EFFECTS OF COUPLED VERTICAL STIFFNESS-STRENGTH IRREGULARITY DUE TO MODIFIED INTERSTOREY HEIGHT

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SUMMARY

Structures may have vertical stiffness or strength irregularity for many reasons. In many practical cases, a change in storey stiffness, results a change in strength at the same storey. In this paper, the effect of a change in interstorey height is quantified. In order to do this, relationships between storey stiffness and strength resulting due to a modified interstorey height for a few common lateral force resisting systems was considered. It was applied to simple shear-type structures of 3, 5, 9 and 15 storeys, assumed to be located in Wellington. All structures were considered to have a constant mass at every floor level. Both regular and irregular structures were designed in accordance with the Equivalent Static method of the current New Zealand seismic design Standard, NZS 1170.5. Regular structures were designed to either (i) produce a constant target interstorey drift ratio at all the storeys simultaneously or (ii) to have uniform stiffness distribution over the height of the structure, with the target interstorey drift ratio at the first storey. An "interstorey height ratio" was defined as the ratio of modified to initial interstorey height, and applied separately at the first storey, mid-height storey and at the topmost storey by amounts between 0.5 and 3. The modified structures were then redesigned until the target interstorey drift ratio was achieved at the critical storey/storeys. Design structural ductility factors of 1, 2, 3, 4 and 6, and target (design) interstorey drift ratios ranging between 0.5% and 3%, were used in this study. Inelastic dynamic time-history analysis was carried out by subjecting these structures to code design level earthquake records, and the maximum interstorey drift ratio demands due to each record were used to compare the responses of regular and irregular structures.

It was found that structural types in which only the storey stiffness was modified due to a change in the interstorey height produced the maximum increase in drift demands rather than structural forms with other stiffness-strength coupling cases. Shorter structures having an increased first storey height, and taller structures with an increased middle storey height generally produced greater interstorey drift demands than regular structures. For cases of increased storey stiffness due to decreased storey heights, the shorter structures with a decreased middle storey height resulted in higher median peak ISDR due to irregularity. A simple equation describing the maximum increase in response due to modifications to a storey height was developed. The equation was used along with the realistic correlations between storey stiffness and strength to obtain the governing code regularity limit.

INTRODUCTION

No real structure is perfectly regular. While some structures are planned to be architecturally irregular, other structures may be irregular due to unplanned effects. One of the common types of irregularity is the stiffness and strength irregularity over the height of the structures. This irregularity generally exists in buildings due to:

- Difference in interstorey height at a particular storey as compared to adjacent storey, as shown in Figure 1(b);
- Modification of member properties, member sizes, material, at a storey, as shown in Figure 1(c);
- Vertical discontinuities of structural members at a particular storey, as shown in Figure 1(c); or
- Lack of infill material or open storey, as shown in Figure 1(d).

Many world-wide earthquake codes define structures to be irregular based on the relative differences in storey structural properties. Regularity limits are set in codes, which determine the analysis method permitted to be used. Such regularity limits are based on heuristic thinking and lack rational justification (Sadashiva *et al.* 2009). For example, the present New Zealand seismic Standard, NZS 1170.5 (*Cl. 4.5.1,* SNZ 2004), defines stiffness and strength irregularities to exist in buildings if:

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- The lateral stiffness of a storey is less than 70% of the stiffness of any adjacent storey, or less than 80% of the average stiffness of the three storeys above or below in the structure.
- The shear strength at a storey is less than 90% that in the storey above.

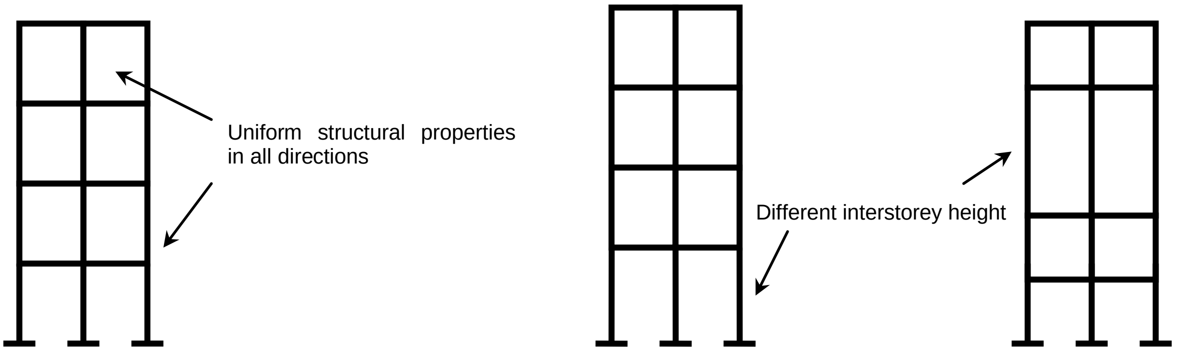
Although separate irregularity limits are defined for stiffness and strength irregularity, in many practical scenarios, a change in the storey stiffness is usually accompanied by a change in the storey strength. For example, when the cross-sectional property is changed at a storey, stiffness and strength at that storey are modified together.

Research on the effects of plan irregularity (e.g., Rutenberg (2002), Chopra and Goel (2004), De Stefano and Pintucchi (2006)) has received more attention than the vertical irregularity effects. Earlier studies on the influence of vertical stiffness and strength irregularity on the performance of structures have been summarised by Sadashiva *et al.* (2011). It includes studies by Valmundsson and Nau (1997), Al-Ali and Krawinkler (1998) and Michalis *et al.* (2006). These previous works neither provide simple methods to determine regularity limits nor justify the suitability of the code regularity limits. The methods used in their designs may result in structures which do not meet the code criteria and they may have unrealistic strength-to-stiffness distribution. An effective methodology to determine vertical regularity limits was developed and then applied on shear-type structures to evaluate the effects of vertical mass irregularity (Sadashiva *et al.* 2009). A study on the effects of coupled vertical stiffness-strength irregularity due to a change in member properties on realistic code-complying structures was conducted (Sadashiva *et al.* 2011) using the same method. The method involved using the NZS 1170.5 *Equivalent Static* (**ES**) method to design both the regular and irregular structures to the same target drift, and comparing the actual drift demands from inelastic dynamic time-history analyses. Simple conservative equations, relating the magnitude of irregularity and the increase in drift demands due to irregularity were derived that then could be used in the design. In this paper, the methodology is applied on structures having vertical stiffness-strength irregularity due to a change in interstorey height and their coupled effects are evaluated. Other member properties are assumed to remain unchanged and equal that of regular structures. The study answers the following questions:

1. What stiffness-strength coupling is likely in realistic structures due to a modified storey height?
2. Which storey/storeys are sensitive to a reduced stiffness only/and strength change due to a taller storey?
3. How do the responses differ when a storey's height is decreased compared to other storeys?
4. How can we estimate the likely increase in response due to stiffness-strength irregularity caused by a modified interstorey height?

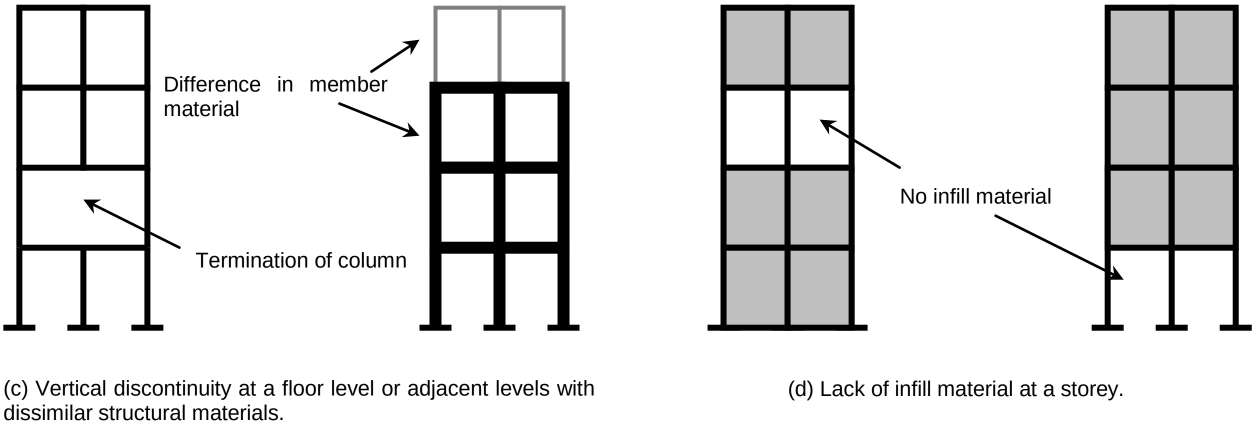
STRUCTURAL CONFIGURATION FOR REGULAR STRUCTURES

Simple models of shear-type structures of 3, 5, 9 and 15 storeys, having uniform mass at every floor, and with equal storey height of 4 m, were used to define the regular (base) structures. Each regular structure was assumed to be located in Wellington (with Hazard Factor, Z = 0.4), and was designed



(a) "Regular" structure with uniform structural property in all directions.

(b) Excessive floor height at a particular storey.



(c) Vertical discontinuity at a floor level or adjacent levels with dissimilar structural materials.

(d) Lack of infill material at a storey.

Figure 1: Examples of vertical Stiffness-Strength irregularities.

according to the NZ 1170.5 ES method. All the structures were designed for structural ductility factors, µ of 1, 2, 3, 4 and 6. A complete description on the design approach adopted in this study is explained in Sadashiva *et al.* (2009).

In order to have an "apples-to-apples" comparison between regular and irregular structures, each regular design model was provided with storey strengths such that a constant strength-to-stiffness ratio was maintained at all the storeys. Any structure having its storey strength-to-stiffness ratio outside the range of 0.3% - 3% was eliminated from this study. Also, structures having the horizontal design action coefficient, $C_d(T_1)$, governed by the lower limit (*Equation 5.2(2) - Cl. 5.2.1.1*, NZS 1170.5) were ignored.

Two classes of regular structures were assumed to define the bounds within which the realistic structures are assumed to have their configuration. The two model types are shown in Figure 2. The structures were provided with:

(a) Decreasing stiffness distribution over the height, with iterations carried out until all storeys simultaneously achieved the *design* (target) *interstorey drift ratio* (**DISDR**). Henceforth, this design model is referred as **CISDR** for *constant interstorey drift ratio*. Since a constant strength-to-stiffness ratio was obtained for CISDR design models at the end of iteration, the shear strength provided at each storey was the minimum required to resist the equivalent static design forces.

(b) Uniform stiffness distribution up the height, with iterations conducted until the first storey (critical) achieved the target interstorey drift ratio. The minimum shear strength required to resist the design force at the first storey was provided for all storeys, thus producing a constant strength-to-stiffness ratio at all the storeys. Henceforth, this design model will be referred as **CS-CSTG** for *constant stiffness* and *constant strength*.

CORRELATIONS BETWEEN STOREY STIFFNESS AND STRENGTH DUE TO MODIFICATION TO A STOREY HEIGHT

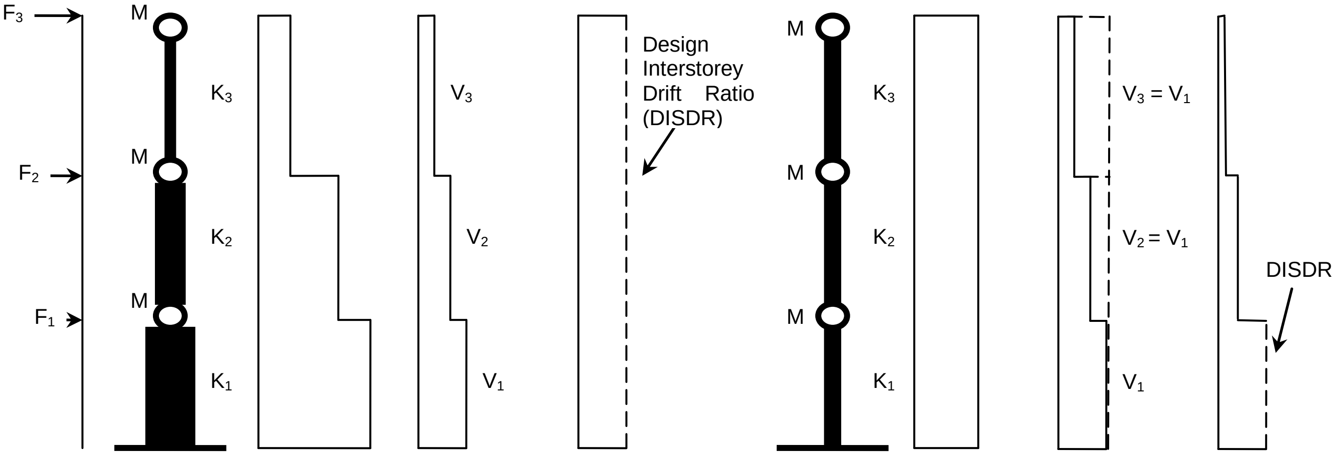
A change in interstorey height from h_o to h_m results in a change in storey stiffness. Relationships between the storey stiffness and strength due to a modified interstorey height can be obtained for various types of *lateral-force-resisting* (**LFR**) systems, as given in Appendix A. Here, the modified lateral stiffness at a chosen storey, K_{hm}, is given by Equation 1 as the product of the *stiffness modification factor* corresponding to the LFR system, $\beta_{k\text{-}LFR}$, and the initial lateral stiffness at the chosen storey, K_o. The corresponding storey strength may remain unchanged, as for a non-buckling steel shear wall, or vary proportionally with stiffness, as for a braced frame, or vary by differing amounts, as for moment frames. Hence, similar to Equation 1, the modified storey strength, V_{hm}, at the storey with modified interstorey height, is given by Equation 2 as the product of *strength modification factor* corresponding to the LFR system, $\beta_{v\text{-}LFR}$, and the initial storey strength provided for the storey, V_o. Thus, a total of four groups with the above correlations between storey stiffness and strength are formed that define the types of irregular structures that are used in this paper to evaluate their coupled effects. In Equations 1 and 2, the modification factors are functions of the parameter *interstorey height ratio*, h_{rat}, which is defined by Equation 3 as the ratio of modified interstorey height, h_m, to the initial interstorey height, h_o.

$$K_{hm} = \beta_{k-LFR} * K_o \tag{1}$$

$$V_{hm} = \beta_{v-LFR} * V_o \tag{2}$$

$$h_{rat} = \left(\frac{h_m}{h_o}\right) \tag{3}$$

The sensitivity of the magnitude of coupled stiffness-strength irregularities on the response of structures were investigated by choosing a set of h_{rat} that resulted in cases of stiffness reduction or enhancement at the storey with the modified interstorey height. For each h_{rat}, the stiffness and strength modification factors were calculated and applied to obtain the modified properties. The modification factors used in this study, calculated for each group and for the set of h_{rat}, are tabulated in Table 1. The ratio of $\beta_{v\text{-}LFR}$ to $\beta_{k\text{-}LFR}$ ranges from unity to h^2_{rat} depending on the system. For the values of h_{rat} chosen in this paper, this ratio of modification factors range between 0.25 and 9.



(a) Design forces (b) CISDR design (c) Stiffness dist. for CISDR design (d) Strength dist. for CISDR design (e) Drift dist. for CISDR design (f) CS-CSTG design (g) Stiffness dist. for CS-CSTG design (h) Strength dist. for CS-CSTG design (i) Drift dist. for CS-CSTG design

Figure 2: Structural configurations defining regular structures.

Table 1(a): Stiffness modification factors due to modified interstorey height.

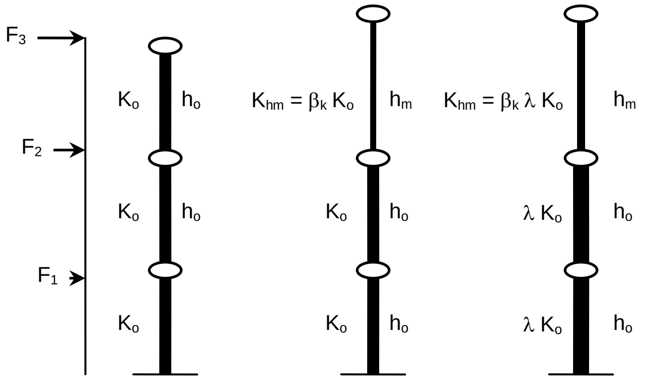
Group	**Stiffness modification factor, β_k**	**Interstorey height ratio, h_{rat}**							
		0.5	**0.75**	**1.25**	**1.5**	**1.75**	**2**	**2.5**	**3**
1	$\left(\frac{1}{h_{rat}}\right)$	2	1.33	0.8	0.67	0.57	0.5	0.4	0.33
2	$\left(\frac{1}{h_{rat}}\right)^2$	4	1.78	0.64	0.44	0.327	0.25	0.16	0.11
3	$\left(\frac{1}{h_{rat}}\right)^3$	8	2.37	0.512	0.3	0.187	0.125	0.064	0.036
4	$\left(\frac{\sqrt{L^2+h_o{}^2}}{\sqrt{L^2+(h_{rat}*h_o)^2}}\right)$	1.14	1.07	0.92	0.85	0.782	0.72	0.62	0.54

Table 1(b): Strength modification factors due to modified interstorey height.

Group	**Strength modification factor, β_v**	**Interstorey height ratio, h_{rat}**							
		0.5	**0.75**	**1.25**	**1.5**	**1.75**	**2**	**2.5**	**3**
1	1	1	1	1	1	1	1	1	1
2	$\left(\frac{1}{h_{rat}}\right)$	2	1.33	0.8	0.67	0.57	0.5	0.4	0.33
3	$\left(\frac{1}{h_{rat}}\right)$	2	1.33	0.8	0.67	0.57	0.5	0.4	0.33
4	$\left(\frac{\sqrt{L^2+h_o{}^2}}{\sqrt{L^2+(h_{rat}*h_o)^2}}\right)$	1.14	1.07	0.92	0.85	0.78	0.72	0.62	0.54

APPLYING COUPLED STIFFNESS AND STRENGTH IRREGULARITIES

The effect of coupled stiffness-strength over the height of the structures was conducted by applying the irregularities separately at the first storey, mid-height storey and at the topmost storey of regular structures. This was done by modifying the interstorey height by h_{rat} at the chosen storey for irregularity. The resulting storey stiffness and strength were obtained for structures with the range of stiffness and strength modification factors according to Appendix A for the particular h_{rat}. The modified structure was then redesigned until the target interstorey drift ratio was achieved at the critical storey/storeys. For example, as shown in Figure 3, consider a regular 3 storey CS-CSTG structure having stiffness distribution resulting in a target interstorey drift ratio (DISDR) of 1% at the first storey. If it is intended to have a taller third storey, the third storey height is modified by h_{rat}. The stiffness at that storey is multiplied by stiffness modification factor, β_k, by an amount corresponding to h_{rat} and the group of LFR system. Upon making this change in storey stiffness, the critical storey would no longer have the chosen DISDR. Therefore, all storey stiffnesses are then uniformly scaled by a scaling factor, λ, and the irregular structure is redesigned until the chosen DISDR is achieved at the critical storey/storeys. Since all storey stiffnesses are uniformly scaled, at the end of iteration, the irregular structure would still maintain the applied β_k at the chosen storey, which is third storey in this example. However, the relative storey stiffnesses at other storeys remain unchanged. To have a meaningful comparison between regular and irregular structures, the strength-to-stiffness ratios at all the storeys other than the irregular storey were kept the same. Therefore, the shear strengths provided over the height of irregular structures were different from the strength demand. Here, the modified strength at the chosen storey for irregularity was provided according to Equation 2, and multiplied by the λ factor. The strengths at other storeys were provided as the product of λ factor and the corresponding regular storey strength. The modified structure design ductility from this process was different from the target design ductility implying that the final drifts were not identical to those obtained with the target ductility. However, the difference in ductility and drifts was always less than 1.2%, and it was generally much smaller than this value. It was considered that this was not enough to significantly affect the results.



(a) Design forces (b) Regular CS-CSTG design with DISDR at first storey (c) Introducing stiffness modification at third storey (d) Irregular structure with DISDR at critical storey

Figure 3: Stiffness irregularity introduced by modifying the interstorey height for CS-CSTG design.

STRUCTURAL MODELLING AND ANALYSIS

All the structures were modelled as a combination of vertical shear and a vertical flexural column. The flexural column, which represents all of the continuous columns in the structure, is necessary to be included, without which high drift concentrations can be expected (e.g., MacRae *et al.* 2004, Tagawa *et al.* 2006, Sadashiva *et al.* (2009)).

Rayleigh damping has commonly been adopted to represent damping effects within multi-degree-of-freedom structures for several decades. A sensitivity study by Sadashiva *et al.* (2009) on the effects of different types of Rayleigh damping model show that the differences in drift responses due to three types of Rayleigh damping models available in RUAUMOKO (Carr 2004) time-history program were minimal. However, the tangent stiffness proportional Rayleigh damping model that uses the absolute form of equation of motion was considered to be more appropriate than other types of Rayleigh damping, to be used in IDTHA. Such a damping model that considers the non-linearity effects of structures, also assures that the damping forces go to zero at the end of excitation, and hence it has been used for all IDTHA conducted in this work. In order to avoid super-critical damping or negative damping, the first mode and the mode corresponding to number of storeys in the structure (Carr 2004) were nominated as the two modes with 5% of critical damping. The RUAUMOKO computer program was used to carry out all the IDTHA considering a post elastic stiffness (bilinear) factor of 1%.

A set of 20 SAC (SEAOC-ATC-CUREE) earthquake ground motion records for Los Angeles, with probabilities of exceedance of 10% in 50 years, were used for the ground motion suite (Sadashiva *et al.* 2009). Response spectra were developed for each of the selected records and the accelerations within each record were scaled so that the single-degree-of-freedom elastic response matched the NZS 1170.5 design acceleration considering a structural ductility factor and a structural performance factor of unity.

Interpretation of Inelastic Dynamic Time-History Analysis Results

The peak *interstorey drift ratio* (**ISDR**) from all the storeys within the structure, from any earthquake record, was obtained. This was obtained for each of the 20 records used. It was assumed that the distribution of ISDR is lognormal (Cornell *et al.* 2002), so the median and dispersion were found to measure the likely and the spread in the results respectively, according to Equations 4 and 5.

$$\hat{x} = e^{\left(\frac{1}{n}\sum_{i=1}^{n}\ln(x_i)\right)} \tag{4}$$

$$\sigma_{\ln x} = \sqrt{\frac{1}{(n-1)}\sum_{i=1}^{n}\left(\ln x_i - \ln\hat{x}\right)^2} \tag{5}$$

where x_i = peak interstorey drift ratio due to i^{th} record; and

n = total number of earthquake records considered.

Comparison between regular and irregular structures – effect of magnitude and location of modified interstorey height ratio:

The median peak interstorey drift ratio (ISDR) obtained for each irregular structure was compared with the corresponding median peak ISDR of the regular structure. The change in median peak ISDR due to the presence of coupled stiffness-strength irregularity was used to show the effects of irregularity. The response plot labels in the following figures have the format "N-L (Q)", where N refers to the number of storeys in the structure, L refers to the location (storey) of h_{rat}, and Q defines the magnitude of interstorey height ratio, h_{rat}. As explained earlier, structures having unrealistic storey strength-to-stiffness ratios and/or having the base shear governed by the code lower limit, were eliminated from this study. Hence, many designs for Group 2-4 structures, and some of the response plots in Figures 4 and 5 for Group 1 structures, were eliminated due to these two conditions imposed in the design. It will be shown later that Group 1 structures, rather than other groups, generally have higher increases in demand due to irregularity. Therefore, typical responses of Group 1 structures are only explained below.

Effect of increased interstorey height and reduced storey stiffness:

The response plots for Group 1 CISDR and CS-CSTG designs are shown in Figures 4(a) and 4(b) respectively for cases of *increase* in interstorey height.

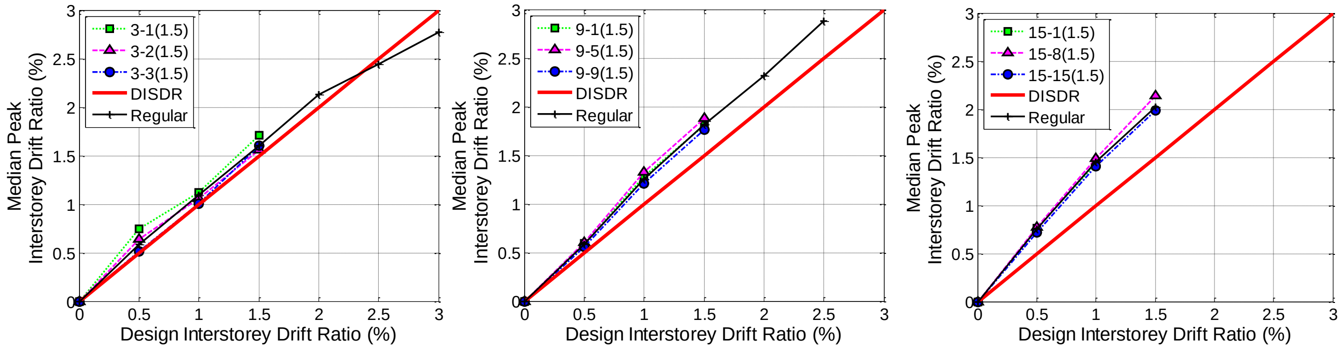
Figure 4(a) shows that for all DISDR when the first storey of 3 storey structures and the mid-height storey of taller structures were increased by h_{rat} of 1.5, the median peak ISDR increased relative to that for the corresponding regular structure. The maximum median peak ISDR increase due to this magnitude of h_{rat} at the first storey was 28%, 1.5% and 1.7% respectively for 3, 9 and 15 storey structures. An increased storey height at the middle storey increased the median peak ISDR by 10% for 3 storey structures and about 6% for 9 and 15 storey structures. For all DISDR, a taller storey at the topmost storey for all structures produced lesser drifts than for the regular structures. On average, the decrease in median peak ISDR due to irregularity at the roof was 7%, 4% and 3% for 3, 9 and 15 storey structures respectively. As the interstorey height ratio was increased from 1.5 to a maximum of 3, the same trends were seen for all structural heights.

Figure 4(b) shows that for Group 1 CS-CSTG designs, an increase in the first storey height by h_{rat} = 1.5, produced a maximum increase in median peak ISDR of 37% for 3 storey structures, whereas for taller structures, the median peak ISDR decreased due to this h_{rat} by upto 4%. The increase in storey height of the topmost storey rather than the mid-height storey was most significant for 3 storey structures with DISDR = 0.5%, producing a maximum of 16% increase in the median peak ISDR over the regular structure, and for taller structures this increase in median peak ISDR was less than 2.5% for the same h_{rat} and DISDR. The responses of CS-CSTG designs were more sensitive to an increase in h_{rat} than CISDR designs. For h_{rat} = 3, the maximum median peak ISDR due to an increased storey height at the first storey for 3 storey structures was 40%, and for taller structures with this h_{rat} of 3 at the first storey, the responses closely matched with those of corresponding regular structures. The effects of an increased storey height by h_{rat} = 3 at the mid-height was more significant than due to irregularity at the topmost storey, producing respectively a maximum increase in median peak ISDR of 26%, 13% and 4% for 3, 9 and 15 storey irregular structures.

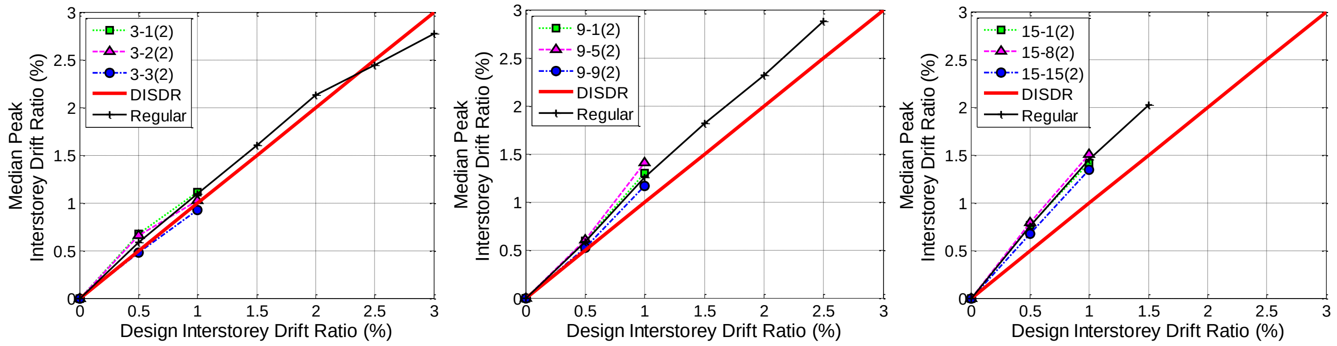
Effect of decreased interstorey height and increased storey stiffness:

The effect of *decreased* interstorey height for Group 1 CISDR designs is shown in Figure 5(a). For h_{rat} = 0.75 at the first storey of any structure height, the regular structures have median peak ISDR higher than the irregular ones. It is the decreased storey height of the mid-height level for 3 storey structures, and the topmost storey for taller structures that has produced increased drifts over the regular structure. For h_{rat} = 0.75 at the mid-height of 3 storey structure, a maximum increase in median peak ISDR of 7% was observed. There was an average decrease of 3% for taller structures with this magnitude of h_{rat} at the mid-height storey. The maximum increases in median peak ISDR due to a shorter topmost storey decreased with the structure height, and were 6%, 3% and 2% respectively for 3, 9 and 15 storey irregular structures. The above effects also occurred for h_{rat} = 0.5, however with slightly higher magnitude.

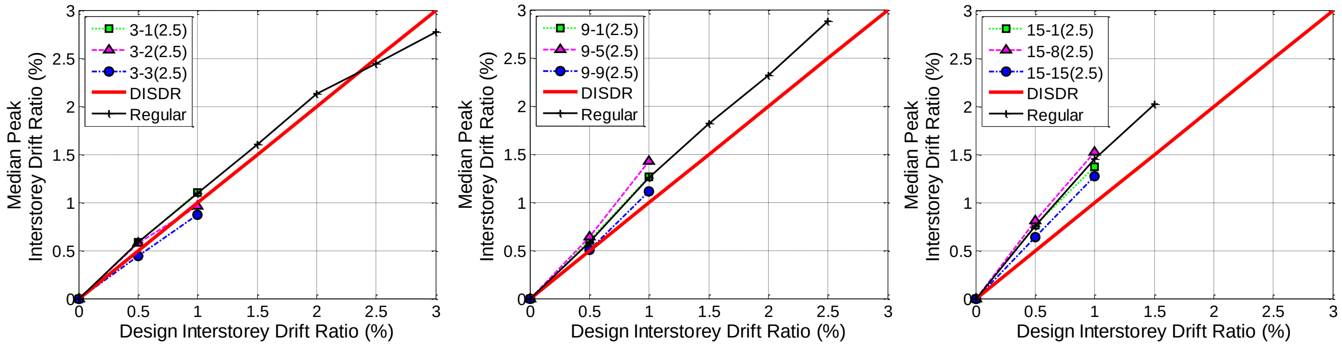
In case of Group 1 CS-CSTG designs, as seen in Figure 5(b), the mid-height of 3 storey structure and the first storey of taller structures with h_{rat} = 0.75, produces higher drift demands due to irregularity than the other two storeys chosen for irregularity. A maximum increase in median peak ISDR due to this h_{rat} at the first storey for 3 storey structures was 17%, and its magnitude decreased with the structure height. Increases in median peak ISDR of up to 5% and 3%, due to h_{rat} = 0.75 at the first storey of 9 and 15 storey structures were respectively obtained. For h_{rat} = 0.75 at the mid-height storey, the increase in median peak ISDR was 22% for 3 storey structures, and less than 2% for 9 and 15 storey irregular structures. Effects of irregularity at the topmost storey were insignificant for all structure heights, and the responses closely matched with the responses of the regular structures. Again, the above observations were generally the same when h_{rat} was decreased from 0.75 to 0.5.



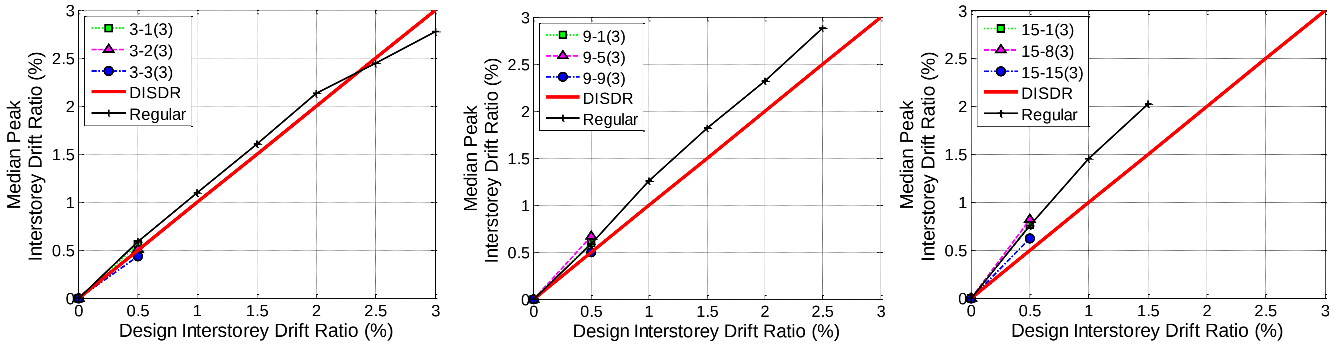
(1) h_{rat} = 1.5



(2) h_{rat} = 2

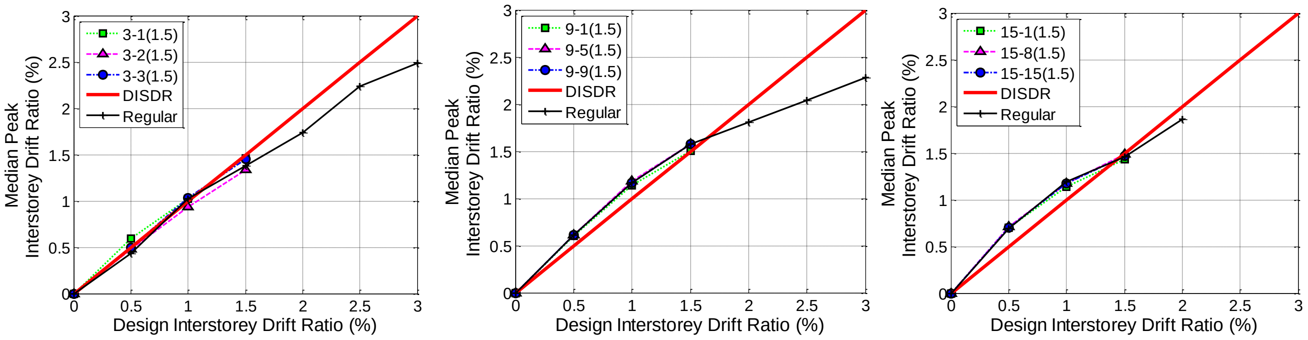


(3) h_{rat} = 2.5

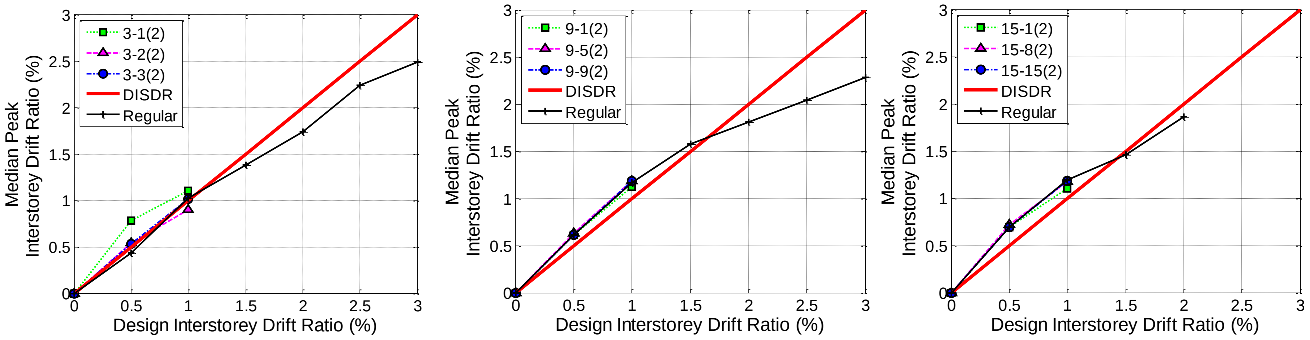


(4) h_{rat} = 3

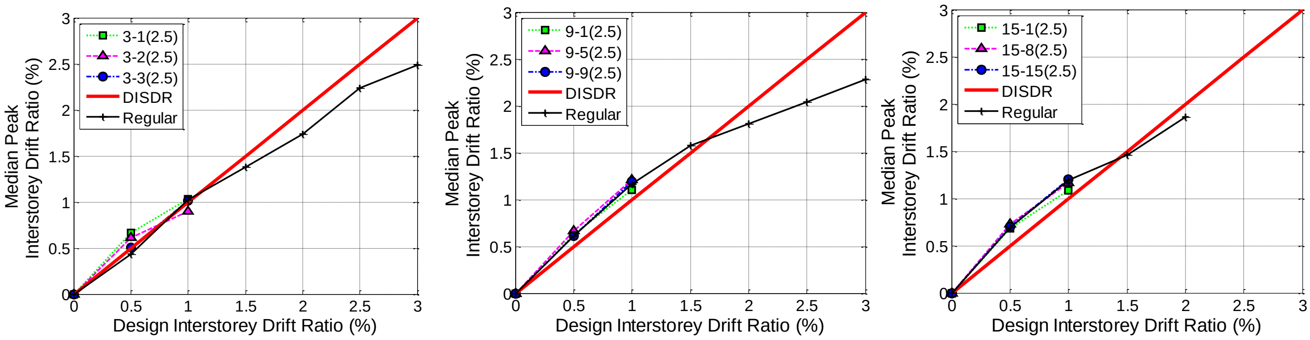
Figure 4(a): Effect of increased interstorey height for Group 1 structures - CISDR design (μ = 3, Z = 0.4).



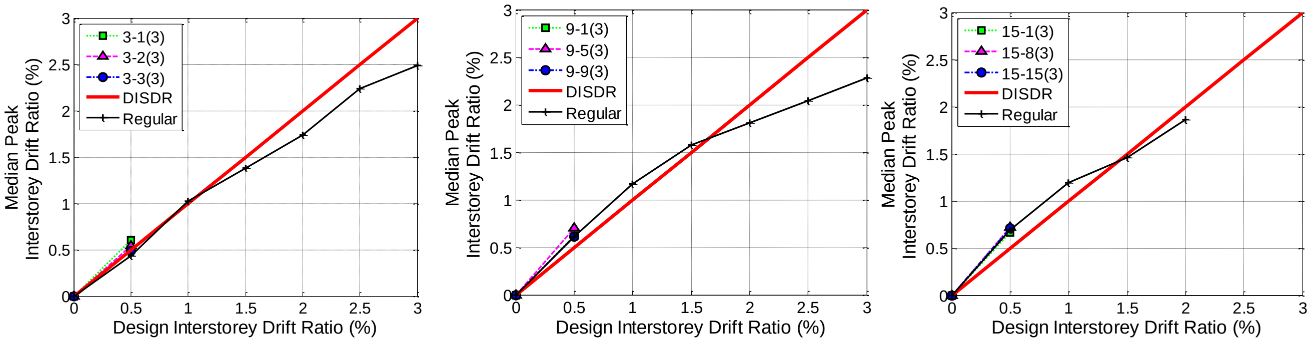
(1) h_{rat} = 1.5



(2) h_{rat} = 2

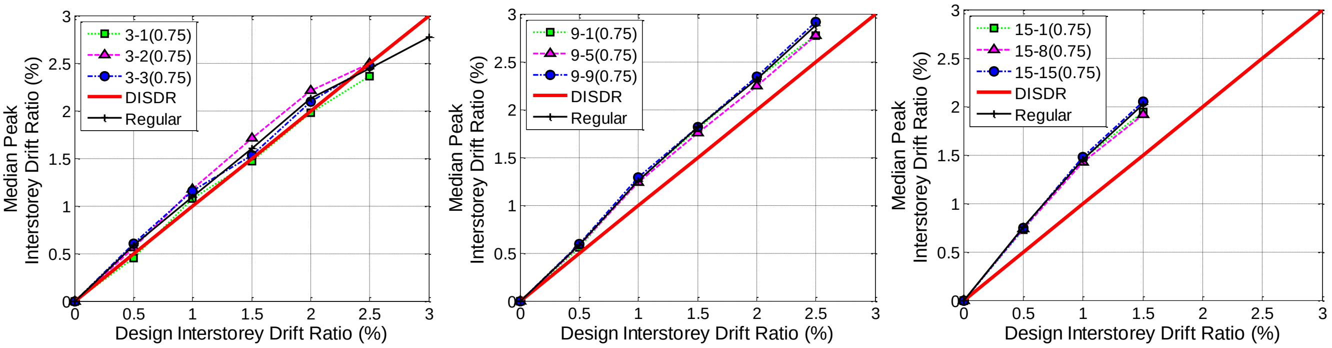


(3) h_{rat} = 2.5

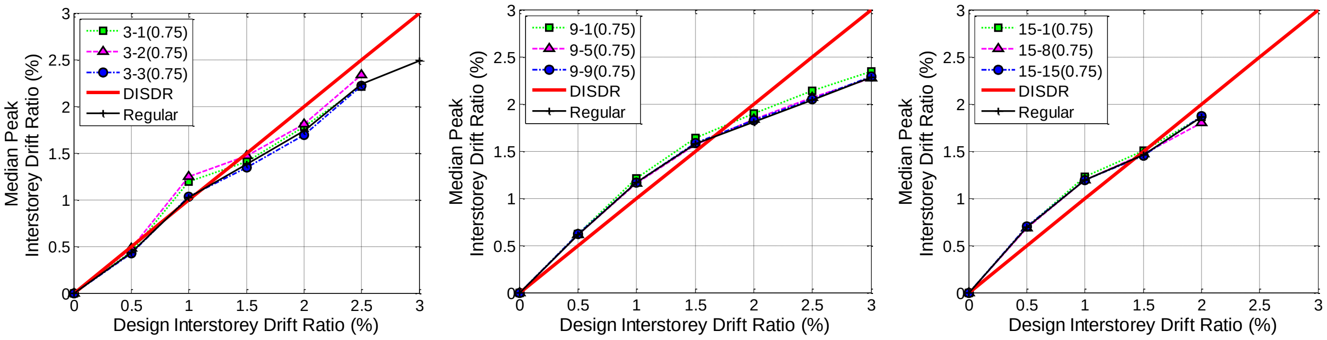


(4) h_{rat} = 3

Figure 4(b): Effect of increased interstorey height for Group 1 structures – CS-CSTG design (μ = 3, Z = 0.4).



(a) CISDR design - h_{rat} = 0.75



(b) CS-CSTG design - h_{rat} = 0.75

Figure 5: Effect of decreased interstorey height for Group 1 structures (μ = 3, Z = 0.4).

DETERMINATION OF ALLOWABLE INTERSTOREY HEIGHT RATIO

The relationship between *increase* in *median peak interstorey drift ratio*, ***ISDR**$_{incr}$*, due to irregularity and magnitude of irregularity, was computed as below:

Step 1. For a combination of structural form, structural ductility factor, design interstorey drift ratio, structure height, and the storey with stiffness-strength irregularity, the median peak ISDR for the regular structure, ***ISDR**$_R$*, and for the irregular structure, ***ISDR**$_I$*, is computed from the results of the structure to the suite of records. The increase in median peak ISDR due to irregularity, *ISDR$_{incr}$*, is calculated by Equation 6.

$$ISDR_{incr} = \left(\frac{ISDR_I}{ISDR_R} - 1 \right) {*}100 \ (\%) \tag{6}$$

Step 2. Step 1 is repeated for all the combinations of structural form, structural ductility factor, design interstorey drift ratio, structure height, storey with stiffness-strength irregularity, and magnitude of irregularity.

Step 3. For each magnitude of irregularity, find the *maximum* value of *ISDR$_{incr}$*. This is labelled as ***ISDR**$_{max\_incr}$*.

For example, Group 1 CS-CSTG structures having h_{rat} = 2 and μ = 3, produce *ISDR$_{max\_incr}$* = 81.4% as shown in Figure 6(a). This maximum value of *ISDR$_{incr}$* is obtained from a three storey structure with its first storey height modified and designed for DISDR = 0.5%. For this example, Figure 4(b) shows that the median peak ISDR for the regular structure, *ISDR$_R$*, is equal to 0.43%, and the median peak ISDR for the irregular structure, *ISDR$_I$*, is equal to 0.78%.

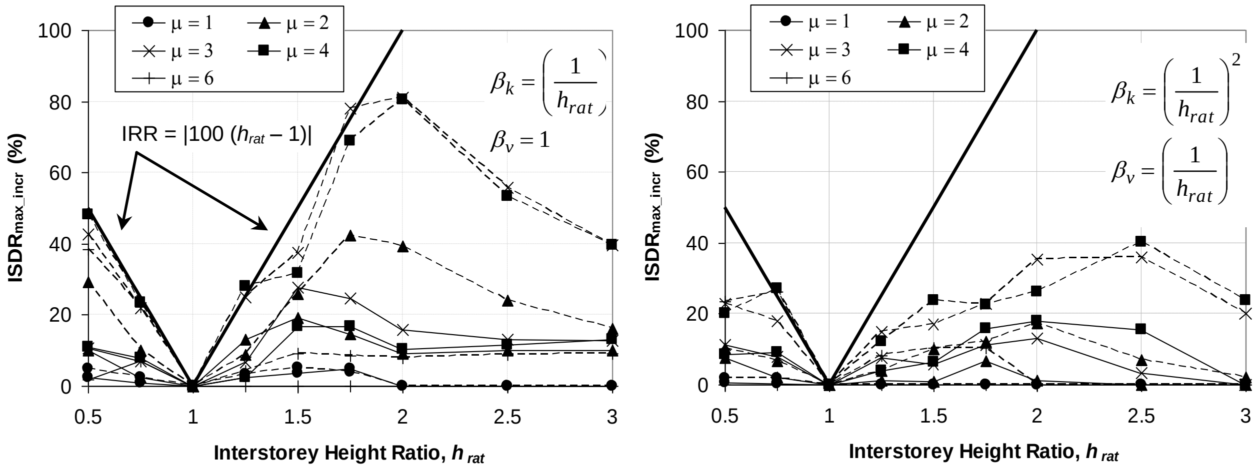
Figure 6 shows *ISDR$_{max\_incr}$* plotted against interstorey height ratio, h_{rat}, for all the coupled stiffness-strength irregularity cases considered in this study. The figure shows that the group of structures having only storey stiffness modified due to a change in storey height produces higher increases in response due to irregularity than other groups. The figure also shows that generally the structures designed to have a uniform distribution of stiffness and strength (CS-CSTG), have greater increases in median peak ISDR due to h_{rat} than the structures designed to produce equal storey drifts (CISDR). However, the absolute responses of CISDR designs are greater than the CS-CSTG designs, as seen in Figures 4 and 5. This observation is consistent with the findings in studies of mass irregularity (Sadashiva *et al.* 2009) and coupled stiffness-strength irregularity due to modified member properties (Sadashiva *et al.* 2011).

Equation 7 is a simple equation that gives a measure of the likely increase in drift response due to modifications to a storey height. This equation is based on Group 1 structures, and it is very conservative for Group 2-4 structures and for those with irregularity at the non-critical storeys, as shown in Figure 6.

$$IRR = \left| 100 \left(h_{rat} - 1 \right) \right| \ \% \tag{7}$$

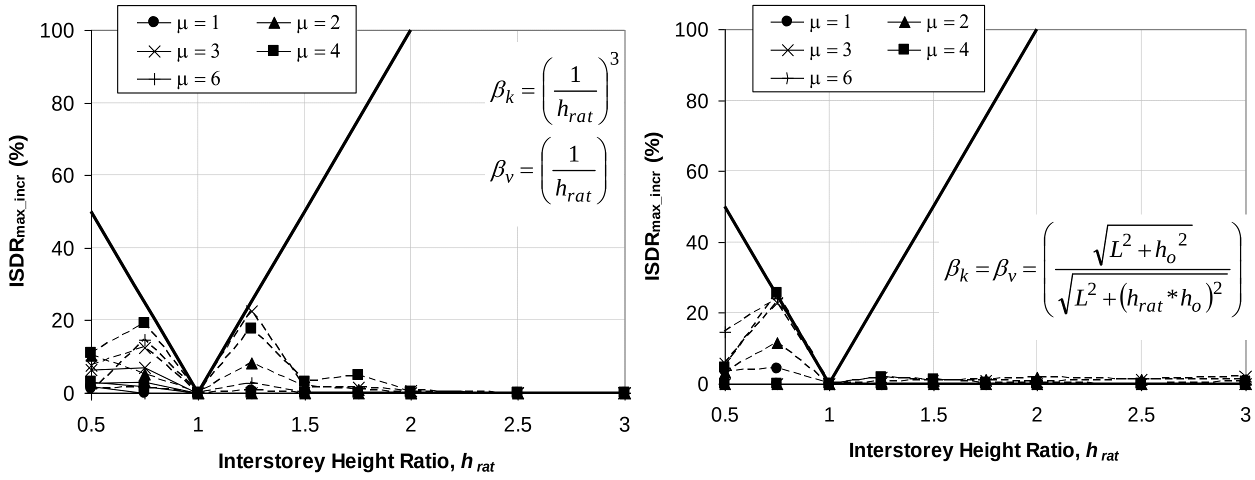
where ***IRR*** is the *irregular response ratio* which specifies how much the irregular response is greater than the regular response; and h_{rat} is the interstorey height ratio.

According to Figure 7, if it is not intended to have responses to be more than 20% due to change to a storey height, then the modified storey height cannot be less than 0.85, or more than 1.2 times the regular storey height. Equation 7 can also be used to calculate *IRR* values due to respective NZS 1170.5 stiffness and strength regularity limits of 0.7 and 0.9. This is done by using the relation between stiffness-strength modification factors and h_{rat}, shown in Figure 6, and applying in Equation 7. The evaluated *IRR* values due to code stiffness and strength regularity limits are shown in Table 2. The governing code regularity limit for each group of structure is also shown in the table by the corresponding *IRR* values in bold.



(a) Group 1

(b) Group 2



(c) Group 3

(d) Group 4

Figure 6: Maximum increase in median peak ISDR due to modified interstorey height.

—— *CISDR* - - - - *CS-CSTG*

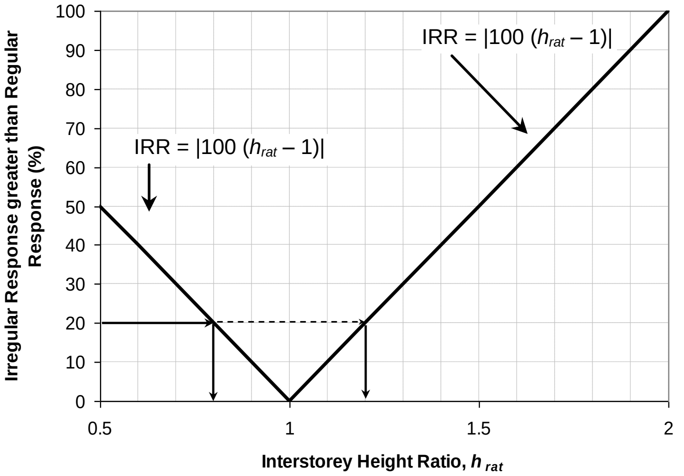


Figure 7: Determination of allowable interstorey height

Table 2: Irregular response ratio (%) due to NZS 1170.5 stiffness and strength regularity limits

Group	1	2	3	4
$\beta_k = 0.7$	**42.85**	19.52	12.62	109.3
$\beta_v = 0.9$	----	**11.11**	**11.11**	**32.72**

CONCLUSIONS

The effects of coupled vertical stiffness-strength irregularities caused in structures due to a modified interstorey height were evaluated and presented in this paper. Regular structures, represented by shear-type structures of 3, 5, 9 and 15 storeys having equal storey height, assumed to be in Wellington and having a constant floor mass at every floor level, were designed for a range of structural ductility factors of 1, 2, 3, 4 and 6 according to the NZS 1170.5 Equivalent Static method. The stiffness distribution over the height was either provided such that it resulted in design (target) interstorey drift ratios (DISDR) at all storeys simultaneously or a uniform stiffness distribution that produced DISDR at the first storey was provided. The strength distribution over the height was provided such that the strength-to-stiffness ratio at each storey was constant. An "interstorey height ratio" was defined as the ratio of modified to initial interstorey height, and applied separately at the first storey, mid-height storey and at the topmost storey. This generally corresponded to a modification of the storey stiffness and strength. The modified structures were then redesigned until the critical storey/storeys achieved the target DISDR. The change in the median peak interstorey drift ratio (ISDR), due to coupled stiffness-strength irregularities, obtained from inelastic dynamic time-history analysis were then used to explain the effects of coupled stiffness-strength irregularity. The conclusions derived from this study can be summarised as below:

1. Realistic correlations between storey stiffness and strength due to modifications to a storey height for a few common lateral force resisting systems were determined. A range of interstorey height ratios that produced cases of stiffness-strength reduction or enhancement were selected to investigate the effects of the magnitude of irregularities;
2. The group of structures having only the storey stiffness modified due to a change to a storey height (Group 1 structures), produced the maximum adverse effects of irregularity. For this group with CISDR or CS-CSTG configuration, a taller first storey for short period structures, and a taller mid-height storey for taller structures, was found to produce median peak ISDR greater than the regular structures. The increase in median peak ISDR due to irregularity generally reduced with the structure height;
3. For Group 1 structures, the effects of a short storey were less than those due to a taller storey in the structure. A shorter mid-height storey of short period CISDR and CS-CSTG designs generally tended to produce higher increases in median peak ISDR due to irregularity than other irregularity positions and structure heights; and
4. A simple equation that can estimate the likely increase in response due to a modified interstorey height was developed. Similar equations can be easily developed for different types of engineering demand parameters and used in design. The equation was also used along with the realistic correlations between storey stiffness and strength to obtain the governing code regularity limit.

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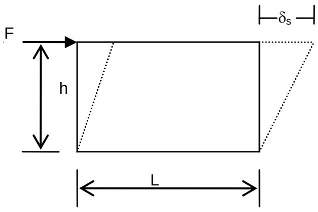
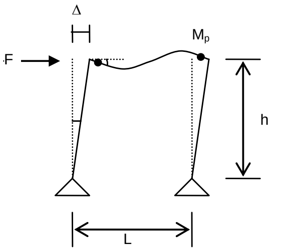
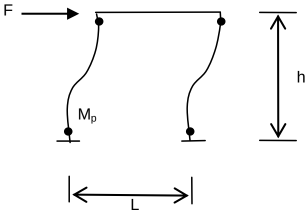
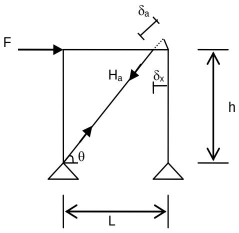
Appendix A: Correlations between storey stiffness-strength due to modified interstorey height.

Lateral-force-resisting (LFR) system	Modified storey stiffness, K_{hm}	Modified storey strength, V_{hm}
(a) ***Non-buckling Steel Shear Wall – Group 1***  $K = \frac{G A_s}{h}$ $V = \frac{A_s \sigma_y}{\sqrt{3}}$	$\frac{K_{hm}}{K_o} = \left(\frac{G A_s}{h_m}\right) * \left(\frac{h_o}{G A_s}\right)$ $K_{hm} = \left(\frac{h_o}{h_m}\right) * K_o$ $K_{hm} = \left(\frac{1}{h_{rat}}\right) * K_o$ $K_{hm} = \beta_{k-1} * K_o$	$\frac{\left(V_{hm}/K_{hm}\, h_m\right)}{\left(V_o/K_o\, h_o\right)} = \left(\frac{\sigma_y}{G\sqrt{3}}\right) * \left(\frac{G\sqrt{3}}{\sigma_y}\right)$ $V_{hm} = \left(\frac{K_{hm}}{K_o}\right) * \left(\frac{h_m}{h_o}\right) * V_o$ $V_{hm} = V_o$ $V_{hm} = \beta_{v-1} * V_o$
(b) ***Moment Frame (strong column weak beam mechanism) – Group 2***  $K = \frac{6 E I}{L h^2}$ $V = \frac{M_p}{h}$	$\frac{K_{hm}}{K_o} = \left(\frac{6 E I}{L {h_m}^2}\right) * \left(\frac{L {h_o}^2}{6 E I}\right)$ $K_{hm} = \left(\frac{h_o}{h_m}\right)^2 * K_o$ $K_{hm} = \left(\frac{1}{h_{rat}}\right)^2 * K_o$ $K_{hm} = \beta_{k-2} * K_o$	$\frac{\left(V_{hm}/K_{hm}\, h_m\right)}{\left(V_o/K_o\, h_o\right)} = 1$ $V_{hm} = \left(\frac{K_{hm}}{K_o}\right) * \left(\frac{h_m}{h_o}\right) * V_o = \left(\frac{h_o}{h_m}\right)^2 * \left(\frac{h_m}{h_o}\right) * V_o$ $V_{hm} = \left(\frac{1}{h_{rat}}\right) * V_o$ $V_{hm} = \beta_{v-2} * V_o$
(c) ***Moment Frame (strong beam weak column mechanism) – Group 3***  $K = \frac{12 E I}{h^3}$ $V = \frac{2 M_p}{h}$	$\frac{K_{hm}}{K_o} = \left(\frac{12 E I}{{h_m}^3}\right) * \left(\frac{{h_o}^3}{12 E I}\right)$ $K_{hm} = \left(\frac{h_o}{h_m}\right)^3 * K_o$ $K_{hm} = \left(\frac{1}{h_{rat}}\right)^3 * K_o$ $K_{hm} = \beta_{k-3} * K_o$	$\frac{\left(V_{hm}/K_{hm}\, h_m\right)}{\left(V_o/K_o\, h_o\right)} = \left(\frac{h_m}{h_o}\right)$ $V_{hm} = \left(\frac{K_{hm}}{K_o}\right) * \left(\frac{h_m}{h_o}\right)^2 * V_o = \left(\frac{h_o}{h_m}\right)^3 * \left(\frac{h_m}{h_o}\right)^2 * V_o$ $V_{hm} = \left(\frac{1}{h_{rat}}\right) * V_o$ $V_{hm} = \beta_{v-3} * V_o$
(d) ***Braced Frame – Group 4***  $K = \frac{A_a E}{\sqrt{L^2 + h^2}}$ $V = \frac{A_a \sigma_y L}{\sqrt{L^2 + h^2}}$	$\frac{K_{hm}}{K_o} = \left(\frac{A_a E}{\sqrt{L^2 + {h_m}^2}}\right) * \left(\frac{\sqrt{L^2 + {h_o}^2}}{A_a E}\right)$ $K_{hm} = \left(\frac{\sqrt{L^2 + {h_o}^2}}{\sqrt{L^2 + {h_m}^2}}\right) * K_o$ $K_{hm} = \left(\frac{\sqrt{L^2 + {h_o}^2}}{\sqrt{L^2 + \left(h_{rat} * h_o\right)^2}}\right) * K_o$ $K_{hm} = \beta_{k-4} * K_o$	$\frac{\left(V_{hm}/K_{hm}\, h_m\right)}{\left(V_o/K_o\, h_o\right)} = \left(\frac{\varepsilon_y L}{h_m}\right) * \left(\frac{h_o}{\varepsilon_y L}\right)$ $V_{hm} = \left(\frac{K_{hm}}{K_o}\right) * V_o$ $V_{hm} = \left(\frac{\sqrt{L^2 + {h_o}^2}}{\sqrt{L^2 + \left(h_{rat} * h_o\right)^2}}\right) * V_o$ $V_{hm} = \beta_{v-4} * V_o$

Nomenclature for Appendix A

A_a = cross-sectional area of the bracing member

A_s = cross-sectional shear area

E = modulus of elasticity

G = shear modulus of elasticity

h = interstorey height

h_o = initial interstorey height

h_m = modified interstorey height

h_{rat} = interstorey height ratio

I = moment of inertia of the member section

K = lateral stiffness of the structure

K_o = initial lateral stiffness of the structure

K_{hm} = modified lateral stiffness of the structure due to modified interstorey height

L = span length of the frame or wall

M_p = section plastic moment

V = lateral strength of the structure

V_o = initial lateral strength of the structure

V_{hm} = modified lateral strength of the structure due to modified interstorey height

β_k = stiffness modification factor

β_v = strength modification factor

ε_y = yield strain of the material

σ_y = yield stress of the material