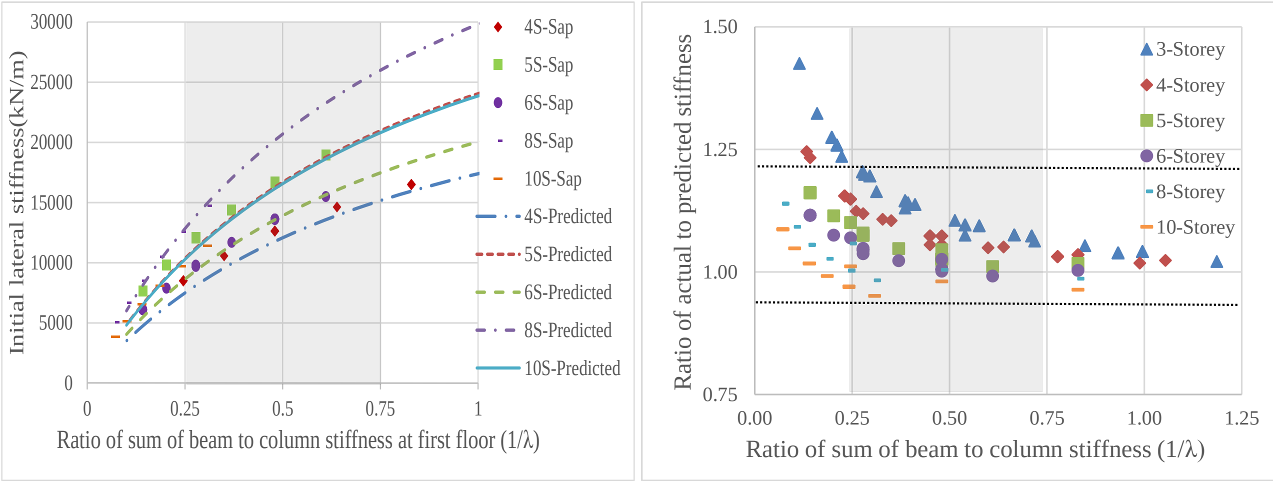
(a) Lateral stiffness vs λ, 1/λ ***(b) flexural deformation (c) shear deformation (d) flexural-shear deformation***

Figure 4: Lateral stiffness vs beam-column , column-beam relative stiffness and lateral deformation of frame buildings [22].



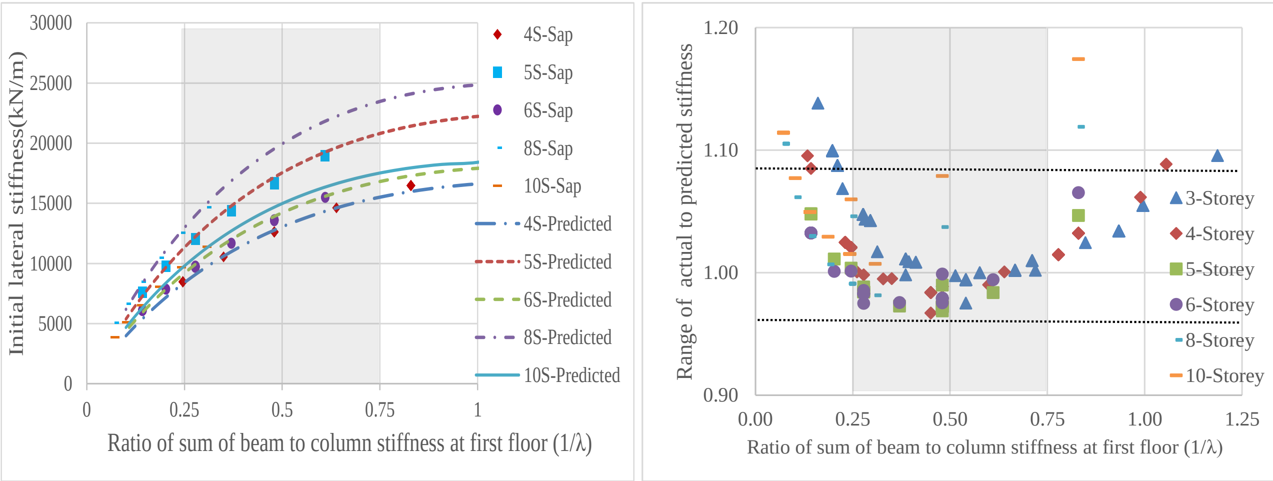
(a) Comparison of predicted and actual stiffness ***(b) Range of actual to predicted stiffness***

Figure 5: Comparison of predicted and actual lateral stiffness with γ=1.



(a) Comparison of predicted with actual stiffness ***(b) Range of actual to predicted stiffness***

Figure 6: Comparison of predicted and actual lateral stiffness with γ=1.27.



(a) Comparison of predicted with actual stiffness ***(b) Range of actual to predicted stiffness***

Figure 7: Comparison of predicted and actual lateral stiffness with use of Ω factor.

Strictly speaking, it is impossible to precisely locate the point of contra-flexure in columns in all storeys without the use of any structural analysis software. The error in the predicted lateral stiffness because of this can be corrected by a correction factor. Here, two approaches are adopted to arrive at a factor which corrects the error shown in Figure 5. In the first approach, the lateral stiffness of the frames is calculated using Equation 7 using different γ values (as listed in Table 3) and finding the value of γ such that Equation 7 results in the lateral stiffness close to the lower bound value. As can be seen in Figure 6, it is found from regression analysis that γ = 1.27 (which corresponds to the point of contra-flexure being at 0.35 times column height) predicts a lower bound lateral stiffness value for most of the cases. Any error resulting in the predicted fundamental period due to remaining discrepancy in the lateral stiffness predicted using Equation 7 with γ = 1.27 will be corrected later with another factor φ which is aimed to account also for the difference between the total seismic mass and the effective mass and between the top deflection based lateral stiffness and the effective lateral stiffness of an equivalent single degree of freedom (SDOF) system under the fundamental mode of vibration.

In the second approach, Equation 7 is used with γ = 1 and later modified with another factor Ω. As the factor Ω is aimed to correct the lateral stiffness predicted by Equation 7 with γ = 1, Equation 7 then turns into:

$$K_{l2} = \frac{12\Omega \sum E_c I_{cef}}{h^3 n_s\left(F_s\,(1-\beta_d)^3 + \lambda F_g\,(1-\beta_c)^3\right)} \qquad (8)$$

Multiple variable regression analyses are carried out to correct the error in predicted lateral stiffness shown in Figure 5. A linear relation between the correction factor Ω with the relative stiffness λ and the number of storeys n_s is developed using data shown in Figure 5b; and the resulting expression is given in Equation 9. After incorporating the correction factor Ω in Equation 8, the comparison of the predicted and actual lateral stiffness are shown in Figure 7a. It is clear from Figure 7b that the accuracy of the predicted lateral stiffness is within ±10% of actual lateral stiffness in the practical beam-to-column relative stiffness range between 0.25 and 0.75.

$$\Omega = 1.25 - \left(\frac{0.3}{\lambda}\right) - 0.027n_s \geq 0.67 \ \text{ with } R^2 = 0.86 \qquad (9)$$

A multi-degree of freedom (MDOF) building frame system has many modes of vibration with different periods. Out of many periods, the period of the first (i.e. fundamental) mode of vibration is commonly used in estimation of seismic demand. The period of the first mode of vibration is generally calculated using Eigenvalue analysis or Rayleigh method by condensing a MDOF system into an equivalent SDOF system.

The period of an equivalent SDOF system is given by Equation 10.

$$T = 2\pi\sqrt{\frac{M_{eff}}{K_{eff}}} \qquad (10)$$

The effective mass M_{eff} in the first mode of vibration depends on the mass participation factor, which in turn depends on the mode shape and mass distribution along the building height. Generally, for low-medium rise frame buildings with uniform stiffness and mass distribution, the mass participation in the fundamental mode will be between 70%-100%. The effective stiffness of an equivalent SDOF system depends on the mode shape and the effective height of the equivalent SDOF system. The lateral stiffness calculated using Equation 7 or 8 is based on the top deflection of a frame, which needs to be modified at the effective height of the frame in the fundamental mode of vibration to get the effective stiffness K_{eff}. A factor φ is introduced herein to account for the effective mass and effective height (i.e. effective stiffness) in the first mode of vibration, which then turns Equation 10 into:

$$T = 2\pi\varphi\sqrt{\frac{M}{K_l}} \qquad (11)$$

In Equation 11, M is replaced with $\frac{W_s}{g}$, where W_s is the seismic weight acting on the chosen frame and g is acceleration due to gravity. Here, the fundamental period of the equivalent SDOF is calculated in two ways; one using the lateral stiffness calculated using Equation 7 and another using lateral stiffness calculated using Equation 8. With the first approach, K_l in equation 11 is replaced with K_{l1}; then the equation to predict the fundamental period can be written as Equation 12a.

$$T = 2\pi\varphi_1\sqrt{\frac{W_s\, h^3 n_s\left(F_{st}1.27\,(1-\beta_d)^3+\lambda F_{gt}\,(1-\beta_c)^3\right)}{g\;12\sum E_c I_{cef}}} \qquad (12a)$$

With the second approach, K_l is replaced with K_{l2}; then the equation to predict the period turns into equation 12b.

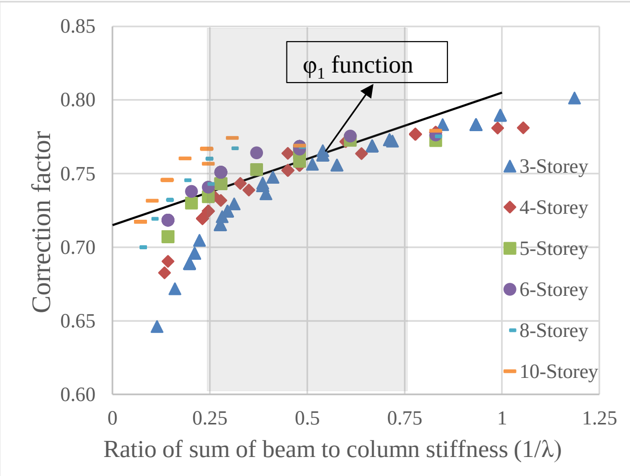
$$T = 2\pi\varphi_2\sqrt{\frac{W_s\, h^3 n_s\left(F_{st}\,(1-\beta_d)^3+\lambda F_{gt}\,(1-\beta_c)^3\right)}{g12\Omega\sum E_c I_{cef}}} \qquad (12b)$$

To arrive at the values of factors φ_1 and φ_2, the fundamental periods computed using Equation 12a with $\varphi_1 = 1$ and using Equation 12b with $\varphi_2 = 1$ are compared with the periods obtained from Eigenvalue analysis using SAP 2000. The correction factors required to multiply the predicted periods to obtain the actual periods for a wide range of frames are plotted in Figure 8. Linear relations between the correction factor φ_1 or φ_2 , the relative stiffness λ and the number of storeys n_s are developed to minimize the error in the predicted fundamental period. The correction factors are given as $\varphi_1 = 0.67 + 0.10/\lambda + 0.005n_s$ with $R^2 = 0.82$ and $\varphi_2 = 0.79 + \frac{0.01}{\lambda} - 0.005n_s$ with $R^2 = 0.63$. The overall variation of φ_1 and φ_2 is between 0.65 and 0.8, and 0.70 and 0.8 respectively. The variation of factor φ_2 with relative stiffness is negligible and can be further simplified to $\varphi_2 = 0.79 - 0.005n_s$. Though overall variation of φ_2 is very less, the linear fit is not good enough. Note that the above expressions have been derived using the F_{st}, F_{gt} values corresponding to a triangular load pattern, if any other load pattern is used to compute the lateral stiffness then these factors have to be recalibrated.

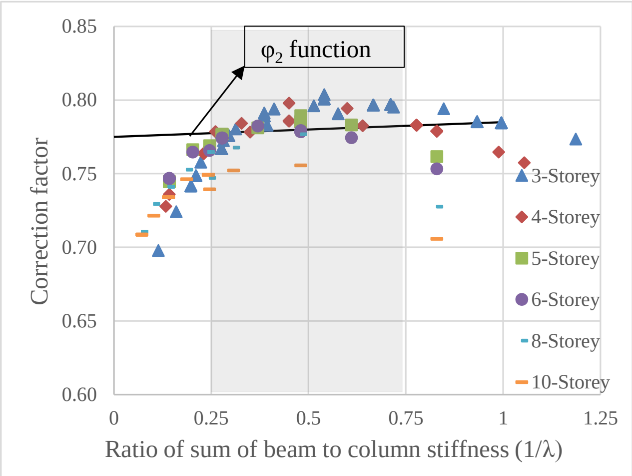
It is realized that both Ω and φ_2 in Equation 12b are functions of the relative stiffness λ and the number of storeys n_s, so a new period correction factor φ_3 $\left(\text{i.e. } \varphi_3(\lambda, n_s) = \frac{\varphi_2(\lambda,n_s)}{\sqrt{\Omega(\lambda,n_s)}}\right)$ is developed by calibrating against actual periods with $\Omega = 1$, then Equation 12b turns into:

$$T = 2\pi\varphi_3\sqrt{\frac{W_s\, h^3 n_s\left(F_{st}\,(1-\beta_d)^3+\lambda F_{gt}\,(1-\beta_c)^3\right)}{g12\sum E_c I_{cef}}} \qquad (12c)$$

From regression analysis, φ_3 is related to λ and n_s as: $\varphi_3 = 0.66 + 0.19/\lambda + 0.008n_s \leq 1.0$ with $R^2 = 0.86$, which is shown in Figure 9a. Although not apparent from the data plotted in the figure, the maximum limit of φ_3 is decided as the ratio of the maximum value of φ_2 (about 0.8 as apparent in Figure 8b) and the square root of the minimum value of Ω specified in Equation 9 (i.e.$\sqrt{0.67} \approx 0.8$). Note that the limiting maximum value is irrelevant in most normal frames and will only come into effect for frames with unusually high values of 1/λ (i.e. frames with very stiff beams and flexible columns). The ratio of actual to predicted periods for varying number of storey's is shown in Figure 9b, and it is clear that the predicted periods are within ±10% of the actual periods. The comparison of the fundamental periods calculated using Equations 12a, 12b and 12c for a four and ten storey frame with the parameters listed in Tables 5 and 6 are plotted as a function of relative stiffness in Figure 10. It can be observed that there is no major difference between the periods predicted by different equations.

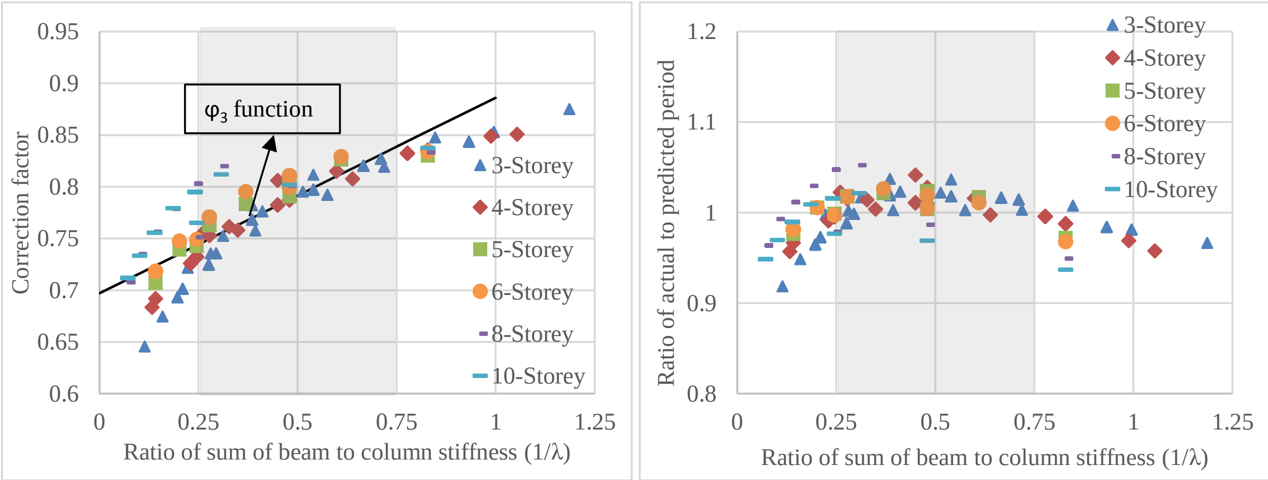


(a) Correction factor for $\varphi_1 = 1$



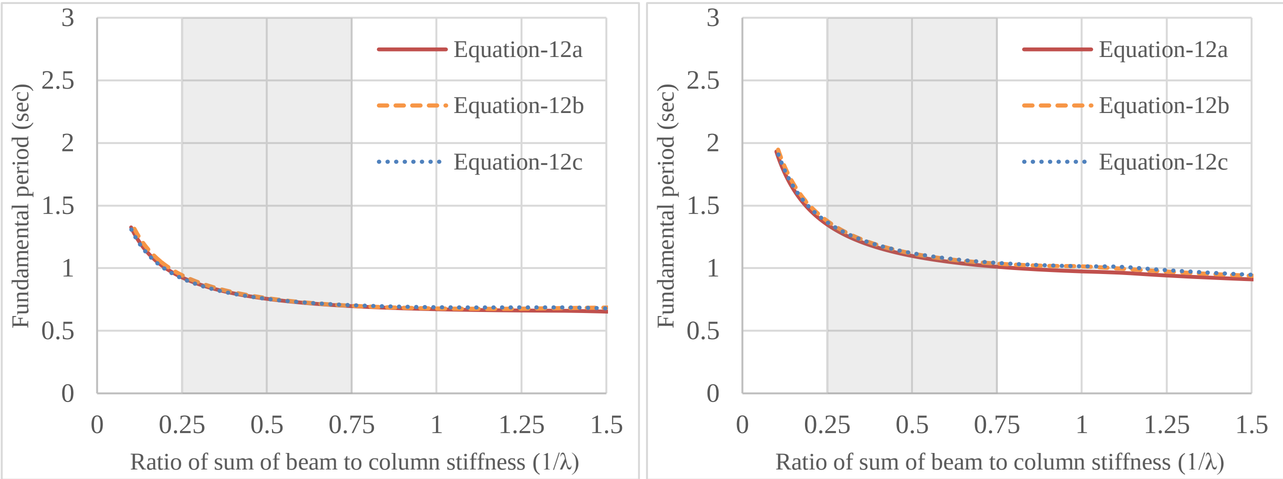
(b) Correction factor for $\varphi_2 = 1$

Figure 8: Correction factors to multiply the predicted period with $\varphi_1 = 1$, $\varphi_2 = 1$.



(a) φ_3 as function of λ and n_s ***(b) Range of actual to predicted period using Equation 12c***

Figure 9: Correction factor φ_3 and ratio of actual to predicted period using Equation 12c.



(a) 4 Storey frame ***(b) 10 Storey frame***

Figure 10: Comparison period plots for a 4 and 10 storey frame using Equation 12a, 12b and 12c.

As Equation 12c needs only one correction factor to be calculated and has high R^2 value compared to other equations, it is proposed for practical applications. The generic Equation 12c can be further simplified for a regular frame with sectional properties don't change with the height of the building (i.e. $F_{st} = F_{gt} = 0.67$ as shown in Table 4 for triangular lateral force distribution). The simplified expression is presented in Equation 13a, where n_b is the number of bays.

$$T = 0.47\varphi_3\sqrt{\frac{W_s\, h^3 n_s\left((1-\beta_d)^3+\lambda\,(1-\beta_c)^3\right)}{(n_b+1)E_c I_{cef}}} \qquad (13a)$$

When clear spans of beam length and column height are used (rather than centre to centre dimension of the frame), the effect of finite size of beam-column joint on the fundamental period can be neglected (i.e. $\beta_d = \beta_c = 0$). In such cases, Equation 13a turns into Equation 13b.

$$T = 0.47\varphi_3\sqrt{\frac{W_s\, h^3 n_s(1+\lambda)}{(n_b+1)E_c I_{cef}}} \qquad (13b)$$

RESULTS AND DISCUSSION

As mentioned earlier, to verify and validate the developed analytical equation to estimate the fundamental period of frame buildings, Eigenvalue analysis is performed on a wide range of low-medium rise RC frame buildings using SAP 2000. The varied parameters and their ranges used in the verification are identified in Table 5. For calculation of the seismic weight on a chosen frame, it is assumed that the building has 4 bays of 6m width in the perpendicular direction unless otherwise specifically mentioned. The tributary seismic weights on the chosen frame is calculated on the assumption that the floor is rigid in plane (i.e. rigid diaphragm) and are distributed proportional to the stiffness of the frames (here it is assumed all frames in a building have equal stiffness). The geometrical dimensions and the seismic weight of the frames used in the parametric investigation are shown in Table 6.

Note that the fundamental periods are calculated based on gross section properties and by neglecting the effect of finite beam-column joint (i.e. $\beta_d = \beta_c = 0$) unless otherwise specifically mentioned. To be consistent with this simplification, the lengths of the beams and columns in the SAP2000 model of the frame used for the Eigenvalue analysis are made equal to their centre to centre spans (without deducting for the joint dimension) and the rigidity provided by the beam-column joint panel is not accounted for.

The fundamental periods predicted using the proposed equation are compared with the periods obtained by using Rayleigh method and measured from experimental tests. The periods predicted using the proposed equation are also used for scrutinizing the limitations of the empirical equations in predicting the fundamental periods. The applicability of the proposed equation to predict the fundamental period of low to medium rise frame buildings with minor irregularity is also investigated.

Table 5: Variables used in a parametric study.

Building characteristics		**Material description**	
Number of storeys	3-10	Grade of concrete	35 N/mm^2
Storey height	3.6 m	Grade of rebar	500 N/mm^2
Span length	5-10 m	**Loading details**	
Number of bays	3-5	Dead load	Roof: 4.75 kN/m^2 Other:4.25 kN/m^2
Young's modulus	19641 N/mm^2	Live load	Roof: 2.5 kN/m^2 Other: 3.0 kN/m^2

Table 6: Seismic weights used in the parametric study.

Number of storeys	Number of bays	Span length (m)	Column dimensions (m×m)	Seismic weight (kN)
3	3	5	0.4x0.5	1501
3	3	5	0.4x0.6	1539
3	3	7	0.4x0.5	2020
3	3	7	0.4x0.6	2057
3	4	5	0.4x0.5	1981
3	4	7	0.4x0.5	2672
4	3	6	0.4x0.5	2358
4	3	6	0.4x0.6	2409
4	4	6	0.4x0.5	3122
4	4	6	0.4x0.6	3185
5	4	6	0.4x0.5	3908
5	4	6	0.4x0.6	4010
6	4	6	0.4x0.5	4698
6	4	6	0.4x0.6	4821
8	4	6	0.4x0.5	6188
8	4	6	0.5x0.7	6484
10	4	6	0.4x0.5	7677
10	4	6	0.5x0.7	8359

Fundamental Period: Proposed Equation vs Eigenvalue Analysis

For the verification of Equation 12c in predicting the fundamental period of a low-medium rise RC frame buildings, the period calculated using Equation 12c is compared with period calculated using Eigenvalue analysis. Initially a six storey and four bay frame building is chosen with the following properties: beam span of 6 m, column section of 0.4×0.6 m, beam section of 0.4×0.45 m, and seismic weight of 4731 kN. The predicted fundamental period using Equation 12c is 1.18 sec whereas the period obtained from Eigenvalue analysis using SAP 2000 is 1.19 sec. As mentioned before, for same frame it is possible to have different fundamental periods depending on the lateral stiffness, which in turn depends on the relative stiffness between beam and column. For a single storey frame, as the beam stiffness is increased from 0 to ∞, the lateral stiffness increases from $\frac{6E_cI_c}{h^3}$ to $\frac{24E_cI_c}{h^3}$ and the period is halved. The variation of the fundamental period of the single storey frame as a function of the relative stiffness of beam to column can be is easily developed and understood. But, for a multi-storey, multi-bay frame the variation of the fundamental period by varying the relative stiffness is not easy to predict and understand. Here, variation of the fundamental period due to variation of the relative stiffness is quantified using the proposed Equation 12c.

To verify the proposed Equation 12c extensively, the fundamental periods are plotted against the beam-to-column relative stiffness ratio for a wide range of 3 to 10 storey frames. The periods predicted by the proposed Equation 12c are compared with those obtained from Eigenvalue analysis; the comparisons are shown in Figures 10 to 12. In the legend of these figures, "C-0.4×0.5" represents the cases with column size 0.4×0.5 m, "S-5" represents the span length of 5 m, "3Bay" represents the frame of 3 bays, "predicted" represents that the periods are calculated using Equation 12c, "Eigen" indicates that the periods are calculated by performing Eigenvalue analysis using SAP 2000. It is important to note that in Figures 11 to 13, the fundamental periods predicted using Equation 12c are based on the seismic weights given in Table 6. Strictly speaking, there will be a slight change in the seismic weight due to change of beam cross-sections (i.e. with the change in relative stiffness between beams and columns), but here the average seismic weight is considered for the analysis and any minor error because of this is neglected.

It is clear from Figures 11 to 13 that as the ratio of beam to column stiffness increases for a given column size, the fundamental period of the frame decreases. This is because the lateral stiffness of the overall frame increases, which is inversely proportional to the period. For the same frame configuration and the same relative stiffness ratio of beam to column, increase in the column dimensions results in an increase in the lateral stiffness and as a consequence the natural period decreases, which can be clearly seen in Figures 11a to 11c, 12 & 13. As the span length of the frame increases for a given frame configuration for a given column size, the lateral stiffness decreases and the period consequently increases, which is evident in Figure 11c. As it can be observed in Figures 11d & 12b, the addition of extra bays of the same span length with given frame configuration does not change the period significantly, because both the lateral stiffness and seismic mass increase when an extra bay is added.

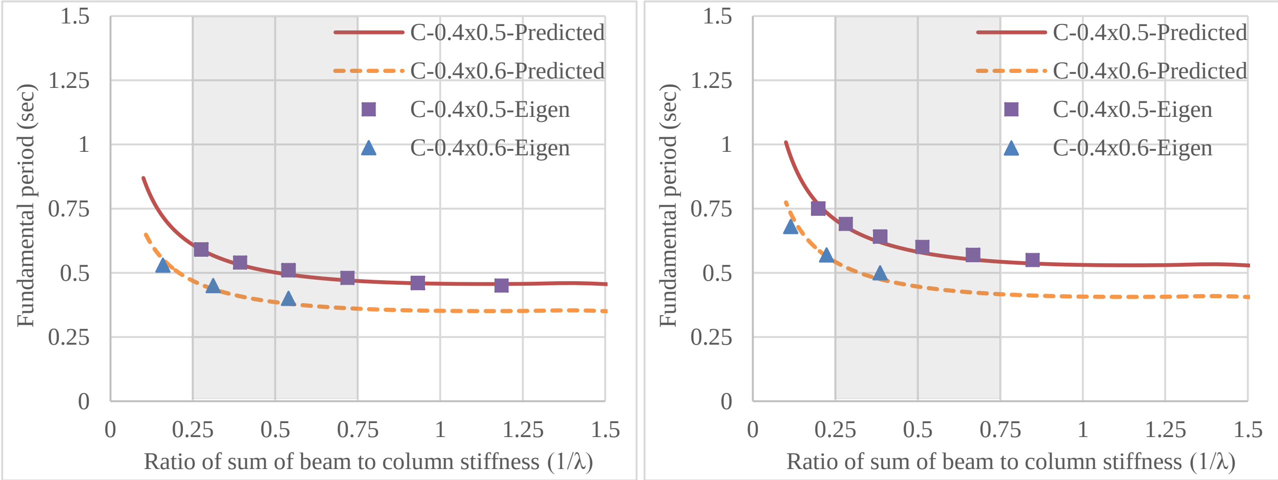
As mentioned before, the ratio of beam to column stiffness is varied up to 1.5 for verification purpose, but in modern frame buildings the beam to column stiffness ratio ranges between 0.25 and 0.75, which is indicated in the figures as the shaded region. The maximum error in the predicted fundamental periods when compared to Eigenvalue analysis results is less than 10% within the shaded zone. Nevertheless, the predicted values in general are very close to Eigenvalue analysis results; and the minor difference is acceptable considering the uncertainties and complexities involved.

Fundamental Period: Proposed Equation vs Rayleigh Method

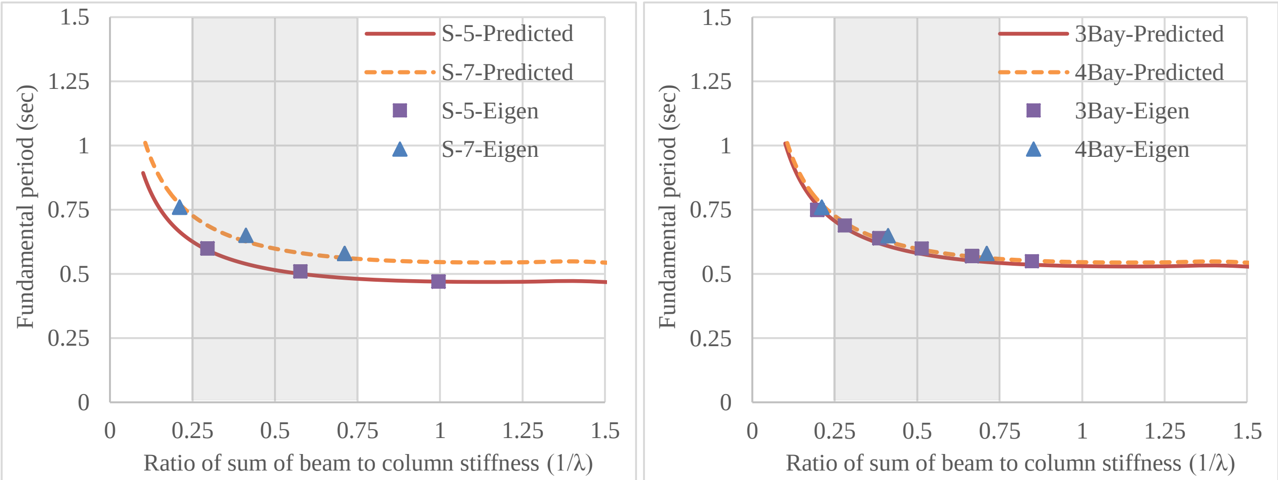
The fundamental period of a frame can be estimated using the Rayleigh equation, which is given in Equation 14, where W_i, δ_i and F_i represent the seismic weight, lateral defection and lateral force at the i^{th} storey level of a frame building.

$$T = 2\pi\sqrt{\frac{\sum_{i=1}^{n} W_i{\delta_i}^2}{g\sum_{i=1}^{n} F_i\delta_i}} \qquad (14)$$

To compute fundamental period with the use of Rayleigh equation, the lateral defections at different storey levels under the triangular load profile need to be computed, for which a structural analysis software is required. If the deflections at different storey levels are computed assuming the building as shear frame building (i.e. storey stiffness= $\sum\frac{12E_cI_c}{h^3}$), the deflections are under predicted (i.e. lateral stiffness is over estimated) and consequently the period estimated will be less than the actual period.

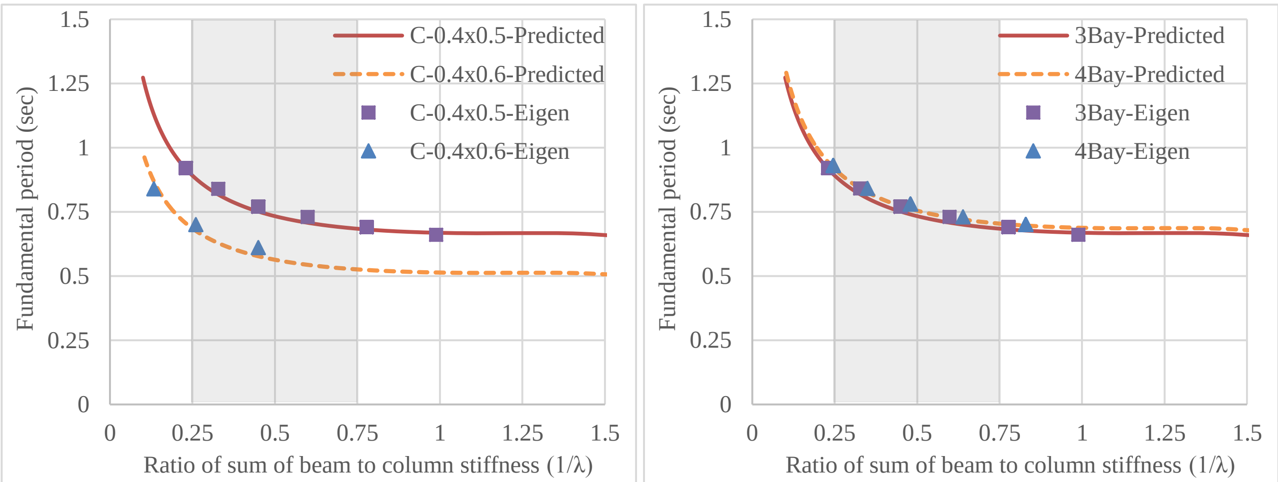


(a) 3 bays of 5m span length ***(b) 3 bays of 7m span length***



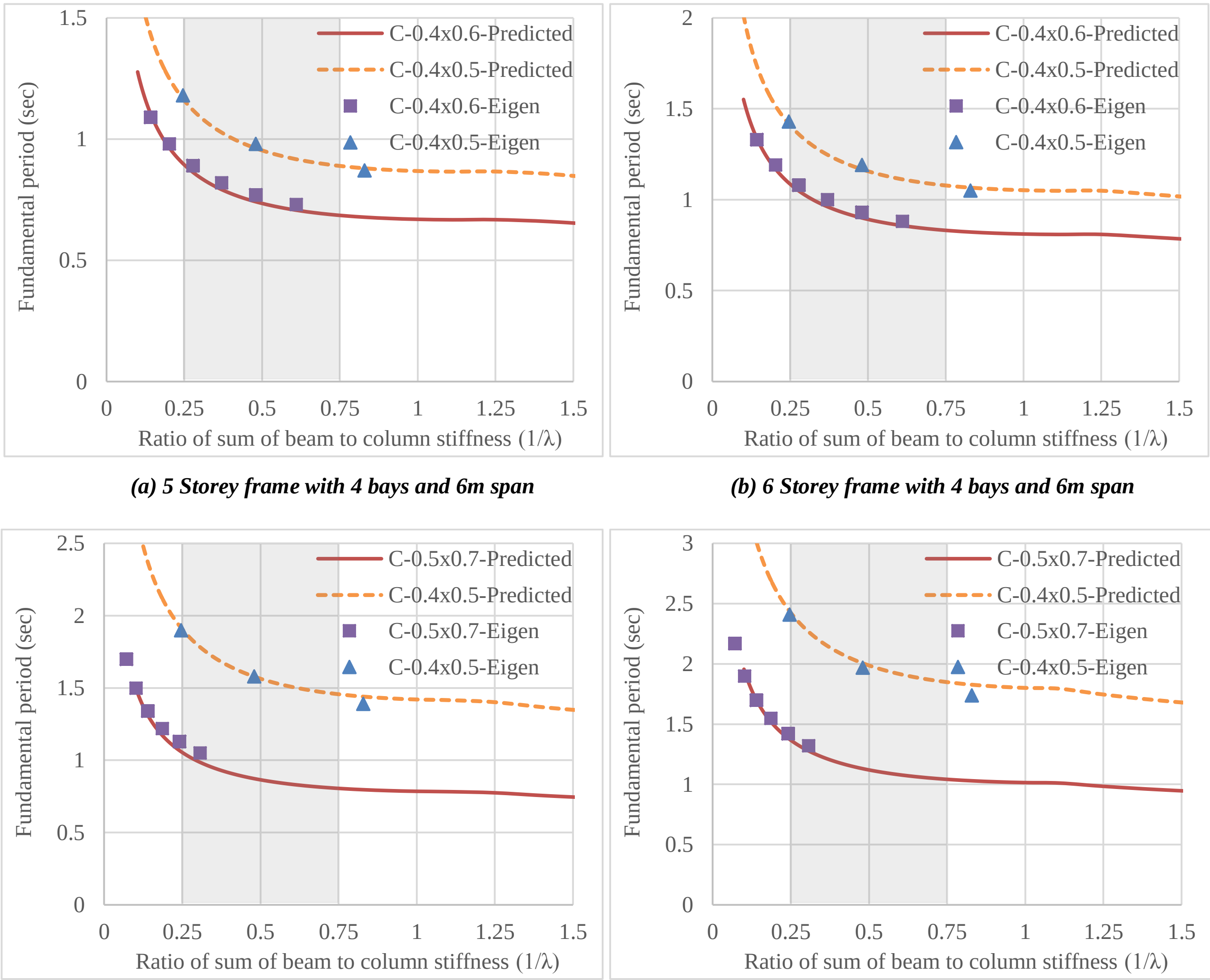
(c) 3 bays of 5&7m span length ***(d) 3 &4bays of 7m span length***

Figure 11: Comparison of periods using proposed equation and Eigenvalue analysis for a 3 storey frame.



(a) 3 bays of 6m span length ***(b)) 3&4 bays of 6m span length***

Figure 12: Comparison of periods using proposed equation and Eigenvalue analysis for a 4 storey frame.



(a) 5 Storey frame with 4 bays and 6m span

(b) 6 Storey frame with 4 bays and 6m span

(c) 8 Storey frame with 4 bays and 6m span

(d) 10 Storey frame with 4 bays and 6m span

Figure 13: Comparison of periods using proposed equation and Eigenvalue analysis for a 5 to 10 storey frames.

For blind prediction of the fundamental period using the developed equations, a five bay seven storey frame (which is not from the original sample of frame buildings used earlier for calibration) is chosen with the following properties: storey height of 3 m, beam span of 6 m, column section of 0.6×0.6 m, beam section of 0.4×0.6 m, and seismic weight of 150 kN at each floor level. The lateral deflections at different storeys of the frame under a triangular lateral load profile are shown in Table 7. The fundamental period calculated using the Rayleigh method is 0.32 sec, which is shown in Equation 15.

Table 7: Seismic weights, lateral force and displacements used in Rayleigh method to estimate fundamental period.

Level	$W_i\ (kN)$	$F_i(kN)$	$\delta_i\ (mm)$	$W_i\delta_i^2$	$F_i\delta_i$
7	150	70	10.88	17775	762
6	150	60	10.18	15544	610
5	150	50	9.02	12214	451
4	150	40	7.44	8303	297
3	150	30	5.54	4605	166
2	150	20	3.41	1748	68
1	150	10	1.29	250	13
				∑=60442	∑=2369

$$T = 2\pi\sqrt{\frac{60442}{9810\times2369}} = 0.32\ \text{sec} \quad (15)$$

On the other hand, as listed in Table 8, the fundamental period's estimated using Equations 12a-12c and 13a vary between 0.30 and 0.32 sec. It is clear that for practical applications the fundamental period estimated using the developed equations are close enough to the fundamental period predicted using the Rayleigh method.

Table 8: Prediction of period using equations 12a-12c, 13a.

Equation	Lamda (λ)	Sigma (Ω)	Phi (φ)	Fundamental period (sec)
12a	3.6	N/A	0.73	0.303
12b	3.6	0.98	0.76	0.308
12c, 13a	3.6	N/A	0.77	0.307

Fundamental Period: Effect of Finite Beam-Column Joint

Note that the previous verifications were conducted for frames with no consideration to the rigid joint panel dimensions. In reality, the joint dimension to the member span ratio (i.e. β_d or β_c in Equations 12c and 13a) in typical frames varies between 0.05 and 0.2. As the flexural stiffness of a beam or column is inversely proportional to the third power of its length, these can result in substantial increase in the stiffness; and consequently noticeable reduction in period. In this section, the validity and accuracy of Equation 12c in predicting the fundamental period including the effect of finite beam-column joint is investigated. For this purpose, a six storey and four bay frame building is chosen with the following properties: storey height of 3.6 m, beam centre to centre span of 6 m, column section of 0.4×0.6 m, beam section of 0.4×0.45 m, seismic weight of 4731 kN, $\beta_d = 0.12$ and $\beta_c = 0.1$. Eigenvalue analysis is conducted by modelling the frame with a combination of beam and column elements of lengths equal to their clear spans/heights and rigid blocks representing the half of the joint dimension at each end of the beam and column elements. The period obtained from Eigenvalue analysis is 1.03 sec (for the same frame, neglecting the effect of finite beam-column joint results fundamental period of 1.18 sec) whereas the predicted period using Equation 12c is 1.0 sec. The authors believe this extent of difference in prediction of fundamental period, which depends on several parameters, is acceptable.

Fundamental Period: Proposed Equation vs Experimental Results

Fundamental period of two RC gravity frames obtained from experimental tests are compared with the fundamental period predicted using the proposed equation. First one is a two storey RC gravity frame with dimensional and cross sectional properties as follows [19]; two bays of 2.85 m c/c span in Y direction and one bay of 4.70 m c/c span in X direction, inter-storey height measured to the centre of the beams of 3.50 m and 3.6 m for the first and second levels, beam cross-section of 0.3× 0.5 m, column cross-section of 0.3×0.3 m, modulus of elasticity of 26672 MPa, $\beta_d = 0.07$ in X and Y direction, $\beta_c = 0.06$ in X direction, $\beta_c = 0.1$ in Y direction, and seismic weight of 330 kN. The fundamental period of vibration measured in the experiment for the two storey un-retrofitted frame in X direction was 3.15 Hz (i.e. 0.317 sec) and in Y direction was 3.30 Hz (i.e. 0.303 sec) [19].

In the estimation of the fundamental period using the developed Equations 12c and 13a, storey height of 3.5 m is used even though there is slight difference in actual storey heights. It is important to note that the tested frame has stronger beams and weaker columns; and the relative column to beam stiffness (λ) is very small compared to a frame building that is designed using modern capacity design based building codes. Hence, the correction factor φ_3 value is dictated by the upper limit, which is expected to induce some error in the prediction. The fundamental periods predicted using the developed Equations 12c and 13a are shown in Table 9. Using the clear spans of beams and columns in Equation 13b also results in the same periods. It is clear from Table 9 the predicted fundamental periods are in comparable range and in reasonable agreement with the actual periods measured from the experimental test. The largest error in the prediction of fundamental period using the proposed Equations in this case is around 20%. As mentioned earlier, this error is mainly due to the column to beam stiffness ratio λ being well outside the range for which φ_3 has been calibrated.

Table 9: Calculation of period using developed equations.

Equation	Lamda (λ)		Phi (φ_3)	Fundamental period (sec)	
	X	Y		X	Y
12c, 13a	0.58	0.26	1.0	0.27	0.24

The comparison between the measured period and periods predicted by different code formulae is also reported by the experimenters [19]. They showed that there is a variation of 100% in the periods predicted by different code formulae, which supports the argument shown in Figure 1. Moreover, it is also obvious that all code formulae have a major shortcoming in their inability to recognize the difference in lateral stiffness (and hence the natural period of vibration) in the two directions whereas the equations proposed herein can capture the do.

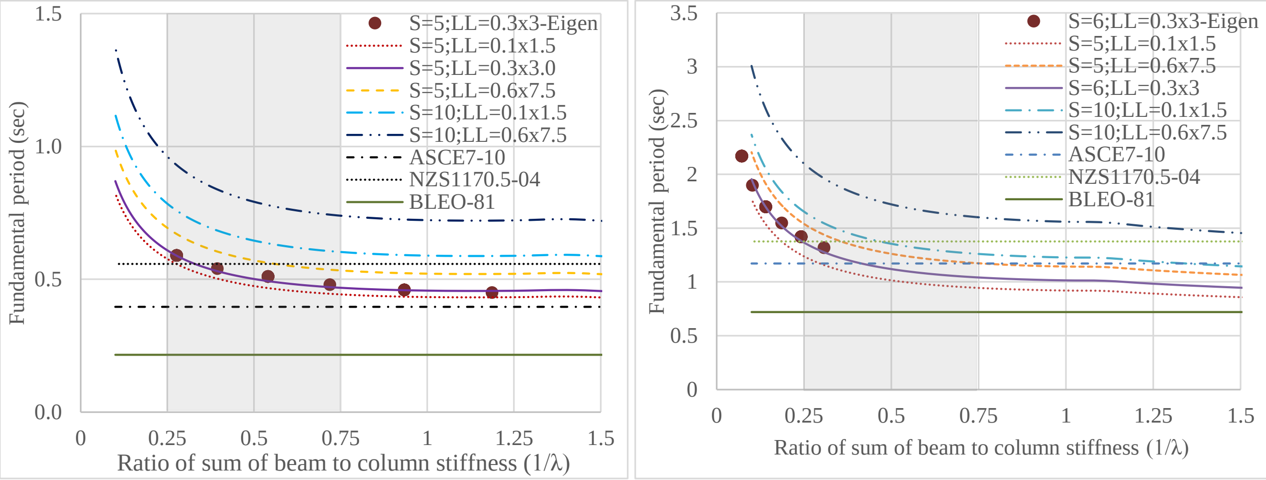
The second experimental test used for verification of the proposed equation is a three storey RC gravity frame with following cross sectional and dimensional properties [20]; three bays of 6 ft c/c span in one direction and one bay of 6 ft c/c span in another direction, inter-storey height of 4 ft, beam cross-section of 3×6 inch, column cross-section of 4×4 inch, modulus of elasticity of 24376 MPa, $\beta_d = 0.12$, $\beta_c = 0.05$, and seismic weight of 360 kN. The three storey frame was tested only in the long (3 bays) direction and the corresponding fundamental period of vibration obtained from white noise test was 0.56 sec [20]. The parameters calculated and used to predict the fundamental period in Equation 12c are $\lambda = 0.79$ and $\varphi = 0.92$. The fundamental period of the frame in the tested direction calculated using Equation 12c and 13a is 0.54 sec. In the other (single bay) direction, the frame should be slightly more flexible, and expectedly the proposed equations predict a longer period (i.e. 0.64 sec) in that direction. Note that some difference between the experimental test and the prediction is inevitable because of the inherent assumptions in the development of the proposed equations, e.g. contribution of slab to the lateral stiffness is not considered, any minor flexibility/slackness in the base connection is not accounted for, the correction factors are developed using data of periods with $1/\lambda$ between 0.25 and 0.75 (i.e. λ between 1.33 and 4), and decoupling of mass in transitional and rotational modes is not completely possible during experimental tests. Despite the above mentioned facts, the predicted periods are close enough to the periods measured in the two series of tests. These verifications prove that the proposed equations can be relied on to predict the fundamental period provided the building properties are well established.

Fundamental Period: Proposed Equation vs Analytical Method vs Empirical Equations

Since the fundamental periods predicted by the proposed Equation 12c are in good agreement with Eigenvalue analysis results, these results are used to scrutinize the limitations of empirical equations in predicting natural period of frame buildings; mainly their inability to capture the change in natural period due to changes in actual seismic mass, lateral stiffness (i.e. effective section properties) at different limit states, and the geometric configuration of a frame building. Firstly, the effect of seismic mass variation, and then the effect of effective section properties on the fundamental period are studied in this section. The seismic mass of a building during seismic excitation is comprised of the dead weight of a building itself and variable imposed load. The imposed load on a building depends on the occupancy class and varies from 1.5 kN/m^2 to 7.5 kN/m^2. In this comparative parametric study, the imposed load is varied by changing the tributary span width from 5 m to 10 m, and the corresponding seismic weights for a 3 storey and a 10 storey frames are shown in Table 10.

Table 10: Seismic weights used to study the effect of mass variation on the fundamental period.

Storeys	Tributary span width = 5 m		Tributary span width = 10 m	
	0.1xLL= 1.5 kN/m^2	0.6xLL= 7.5 kN/m^2	0.1xLL= 1.5 kN/m^2	0.6xLL= 7.5 kN/m^2
3	1347	1947	2492	3745
10	6866	10624	12251	19768



(a) 3 Storey –Live load variation ***(b) 10 Storey- Live load variation***

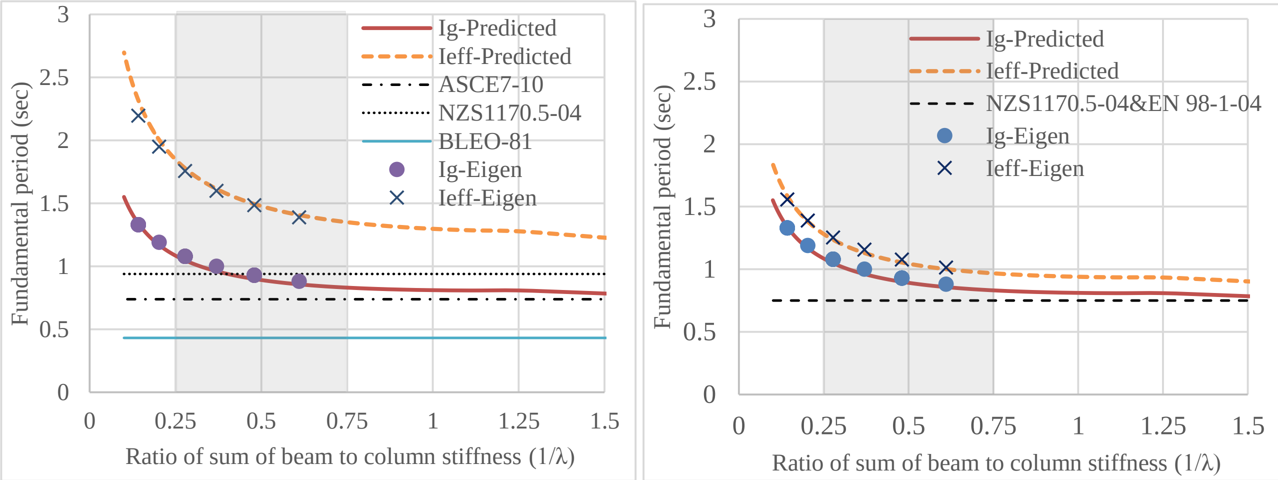
Figure 14: Limitations of empirical equations in prediction of period (i.e. with mass variation).

Building codes prescribe that seismic mass includes only a fraction of live load, generally 30% of characteristic live load. Here, periods are compared for cases including 10% and 60% of the live load to understand its effect on the period. In the legend of Figure 14, "S=5" indicates that the tributary span width is 5 m, and "LL=0.1x1.5" corresponds to live load of 0.1 times 1.5 kN/m^2. It is clear from Figure 14 that the empirical equations prescribed in different building codes do predict the period with reasonable accuracy for certain range of beam to column stiffness ratios and for low to medium range of imposed loads. For the frames with high imposed loads, the empirical equations under-predict the period. Moreover, it is already discussed and shown in the previous section that for different column size the natural period will be different, but the empirical equations account only for the overall building height.

As a case study, a six storey frame building is chosen to investigate the effect of effective section properties on the fundamental period. The periods are calculated using the effective section properties reported in Table 3 and using the proposed Equation 12c and compared with the periods calculated using empirical equations from different building codes. It is assumed that at the ultimate limit state (ULS) the effective moment of inertias are 0.5 and 0.32 times gross moment of inertia and at the serviceability limit state (SLS) the effective moment of inertias are 0.8 and 0.7 times gross moment of inertia for columns and beams respectively (i.e. $\alpha_c = 2; \alpha_b = 3.125$ -ULS $\alpha_c = 1.25; \alpha_b = 1.43$-SLS). It is observed that in most cases, the fundamental periods obtained using analytical methods (i.e. proposed equation and Eigenvalue analysis) with use of effective section properties (with the assumption that the cross sections chosen here are safe and adequate) are higher than the periods calculated using empirical equations at all limit states, which is shown in Figure 15. In legend of Figure 15, "I_g" and "I_{eff}" represent that the fundamental periods are calculated using gross and effective section properties respectively. It is clear from the plotted comparison in Figures 14 and 15 that empirical equations prescribed in the building codes significantly under predict the fundamental period; and cannot be used in assessment of an existing building. Hence, it can be concluded that the empirical equations are unable to capture the effect of lateral stiffness (i.e. wide range of combination of beam and column sizes) and imposed loads on a frame building's fundamental period. Since the proposed equation is simple, reasonably accurate, and considers all parameters known to affect the natural period, it can be reliably used in design or assessment of a building. As mentioned before, design engineers can also use the proposed equation for comparison with computer analysis results.

Application to Buildings with Minor Irregularity

The applicability of the proposed equation in predicting the fundamental period for slightly irregular frame buildings is scrutinised herein. For this purpose, the bay lengths are assumed to differ through the building width, the bottom storey is assigned a greater height, and the seismic weight is assumed to vary across different storeys; but the variation is restricted to 15% to be within the limits posed by the New Zealand seismic loading standard [13] for buildings which do not need an advanced dynamic analysis to assess the design seismic actions. A six storey four bay frame building is selected with both beam and column cross-sections of 0.4×0.5 m but other properties varying as listed in Table 11. It is assumed that the bottom storey height is 1.15 times other storeys height of 3.6 m (i.e. all other storeys are of the same height). The effective height h_{ef} and λ to be used in Equation 12c to predict the period is given by $\frac{1.15h+(n_s-1)h}{n_s}$ and $\frac{\frac{(n_b+1)E_cI_c}{h_{ef}}}{\frac{E_bI_b}{l1}+\frac{E_bI_b}{l2}+\frac{E_bI_b}{l3}+\frac{E_bI_b}{l4}}$ respectively. In Table 11, "L1-4=6" represents the length of all four bays is 6 m and "S1-6=w" indicates the seismic weight of storeys 1 to 6 is "w" kN. The fundamental period from Eigenvalue analysis and Equation 12c and the prediction error are reported in Table 11. For this case, the error is less than 5%, and it is believed that the error for other low to medium rise irregular frame buildings will be of similar magnitude. Even though the proposed equation is developed based on the assumption of equal storey heights, it can used to estimate period of slightly irregular frame buildings using the effective storey height h_{ef}.



(a) 6 Storey –Ultimate limit state ***(b) 6 Storey –Serviceability limit state***

Figure 15: Limitations of empirical equations in prediction of period (i.e. effective properties).

Table 11: Variation of bay lengths , seismic weights, and fundamental periods

Case	Bay length (m)	Seismic weight (kN)	Seismic weight (kN)	Eigen value-Period (sec)	Lamda (λ)	Period correction factor (φ_3)	Equation 12c- period (sec)	Error (%)
1	L1-4=6	S1-6=w	4756.77	1.248	2.03	0.801	1.24	0.76
2	L1-2=6 L3-4=5	S1=1.15w S2-6=w	4485.31	1.172	1.85	0.811	1.18	0.60
3	L1-2=6 L3-4=7	S1-2=1.15w S3-6=w	5302.63	1.327	2.19	0.795	1.33	0.21
4	L1-2=6 L3-4=7	S1-3=1.15w S4-6=w	5399.03	1.337	2.19	0.795	1.34	0.36
5	L1-2=6 L3-4=7	S1-3=w S4-6=1.15w	5399.03	1.380	2.19	0.795	1.34	2.77
6	L1-2=6 L3-4=7	S1-4=w S5-6=1.15w	5302.63	1.365	2.19	0.795	1.33	2.58
7	L1-2=6 L3-4=7	S1-5=w S6=1.15w	5206.22	1.345	2.19	0.795	1.32	2.04

CONCLUSIONS

The original Macleod's model is modified to more accurately predict the top deflection and the lateral stiffness of regular frame buildings in seismic regions. The error in the predicted lateral stiffness is reduced from ±25% in the original equation to ±10% in the modified equation. A new approach to predict the fundamental period of regular frame buildings with constant storey height and uniform mass along the building height is developed and a versatile generic formula is proposed. According to this formula the fundamental period can be calculated as:

$$T = 0.47\varphi_3\sqrt{\frac{W_s\, h^3 n_s(\,(1-\beta_d)^3 + \lambda\,(1-\beta_c)^3)}{(n_b+1)E_c I_{cef}}}$$

where W_s is the total seismic weight of the frame, h is the storey height (measured between top of successive floors), n_s is the number of storeys, n_b is the number of bays and φ_3 is the correction factor to account for effective mass and effective height (calculated as $\varphi_3 = 0.66 + 0.19/\lambda + 0.008 n_s \leq 1.0$). Similarly, λ is the ratio of total effective stiffness of the columns to the total effective stiffness of beams at the first floor level calculated as $\lambda = \frac{\Sigma \frac{E_c I_{cef}}{h}}{\Sigma \frac{E_b I_{bef}}{l}}$ where E_b and E_c are elastic modulus of beams and columns, I_{bef} and I_{cef} are the average effective moment of inertia of the beams and columns (to be replaced with gross moment of inertia I_b and I_c if gross section properties are to be used), and l is the average bay length (measured between the column centre lines). The factors $\beta_d = \frac{D}{h}$ and $\beta_c = \frac{C}{l}$ are to account for the finite size of beam-column joint; D and C are the beam and column depths, respectively. When clear spans of beam length and column/storey height are used (rather than centre to centre dimensions of the frame), the effect of finite size of beam-column joint on the fundamental period can be neglected (i.e. $\beta_d = \beta_c = 0$), then the fundamental period can be calculated as:

$$T = 0.47\varphi_3\sqrt{\frac{W_s\, h^3 n_s(\,1+\lambda\,)}{(n_b+1)E_c I_{cef}}}$$

The proposed equation captures the effect of all parameters that are known to affect the fundamental period of a frame building. As a distinct advantage over other existing empirical equations used in codes/standards throughout the world, the proposed equation recognises the difference between the periods of vibration in the two orthogonal directions of response of a frame building. It is shown to reliably capture the period of frame buildings with all configurations and characteristics. The fundamental periods predicted using the proposed equation are shown to be in good agreement with periods obtained from Eigenvalue analysis for a large number of frame buildings covering a wide range of building characteristics. Also, the predicted periods are found to be reasonably close to the periods obtained using Rayleigh method and those measured from experimental tests.

Although developed for regular frame buildings, the proposed equation has also been found to be able to predict the fundamental period of frame buildings with minor irregularity without inducing much additional error. It provides a viable tool to investigate the actual variation of the period with change in frame characteristics; e.g. number of bays and storeys, beam and column dimensions, bay length, storey height, seismic mass etc. Moreover, the proposed equation requires input parameters that are not difficult to determine, and is simple enough to be easily implemented into building design codes. Hence, this can be readily used by practicing engineers in designing new buildings as well as in the assessment of existing buildings.

REFERENCES

1 Goel RK and Chopra AK (1997). "Period formulas for moment-resisting frame buildings". *Journal of Structural Engineering,* 123(11): 1454-1461.

2 Günaydın E and Topkaya C (2013). "Fundamental periods of steel concentrically braced frames designed to Eurocode 8". *Earthquake Engineering and Structural Dynamics,* 42(10): 1415-1433.

3 Crowley H and Pinho R (2004). "Period-height relationship for existing European reinforced concrete buildings". *Journal of Earthquake Engineering,* 8(spec01): 93-119.

4 JapanBLEO (1981). "*Part 1, Earthquake Resistant Design of Structures and Part 2, Earthquake Resistant Design of Buildings*". Tokyo, Japan.

5 Aoyama H (1981). "*Outline of Earthquake Provisions in the Recently Revised Japanese Building Code*". Tokyo University, Japan.

6 Bangash M (2011). "*Earthquake Resistant Buildings: Dynamic Analyses, Numerical Computations, Codified Methods, Case Studies and Examples*". Springer Science & Business Media.

7 Kelly CY (2011). "*An Investigation of the Fundamental Period of Vibration of Irregular Steel Structures*". Master’s Thesis, The Ohio State University, USA.

8 Günaydn E (2012). "*Natural Periods of Braced Steel Frames Designed to EC8*". Master’s Thesis, Middle East Technical University, Turkey.

9 Shon BA (2015). "*Study on the Fundamental Period of Vibration for Buildings with Different Configurations*". Atilim University, Turkey.

10 Chopra AK and Goel RK (2000). "Building period formulas for estimating seismic displacements". *Earthquake Spectra,* 16(2): 533-536.

11 EN1998 (2004). "*Eurocode 8: Design of Structures for Earthquake Resistance-Part 1: General Rules, Seismic Actions and Rules for Buildings*". in *London: BSi*. pp. 10-12.

12 ASCE (2013). "*Minimum Design Loads for Buildings and Other Structures*". American Society of Civil Engineers.

13 NZS1170 (2004). "*NZS1170: Structural Design Actions. Part 5: Earthquake Actions- New Zealand*". Wellington, NZ.

14 ACI (2008). "*Building Code Requirements for Structural Concrete (ACI 318-08) and Commentary*". American Concrete Institute, Farmington Hills, USA.

15 NZS3101 (2006). "*NZS3101: The Design of Concrete Structures*". Standards New Zealand, Wellington.

16 Adeli H (1985). "Approximate formulae for period of vibrations of building systems". *Civil Engineering for Practicing and Design Engineers,* 4(1): 93-128.

17 Chrysanthakopoulos C, Bazeos N and Beskos D (2006). "Approximate formulae for natural periods of plane steel frames". *Journal of Constructional Steel Research,* 62(6): 592-604.

18 CSI (2013). "*SAP*". Computers and Structures Inc, Berkeley, CA, USA.

19 Di Sarno L and Manfredi G (2012). "Experimental tests on full-scale RC unretrofitted frame and retrofitted with buckling-restrained braces". *Earthquake Engineering & Structural Dynamics,* 41(2): 315-333.

20 Bracci JM, Reinhorn AM and Mander JB (1995). "Seismic resistance of reinforced concrete frame structures designed for gravity loads: performance of structural system". *ACI Structural Journal,* 92(5): 597-610.

21 Iain A (1971). "Simplified equations for deflection of multistory frames". *Building Science,* 6(1): 25-31.

22 Miranda E (1999). "Approximate seismic lateral deformation demands in multistory buildings". *Journal of Structural Engineering,* 125(4): 417-425.