

# PLASTIC HINGE LOCATION IN COLUMNS OF STEEL FRAMES SUBJECTED TO SEISMIC ACTIONS

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## SUMMARY

Several steel structure standards around the world contain an equation to encourage any column flexural yielding during earthquake shaking to occur at the column ends, rather than along the column length. The accuracy of these equations and their applicability to columns of both moment frames and braced frames are examined in this paper. It is shown, using an analytical procedure developed from first principles considering the reduction in member stiffness from axial force due to geometric and material nonlinearity, that the existing code equations are conservative. Less conservative empirical equations are developed based on the analysis results. It is found that these equations are applicable to frames with a braced connection, rather than a moment connection into the column. Time-history analysis of eccentrically-braced frames with inverted V-bracing, where the active link occurs at the centre of the beam, is carried out. The likely column end moment ratio needed for the new equations is determined. The analysis also shows that yielding often did not occur in the bottom story columns during earthquake excitations. A simple check is proposed to relate the axial force limit and the design drift to flexural yielding of columns which can be used in conjunction with the proposed equations.

## 1. INTRODUCTION

New Zealand and Australian steel design standards [12, 11] have a seismic provision to define the plastic hinge location in columns of steel frames that is not present in the codes of other countries. This provision aims to provide rotational capacity to the steel frames primarily by ensuring that flexural yielding would occur at the column ends rather than along the column length. Yielding along the column length under seismic actions is considered to be less desirable because:

- The rotational capacity is likely to be less reliable than that determined from the tests, where columns were designed to yield at the member ends, where the member ends were effectively braced.
- The correct collapse mechanism and hinge rotational demands are harder to predict.
- It is difficult to effectively brace along the member to restrict local and lateral buckling.
- Cumulative hinge rotations in one direction may occur during cyclic loading.

Clause 12.8.3.1(b) in the New Zealand steel structures standard, NZS3404, specifies that the seismic design axial compression force,  $N^*$ , for columns shall satisfy Equation 1, where  $\phi$  is the safety reduction factor,  $N_S$  is the nominal section capacity given by Equation 2,  $\beta_m$  is the end moment ratio which is positive in double curvature,  $\lambda$  is the member in-plane slenderness factor given by Equation 3,  $A$  is the

nominal cross-sectional area,  $f_Y$  is the material nominal yield stress,  $N_{OL}$  is the Euler in-plane buckling force given in Equation 4,  $I$  is the member second moment of area for bending in the direction considered,  $E$  is the elastic modulus, and  $L$  is the member length. The end moment ratio,  $\beta_m$ , is taken as zero for columns forming part of a seismic-resisting system, based on a study of moment resisting frames [8], and 0.5 for columns forming part of an associated structural system. Figure 1 shows that the maximum permitted axial force ratio,  $N^*/(\phi N_S)$ , is greater for columns in double curvature (i.e. greater  $\beta_m$ ) for a given slenderness ratio,  $\lambda$ .

$$N^* \leq \phi N_S \left[ \frac{1 + \beta_m - \lambda}{1 + \beta_m + \lambda} \right] \quad (1)$$

$$N_S = A \times f_Y \quad (2)$$

$$\lambda = \sqrt{\frac{N_S}{N_{OL}}} \quad (3)$$

$$N_{OL} = \frac{\pi^2 EI}{L^2} \quad (4)$$

According to Steel Construction New Zealand, Equation 1 often governs the size of columns in eccentrically braced frames [9]. However, the use of Equation 1 in the NZ steel structures standard may not be appropriate for the following reasons:

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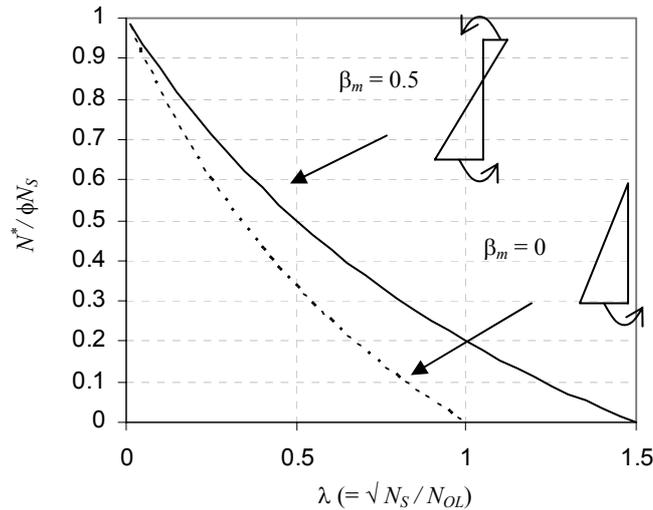


Figure 1: This Axial force limits for different member slenderness and end moment ratios (NZS3404 Clauses 12.8.3.1(b)).

- Equation 1, henceforth referred to as “Lay’s equation”, was originally developed as part of Lay’s doctoral work [6] by curve-fitting column deflection data for axially-loaded braced members in conjunction with an analytical approach to guarantee rotational capacity by keeping the hinges at the member ends [7]. However, the degree of rotational capacity used in the development of these equations was not stated and their development is not clear. The conservatism of Lay’s equation, or lack thereof, is therefore unknown.
- In addition to Lay’s equation, the New Zealand steel structures standard has specific provisions to ensure adequate flexural rotation capacity of a plastic hinge by defining absolute limits on the maximum axial force ratio. Because Lay’s equation was developed to provide rotation capacity as well as to encourage yielding at the member ends, they may be more conservative than the requirements to cause yielding at the member ends alone.
- According to NZS3404, Lay’s equation is currently applied to columns in moment resisting frames, MRFs, as well as to eccentrically braced frames, EBFs, and concentrically braced frames, CBFs. This is despite the fact that the column loading is quite different in sway (e.g. MRF) and braced (e.g. EBF, CBF) structures.
- In NZS3404, Lay’s equation is applied to columns irrespective of whether or not they actually yield. Location of maximum moment is only important if the column yields, therefore existing equations will be unnecessary and conservative in cases when no column yielding is expected.
- The critical end moment ratio,  $\beta_m$ , of zero, was based on the results of analyses of MRFs and it is not clear whether it is applicable to columns of braced frames.

This study was therefore initiated in order to answer the following questions:

1. Is Lay’s equation appropriate to estimate whether or not the maximum moment will occur at the column ends?
2. Is Lay’s equation applicable to both MRFs and EBFs?
3. Do typical EBFs suffer column yielding under design level earthquakes?
4. Are the end moment ratios,  $\beta_m$ , of zero developed for MRFs applicable to EBFs?
5. Can revised guidelines be developed based on the information obtained?

## 2. ANALYTICAL METHODS FOR EVALUATION OF LAY’S EQUATION

The computer program ‘Dr. Frame’ [4] and an analytical approach developed considering stability functions and residual stress effects are used to determine when the maximum moment will occur at the column ends.

### 2.1. Dr. Frame Analysis

Dr. Frame was used to perform a second order analysis using stability functions with load-dependent stiffness based on the AISC column curve [2]. The procedure used to determine the axial force ratio,  $N_C / N_S$ , to cause the maximum moment to move away from the member ends as a function of end moment ratio,  $\beta_m$ , and slenderness limit,  $\lambda$ , is listed below:

1. A simply supported member with specific section properties is set up.
2. The ‘Second Order Geometric Effects’ and ‘EI Dependency’ are turned on and ‘Resistance Factors’ is turned off.
3. The end moments are applied for a specified  $\beta_m$ , where the magnitude of the moment is not important as long as it is less than the yield moment.
4. A member length is chosen initially so that  $\lambda$  is equal to 0.1.
5. A small axial force is applied and increased gradually until the maximum moment moves away from the member ends. This is the axial force causing the maximum moment to move away from the member ends.
6. Step 4 and 5 are repeated with different member length so that  $\lambda$  increases in increments of 0.1 until a value of 3 is reached. This gives a relationship between  $N_C / N_S$  and  $\lambda$  for a chosen  $\beta_m$ .
7. Steps 3 to 6 are repeated for end moment ratios of 0 and 0.5 to obtain a relationship between  $N_C / N_S$ ,  $\lambda$ , and  $\beta_m$ .

### 2.2. Analytical Model

The analytical model was developed from first principles to independently verify Lay’s equation. The model is based on stability functions which take into account the reduction in member stiffness from axial force due to geometric non-linearity from second-order effects. It is also based on the New

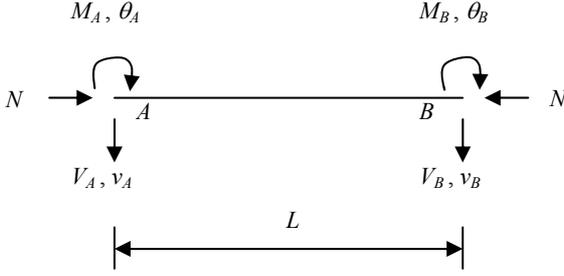


Figure 2: Beam-column member.

Zealand column design curves which consider the reduction in flexural stiffness due to material non-linearity that arises from initial residual stress effects, member out-of-straightness and accidental non-concentric loading. The algorithm was developed in the computer program MATLAB [1]. The analytical procedure used in this study [9] is summarised below.

### 2.2.1. Stability Functions

The stability functions which consider the effect of axial compression force,  $N$ , on the elastic member stiffness for a simply supported column as illustrated in Figure 2, are expressed in Equation 5 where  $V_A$ ,  $M_A$ ,  $V_B$ ,  $M_B$ ,  $v_A$ ,  $v_B$ ,  $\theta_A$ , and  $\theta_B$  are the shear force, moment, lateral displacement and rotation at ends  $A$  and  $B$  respectively.

$$\frac{EI}{L} \begin{bmatrix} f/L^2 & g/L & -f/L^2 & g/L \\ g/L & s & -g/L & r \\ -f/L^2 & -g/L & f/L^2 & -g/L \\ g/L & r & -g/L & s \end{bmatrix} \begin{Bmatrix} v_A \\ \theta_A \\ v_B \\ \theta_B \end{Bmatrix} = \begin{Bmatrix} V_A \\ M_A \\ V_B \\ M_B \end{Bmatrix} \quad (5)$$

where

$$r = \frac{\phi^2 - \phi \sin \phi}{2(1 - \cos \phi) - \phi \sin \phi} \quad (6)$$

$$s = \frac{\phi(\sin \phi - \phi \cos \phi)}{2(1 - \cos \phi) - \phi \sin \phi} \quad (7)$$

$$g = s + r \quad (8)$$

$$f = 2(s + r) - \phi^2 \quad (9)$$

$$\phi = \sqrt{NL^2/EI} \quad (10)$$

If the member is now broken into two sub-members as illustrated in Figure 3 where  $L_1$  is much smaller than  $L_2$  and  $M_A$  is greater than  $M_C$ , then the moment at node  $B$  can be determined as follows:

1. A 6x6 global stiffness matrix is assembled and reduced to a 4x4 matrix as shown in Equation 11 as the support displacements,  $v_A$  and  $v_C$  are zero.
2. For a specified end moment ratio where  $M_A$  and  $M_C$  are known, and the external forces applied at  $B$  are zero, the degrees of freedom,  $\theta_A$ ,  $\theta_C$ ,  $v_B$ ,  $\theta_B$ , in Equation 11 can be solved.
3. These degrees of freedom can be substituted into a sub-member stiffness matrix, such as Equation 12 to determine the moment at  $B$ .

The axial force which causes  $M_B$  to be greater than  $M_A$  is identified as that makes the maximum moment move away from the member ends. A sensitivity study has shown that when  $L_1$  is less than  $0.01L$ , the axial force causing the maximum moment to migrate was not sensitive to  $L_1$  but the

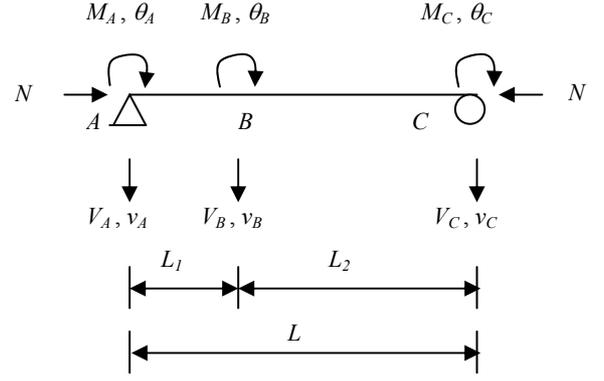


Figure 3: Simply supported beam-column member containing two sub-members and two internal degree of freedoms.

computational time became excessive as  $L_1$  becomes smaller. Consequently,  $L_1$  was set as  $0.01L$  for all the analyses.

$$EI \begin{bmatrix} s_1/L_1 & 0 & -g_1/L_1^2 \\ 0 & s_2/L_2 & g_2/L_2^2 \\ -g_1/L_1^2 & g_2/L_2^2 & f_1/L_1^3 + f_2/L_2^3 \\ r_1/L_1 & r_2/L_2 & -g_1/L_1^2 + g_2/L_2^2 \end{bmatrix} \begin{Bmatrix} \theta_A \\ \theta_C \\ v_B \\ \theta_B \end{Bmatrix} = \begin{Bmatrix} M_A \\ M_C \\ V_B \\ M_B \end{Bmatrix} \quad (11)$$

$$\frac{EI}{L_2} \begin{bmatrix} s_2 & g_2/L_2 & r_2 \\ g_2/L_2 & f_2/L_2^2 & g_2/L_2 \\ r_2 & g_2/L_2 & s_2 \end{bmatrix} \begin{Bmatrix} \theta_C \\ v_B \\ \theta_B \end{Bmatrix} = \begin{Bmatrix} M_C \\ V_B \\ M_B \end{Bmatrix} \quad (12)$$

### 2.2.2. Effective Flexural Stiffness

The inelastic column curve was interpreted as an elastic buckling curve with a reduced flexural stiffness,  $(EI)_i$ , for a given axial force,  $N_C$ , as shown in Figure 4. This reduced flexural stiffness was calculated using the ratio between the effective lengths corresponding to the inelastic column curve and the Euler buckling curve,  $(kL)_i$  and  $(kL)_e$  respectively, according to Equation 13, by setting  $N = N_{OL} = N_C$  in the equations in Figure 4. The value of  $(kL)_e$  was found from the Euler buckling expression in Equation 14, and  $(kL)_i$  was found from the nominal column design curve in the New Zealand steel code.

$$(EI)_i = EI \left( \frac{(kL)_i}{(kL)_e} \right)^2 \quad (13)$$

$$(kL)_e = \sqrt{\frac{\pi^2 EI}{N_{OL}}} \quad (14)$$

The New Zealand steel structures standard contains five nominal column design curves representing different section types, described by section constant,  $\alpha_b$ , as illustrated in Figure 5. These nominal column curves were developed not only considering the initial residual stress effects but also the effect of accidental non-concentric loading and member out-of-straightness. Consequently, the effective flexural stiffness obtained in this study is likely to be more conservative than that considering the initial residual stress effects alone.

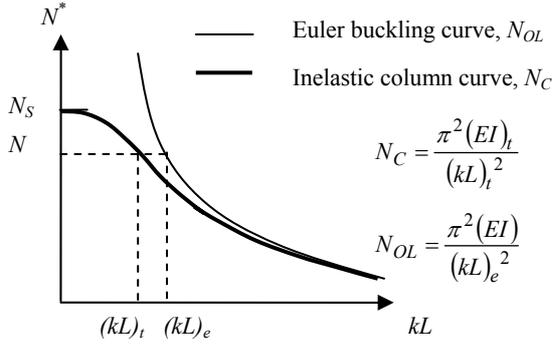


Figure 4: Schematic inelastic and elastic buckling curves for a steel column.

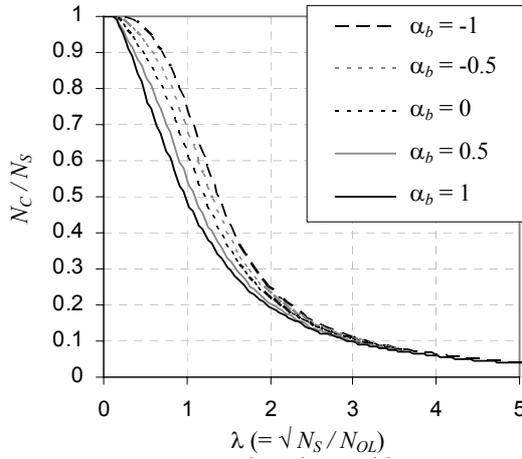


Figure 5: NZS3404 inelastic column design curves.

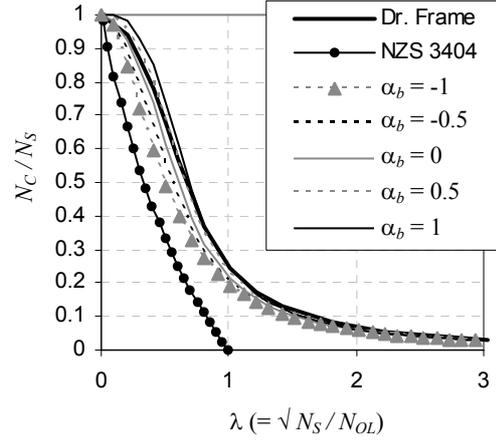
### 2.2.3. Analytical Procedure

The overall analysis procedure for determining the end yielding criteria is iterative. It requires that the flexural stiffness be updated every time the axial force is changed. The procedure is outlined below:

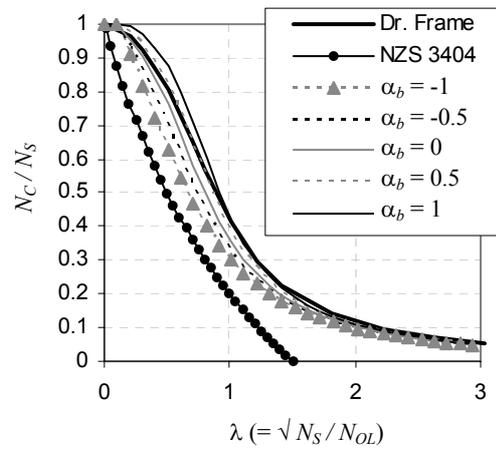
1. A simply supported member with specific properties such as end moment ratio,  $\beta_m$ , slenderness ratio,  $\lambda$ , and section constant,  $\alpha_b$ , is chosen.
2. A small axial force is applied and the effective flexural stiffness,  $(EI)_t$ , is calculated using Equation 13. Iteration is required to obtain the slenderness for a given axial force,  $N_C$ , in the NZS3404 steel code.
3. The effective flexural stiffness is combined with stability functions given by Equations 11 and 12 to find the moment at  $B$ .
4. If the moment at  $B$ ,  $M_B$ , is less than the moment at  $A$ ,  $M_A$ , the axial force is gradually increased and Steps 2 and 3 are repeated until  $M_B$  is greater than  $M_A$ .
5. The process is repeated for different  $\alpha_b$ ,  $\lambda$ , and  $\beta_m$  so that a relationship between the axial force ratio that cause the maximum moment to occur away from the member ends,  $N_C/N_S$ , section constant,  $\alpha_b$ , slenderness limit,  $\lambda$ , and end moment ratio,  $\beta_m$ , can be obtained.

### 3. EVALUATION OF LAY'S EQUATION

A comparison between the Dr. Frame results, analytical results for different section constants,  $\alpha_b$ , and existing seismic



(a)  $\beta_m = 0$



(b)  $\beta_m = 0.5$

Figure 6: Comparison of Dr. Frame, NZS3404 and analysis results for end moment ratios of 0 and 0.5.

provision in NZS3404, for the axial force ratio that causes the maximum moment to move away from the member ends, for different  $\beta_m$  are illustrated in Figure 6. It may be seen that:

1. Lay's equation (from NZS3404) is more conservative than the results obtained from Dr. Frame and the analysis.
2. Dr. Frame results matched well with the analysis. Some difference is expected because Dr. Frame is based on the AISC column curve and the analytical model is based on the NZS3404 column design curves.
3. As  $\alpha_b$  increases, the axial force limit decreases. This is expected, a higher  $\alpha_b$  corresponds to a section with larger stress variations and initial stress effects. Hence, as  $\alpha_b$  increases the ability of columns to carry axial force decreases.

### 4. REVISED END YIELDING EQUATIONS

Empirical equations are proposed to match the  $N_C/N_S$ ,  $\alpha_b$ ,  $\lambda$ , and  $\beta_m$  relationship computed in the previous section. The exponential function given in Equation 15 is used where three constants,  $A$ ,  $B$  and  $C$  vary with section constant,  $\alpha_b$ . The recommended values of these constants for different values of  $\alpha_b$  are given in Table 1.

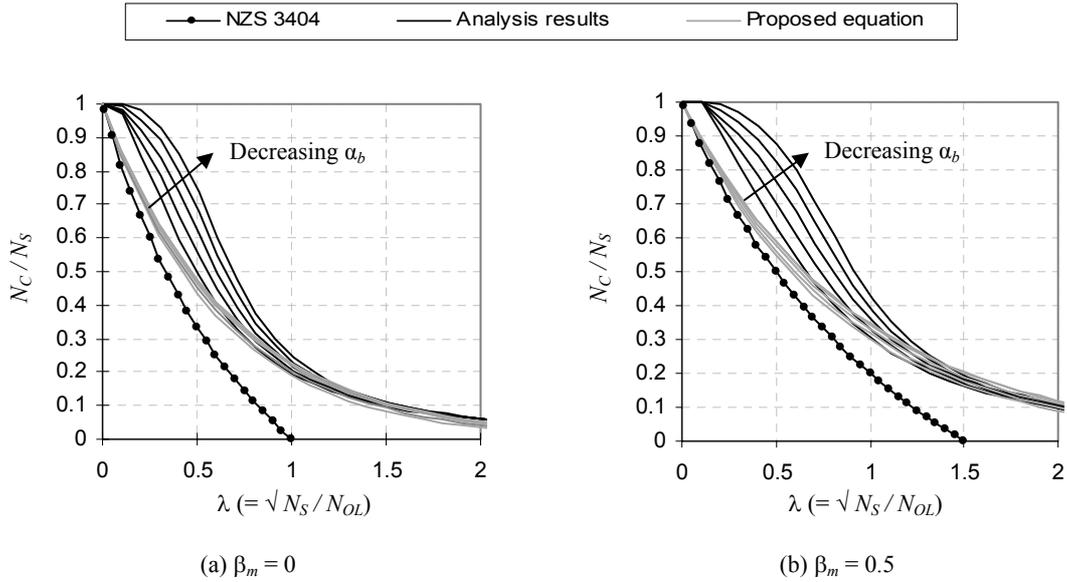


Figure 7: This Axial force limits for different member slenderness and end moment ratios.

$$\frac{N^*}{\phi N_S} \leq \left\{ \frac{A \times (\beta_m + 1)^B}{e^{(C/\beta_m + 1)}} \right\}^\lambda \quad (15)$$

The proposed equations together with the analysis results and the NZS3404 equation are plotted in Figure 7. It may be seen that the proposed equations are much closer to the actual end yielding criteria (EYC) curves and less conservative than the current NZS3404 provision. For columns with axial force ratios greater than 0.5, the proposed equations are still significantly more conservative than the actual values. However, the design axial force ratio in a column seldom exceeds this value and the equations are only slightly conservative in the normal design range.

Table 1. Coefficients for Different Section Types

$\alpha_b$	$A$	$B$	$C$
1	0.235	0.95	0.21
0.5	0.247	0.91	0.19
0	0.263	0.88	0.19
-0.5	0.265	0.92	0.17
-1	0.276	0.87	0.19

## 5. RELEVANCE OF EYC EQUATIONS TO EBFS AND MRFS

The EYC equations developed in this study and the one proposed by Lay were based on the assumption that the ends of the member do not move laterally. That is, they were developed for a braced column where the forces are applied axially along the member. It may be seen in Figure 8 that the second-order moments follow the deformed shape and consequently, it is possible for yielding to occur away from the member ends.

The forces on the columns of a MRF (sway-type frame) are examined below. The deformed shape of the bottom story of

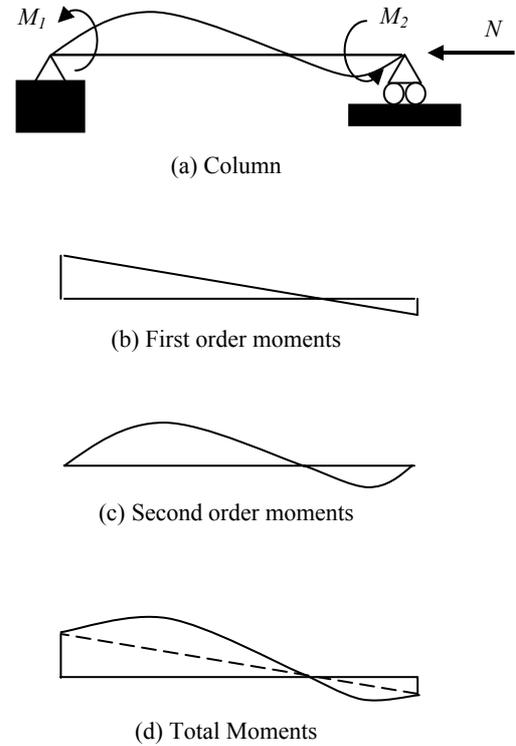


Figure 8: Model use for development of EYC curves.

the MRF, assuming  $M_1 = M_2 = 0.5M_{base}$  and no axial force in the beam, is shown in Figure 9.

The moment diagram associated with the right-hand column is shown in Figure 10. It can be seen that both the first-order and the second-order moments are maximum at its base. Therefore, it is not possible for yielding to occur along the column length. Similar arguments may be made for other moment values at the top of the column which put the column into double curvature. Since the end restraints and the loading configuration are different to the ones used in deriving the EYC equations, the EYC equations are not appropriate for moment-resisting frames in general, where the column deformed in double curvature.

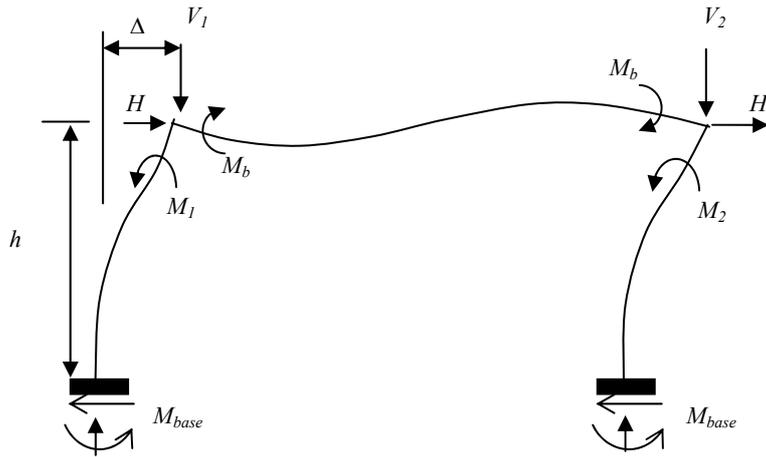


Figure 9: Forces on moment frame in deformed configuration.

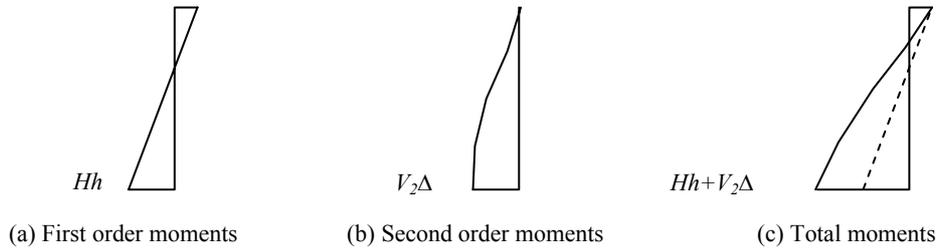


Figure 10: Bending moment profile for the right hand column.

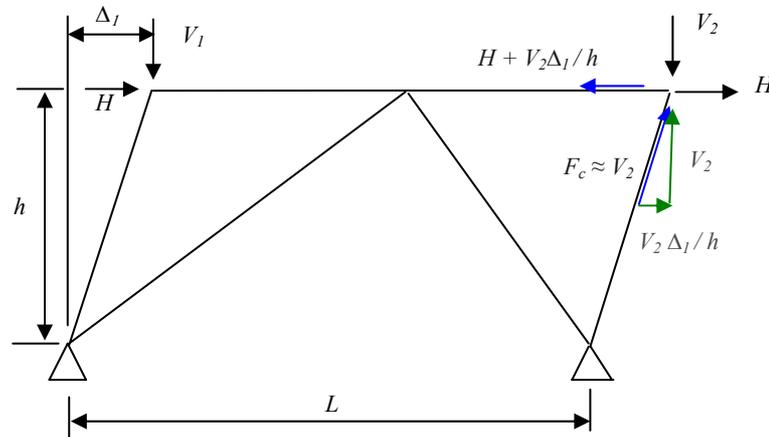


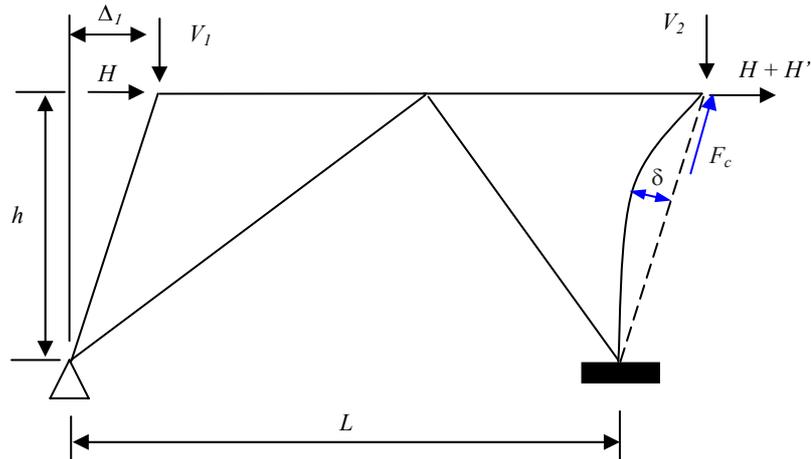
Figure 11: Forces on braced pin jointed frame in deformed configuration.

The forces on a column of an CBF, where the connections are assumed to be pinned are examined. All the members in the bottom story of the braced frame in Figure 11 are assumed to behave like a truss, carrying only axial tension or compression force. It may be seen that the column axial force,  $F_c$ , is equal to  $V_2$  if the drift angle is small. The  $P-\Delta$  force,  $V_2\Delta_1/h$ , is resisted solely by the beam, and the column does not provide any restraint to the  $P-\Delta$  force. This is quite different to the column in MRFs where the column resists its own  $P-\Delta$  force by bending.

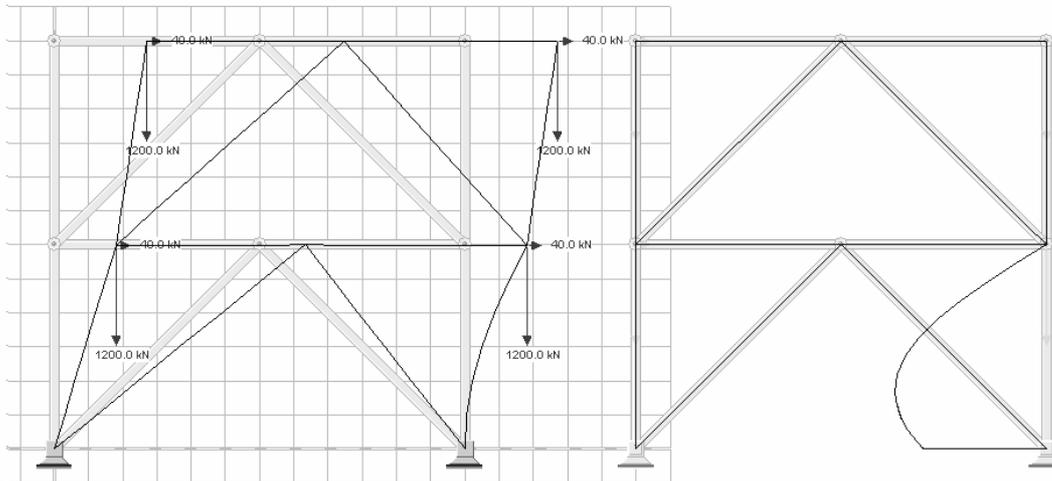
For the same frame as above, if the right hand column is now provided with a small flexural stiffness and fixed at the base, an additional force,  $H'$ , is required at the top of the first story

right hand column as shown in Figure 12 to obtain identical lateral displacements to the truss frame above. In this case, the forces in all members, except the right hand column, are identical to those in Figure 11.

The force  $H'$  depends on the lateral resistance of the right hand column. In the extreme case where  $EI$  of the column tends to zero, the value of  $H'$  also tends to zero, and the force on the column will be  $F_c$  and the deflection of the column from its straight position will be  $\delta$  as shown in Figure 12. In this case, the maximum  $P-\delta$  moment,  $F_c\delta$ , will occur along the member length and the maximum moment can move away from the column ends. In the case where  $EI$  is greater than zero,  $H'$  will be greater than zero, and there will be a moment



**Figure 12:** Forces on braced frame in deformed configuration where all members are pin-jointed except the base of the right hand column is fixed.



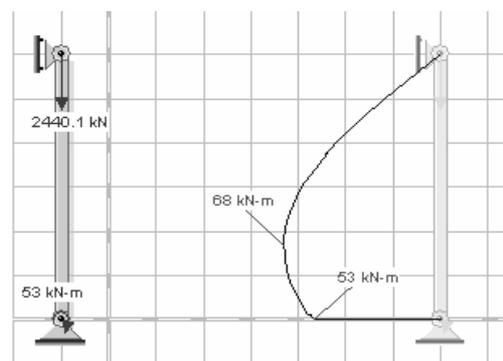
**Figure 13:** Braced frame analysis with moment fixity only at the base of the right hand ground story column [4].

at the base of the column equal to  $H'h$  with a triangular shape up the height in addition to  $F_c\delta$ . For either case, the boundary conditions are identical to that used in the development of EYC equations. Therefore, columns in this braced frame configuration are capable of yielding along their length and EYC equations should be applied for design.

It should be noted that in D-type EBFs, where the active link directly frames into one side of column and does not carry any horizontal force, the columns behave similarly to the ones in MRFs and the EYC equations should not be used for design.

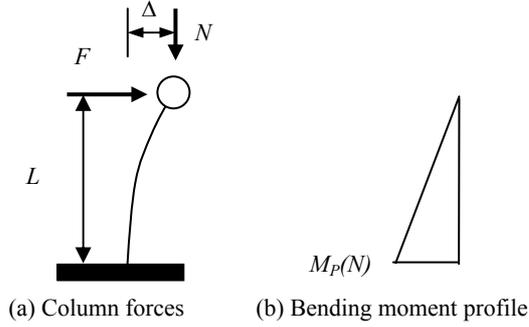
In order to verify that maximum moment may occur along the member length in columns of EBFs and CBFs, a two story CBF was analyzed with Dr. Frame. Initially, the frame was analyzed as a truss considering second-order geometric effects and no moments were obtained in the members. Then, the right hand column in the ground story was provided with a low flexural stiffness and fixed at the base. The deformed shape and moment patterns of the frame are shown in Figure 13. Here, the axial force in the column is 2440.1kN, the moment at the fixed base is  $53.4 \times 10^6$  kNm, and the maximum moment of  $65.5 \times 10^6$  kNm occurred away from the member ends.

To check that EYC is directly applicable to the column in this frame, a single column, restrained against movement at each



**Figure 14:** Dr. Frame analysis for first story right hand column.

end, as was assumed for the development of the EYC equations, is subjected to the same axial force and end moments. The results on the Dr. Frame plot do not show the moment to sufficient precision. Subsequently, the applied moment was magnified by  $10^6$  as shown in Figure 14. The maximum moment obtained in this case was approximately 68kNm. This is close to the value obtained in the braced frame analysis indicating that the column load conditions are similar



**Figure 15:** Schematic column under axial and lateral forces.

to that for the development of the EYC equations, thereby confirming the applicability of the EYC equations to columns in braced frames.

## 6. YIELD CONSIDERATIONS

For yielding to occur away from the column ends, the member must not only violate the proposed EYC equations (Equation 15) but it must also yield in flexure. Columns which do not satisfy the EYC equations but which are not expected to yield do not need to be increased in size to prevent yielding occurring away from the member ends. A simple approach to relate flexural yielding of the member to the axial force limit and the design drift is given below.

Figure 15a shows a schematic diagram of column with length,  $L$ , under axial compression force,  $N$ , and lateral force,  $F$ . The expected bending moment profile from lateral force alone is shown in Figure 15b, where  $M_P(N)$  is the plastic moment capacity reduced for axial force.

For the column to yield in flexure the moment capacity,  $M_P(N)$ , must be less than the moment demand,  $M^*$ , as shown in Equations 16 and 17 where  $K$  is the stiffness of column which varies with the end moment ratio and  $\Delta$  is the lateral displacement.

$$M_P(N) \leq M^* \quad (16)$$

$$= K\Delta L \quad (17)$$

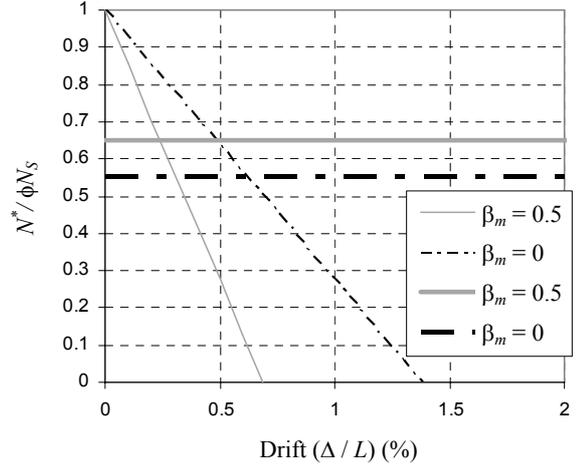
For an end moment ratio of zero,  $K$  may be approximated as  $3EI/L^3$  according to first order flexural analysis and the yielding equation can be written as Equation 18 or 19 where  $\Delta/L$  is the applied drift. Here, for simplicity and convenience, it is conservatively assumed that the column stiffness remains constant (i.e.,  $EI_{eff} = EI$ ), shear deformations and second order effects can be neglected, and a linear moment - axial force relationship specified in NZS3404 can be used.

$$M_P \left( 1 - \frac{N^*}{\phi N_s} \right) \leq \left( \frac{3EI}{L^3} \Delta \right) L \quad (18)$$

$$\frac{N^*}{\phi N_s} \geq 1 - \frac{3EI}{M_P L} \left( \frac{\Delta}{L} \right) \quad (19)$$

### 6.1. EXAMPLE

A 310UC240 section is chosen with the following properties,  $\alpha_b = 0$ ,  $M_P = 1062 \text{ kNm}$ ,  $I = 642 \times 10^6 \text{ m}^4$ ,  $L$  is 5m so  $\lambda$  is 0.3885. The axial force limit for the EYC is plotted in Figure 16 via the thick horizontal line and the yield requirement is plotted using the thin diagonal line. The region



**Figure 16:** Indication of plastic hinges occurring away from the member ends for different  $\beta_m$ .

on the right of the thin diagonal lines corresponds to yielding for different end moment ratios. It may be seen that the member yields at a lower drift when it is in double curvature (i.e. when  $\beta_m > 0$ ) because the stiffness for the member in double curvature is higher. The region above the thick horizontal lines corresponds to the maximum moment occurring away from the column ends for different end moment ratios. It may be seen that a higher force is required to move the maximum moment away from the column ends when it is in double curvature. This matches with the behaviour observed in Figure 6 for different  $\beta_m$  values. Based on Figure 16, the design region for an end moment ratio of zero is that below the horizontal dashed line or to the left of diagonal dashed line. The design region for an end moment ratio of 0.5 is that below or to the left of the two continuous lines.

## 7. EBF COLUMN BEHAVIOUR DURING EARTHQUAKE SHAKING

Time-history analysis was carried out with RUAUMOKO [3] to examine the column end moment ratios and to determine whether plastic hinges would form away from the end of columns in EBFs. A total of 14 earthquakes, obtained from PEER Strong Motion Database [10], were scaled according to the New Zealand Loading Standards NZS1170.5:2004 [13] assuming buildings are located in Wellington. A 7 story EBF, designed by Steel Construction New Zealand, as described by Peng [9], was used. The capacity design method proposed by Hyland [5] was used to check the structural strength. In addition, the first story columns were also checked against the EYC equations developed in this study and the one proposed by Lay.

The columns satisfied the capacity design method but failed under the current NZS3404 seismic provision with  $\beta_m = 0$  and the proposed EYC equations (Equation 15). The end moment ratio of the bottom story columns, under compression, when either i) the maximum moment occurred or ii) the maximum drift occurred in compression is plotted in Figure 17 for the 14 different earthquakes. It may be seen that the end moment ratios are between 0.0 and 0.4 indicating that the columns are in double curvature.

The axial force ratio and drift at these critical points are also plotted against the limiting line for yielding and EYC as shown in Figure 18. The results from RUAUMOKO show that the columns remained elastic in all 14 earthquakes. However, one point in Figure 18 lies slightly outside the elastic region

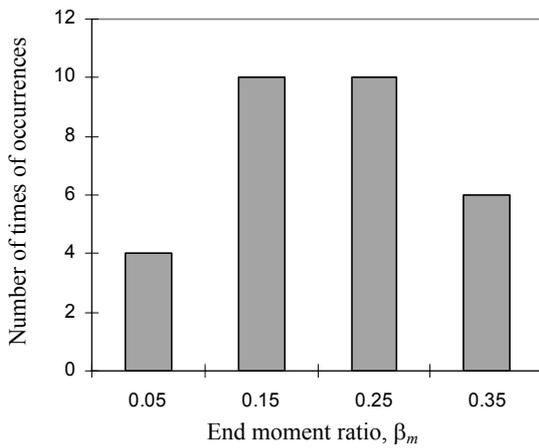


Figure 17: End moment ratio,  $\beta_m$ , at the maximum drift and maximum moment.

calculated using the yield equation approximation, Equation 19. This is due to the conservative assumption associated with the approximate yield line as mentioned earlier.

## 8. CONCLUSIONS

The accuracy and applicability of the seismic provision in the New Zealand steel structures standard, NZS3404, to discourage yielding to occur away from the column ends in steel frames was investigated. It was found that:

1. The NZS3404 provision conservatively estimates the axial force corresponding to maximum moment occurring away from the column ends. This currently results in column sizes being larger than necessary.
2. New end yielding criteria, EYC, equations were proposed to estimate the axial force corresponding to maximum moment occurring away from the column ends. This equation has been approved for inclusion in the 2007 amendment to NZS3404.
3. The EYC equations are generally applicable to columns in both eccentrically and concentrically braced frames. However, the code requirements to consider EYC for columns in moment frames or in braced frames where the active link frames into the column are generally not appropriate.
4. A simple approach to relate the axial force limit and the design drift to flexure yielding of a column is described. This can be used in conjunction with the proposed EYC equations. An example of the assessment of a column is also provided.
5. Based on the analysis of one 7 story EBF, an end moment ratio,  $\beta_m$ , of zero is found to be appropriate for use with the EYC equations.

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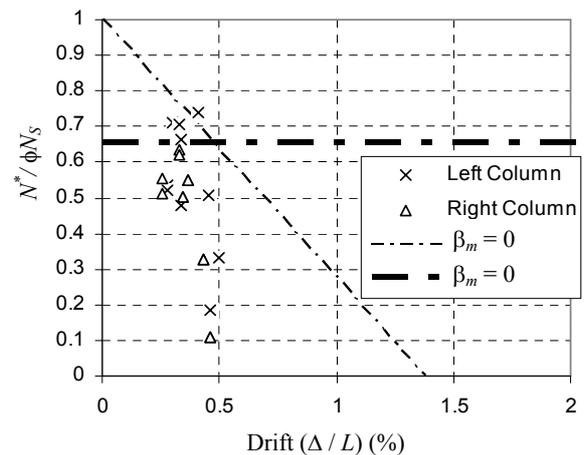


Figure 18: Axial forces vs drifts at maximum moment.

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