

EFFECT OF VERTICAL DISTRIBUTION OF MASS AND TRANSLATIONAL STIFFNESS ON DYNAMIC ECCENTRICITIES FOR A SPECIAL CLASS OF MULTI—STOREY BUILDINGS

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SUMMARY

The results from analyses which considered dynamic torsional coupling in single storey buildings have commonly been extrapolated to a special class of multi-storey buildings. Recently developed probabilistic methods are used in this paper to study the influence of various realistic vertical mass and stiffness distributions on the characteristics of torsional coupling in multi-storey buildings. Qualitative and quantitative differences between the results for single and multi-storey models are identified.

INTRODUCTION

The key parameters affecting the seismic response of torsionally coupled buildings have been well established from consideration of idealised single storey structural models [1,2,12]. These studies were limited by the assumption that their results are also valid, in both qualitative and quantitative terms, for a certain special class of multi-storey buildings [5,6,10,11]. The distribution of the structural properties of the special class of buildings is such that the loci of the storey centres of mass and rigidity lie on two vertical straight lines, and that the distributions of the storey translational and torsional stiffnesses are proportional. Aspects of the validity of the extrapolation from results of single storey models to multi-storey cases are considered in this paper. In particular, the effect of various realistic vertical distributions of mass and stiffness are examined, with a view to establishing a combination of these distributions which may be used for more extended parametric studies of torsional coupling in multi-storey models with vertical irregularities [3].

The results for an idealised 20-storey building model with the above eccentricity characteristics are presented in this study. A further detailed parametric study including evaluation of appropriate design provisions is in preparation [4]. References [3] and [4] include comparative results for single and multi-storey buildings with 5,10,15 and 20 floors. The effect of irregular vertical distributions of static stiffness eccentricities are also considered [3].

In the following sections the analytical techniques are summarised and the salient dynamic properties are specified. A comparison of torsional coupling effects for representative 20-storey buildings are presented and the implications for further parametric studies are considered.

Ground Motion Model

Probabilistic normalised power spectrum methods for seismic analysis have been extended [7] to allow for uncertainty in ground motion intensity and its correlation with ground motion frequency content. This new method involving the intensity-correlated probabilistic power spectrum (ICPPS) was used [8] to confirm that earlier findings relating to torsional coupling effects in single storey models are also valid for the generalised and more realistic earthquake representation offered by the probabilistic approach. In this study the ICPPS method is employed to establish the extent to which the general trends previously observed for simple single storey systems can be extrapolated to a representative 20-storey structure with and without vertical mass and stiffness irregularities.

Analytical Model

In this paper, a representative shear beam building model of 20 storeys ($N = 20$) is idealised by a partially symmetric multi-storey structure (Fig. 1) with rigid floor diaphragms. The static eccentricity, e_i , is defined as the distance between the centres of mass and stiffness at that floor; the latter point may (for the special class of buildings studied herein), or may not be the same as the centres of rigidity [9,11] and rotation. For this special class of building, e_i equals a constant e ($i = 1, 2, \dots, N = 20$), where e is a characteristic value for defining the structural eccentricity of the building. As illustrated in Fig.1, the centres of stiffness need not lie on the same vertical line. It is noted, however, that the emphasis in the present paper is on buildings with centres of stiffness and centres of mass lying on two parallel vertical lines. The i th floor translational stiffness K_{xi} and torsional stiffness $K_{\theta i}$ taken about the centre of mass (or $K_{\theta si}$, taken about the centre of stiffness), together with the static stiffness eccentricity e_i , are defined for each level on a floor-to-floor basis.

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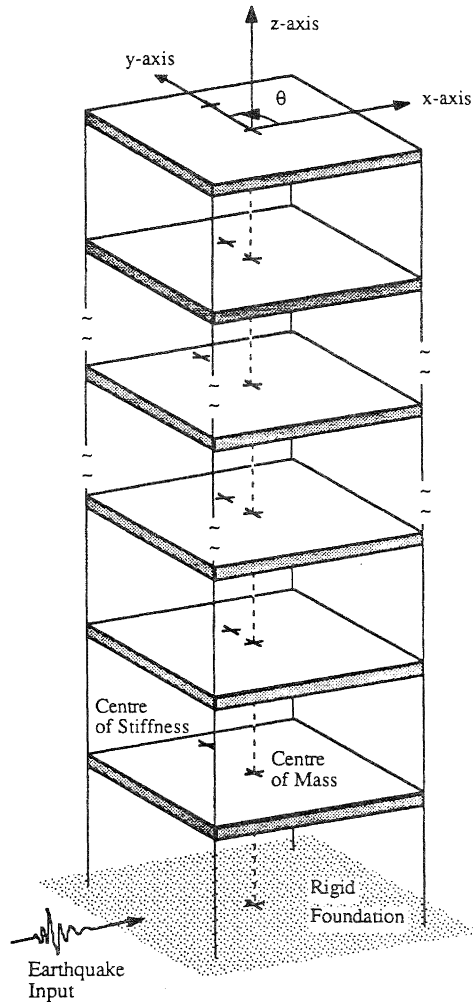


Figure 1 Idealised multi-storey building model.

The building model shown in Fig.1 is subjected to a unidirectional earthquake base motion which is applied uniformly (across the y -axis of the structure) in the x -direction. Consequently each floor i has two degrees of freedom, a translation in the x -direction, u_{xi} , and a rotation, $u_{\theta i}$, about the centre of mass for that floor.

Normalised Mass and Stiffness Distributions

For this study, the vertical distributions of floor mass and storey stiffnesses are expressed in the following dimensionless forms: $\mu_i = m_i/m_1$, where m_i is the lumped mass at the i th floor; $\sigma_{xi} = K_{xi}/K_{x1}$ and $\sigma_{\theta i} = K_{\theta i}/K_{\theta 1}$, where K_{xi} and $K_{\theta i}$ are the translational and torsional storey stiffnesses, the latter defined about the floor centre of mass. For this special class of building, the proportional stiffness characteristics can be expressed as $\sigma_{xi} = \sigma_{\theta i}$ ($i = 1, 2, \dots, N = 20$). It is important to note that the assumed equality between σ_{xi} and $\sigma_{\theta i}$ leads to the result that the parameter $R_{\theta i}$, as defined in the following section, is equal to the ratio of the uncoupled torsional frequency to the uncoupled translational frequency, for corresponding pairs of all the $2N$ modes of the N -storey building.

System Parameters

The parameter $e_r = e/r$ is the characteristic static eccentricity ratio, where r is the mass radius of gyration of each floor, taken about the centre of mass. For the equivalent uncoupled multi-storey building, $e_r = 0$ for all floors. The frequency parameters $f_{x1} = (1/2\pi)(K_{x1}/m_1)^{1/2}$ and $R_{\theta 1} = (K_{\theta 1}/r^2 K_{x1})^{1/2}$ are the uncoupled translational frequency and the corresponding uncoupled torsional to translational frequency ratio, respectively, and are defined for the first storey assuming that it is, hypothetically, isolated from the rest of the building. A realistic numerical value for the fundamental natural frequency f_{xM} of the multi-storey building is prescribed by the empirical codified relationship [13] $f_{xM} = (10/N)$ Hz.

The response parameter of primary interest in the study of torsional coupling effects in the elastic range of earthquake structural response is the dimensionless ratio of the mean peak coupled to uncoupled torque response $T_{Si}/eV_{\theta xi}$, where $V_{\theta xi}$ is the dynamic shear force for storey i of the equivalent uncoupled multi-storey building, and T_{Si} is the dynamic torque for storey i , calculated about the centre of stiffness. For floor i , the response ratio given above represents the dynamic amplification of eccentricity at that floor.

The structural parameters which influence the response parameter defined above can be reduced [3,7] to e_r , $R_{\theta 1}$, f_{xM} , μ_i and σ_{xi} , assuming that the modal damping is constant for all modes. The damping is taken to be 5% of critical damping, a value which is representative of prototype buildings of the type studied herein.

In this study, the values of e_r , $R_{\theta 1}$ and f_{xM} (for $N = 20$) are taken to be 0.1, 1.1 and 0.5 Hz, respectively. The results of analyses of single storey buildings [1,12] have shown that this combination of e_r and $R_{\theta 1}$ leads to significant torsional coupling effects, particularly in relation to the displacements of critical edge elements of the structure resulting from a combination of dynamic shear and torque loadings.

Effect of Vertical Irregularities

Definitions

To investigate the effect of the vertical distribution of storey mass, μ_i , on the amplification of storey dynamic eccentricity, $T_{Si}/eV_{\theta xi}$, for the 20-storey model, the translational stiffness distribution, σ_{xi} , is equated to unity. Three realistic vertical mass distributions are considered (see Fig.2).

It is noted that the factor of 5, by which the mass of a heavy floor in distributions (2) or (3) is greater than the mass of a typical floor, represents an extreme case of vertical irregularity.

To examine the effect of variation in the vertical distribution of translational stiffness, σ_{xi} , on the amplification of storey dynamic eccentricity for the representative 20-storey model, a uniform mass distribution is assumed and three translational stiffness distributions are considered (see Fig.3).

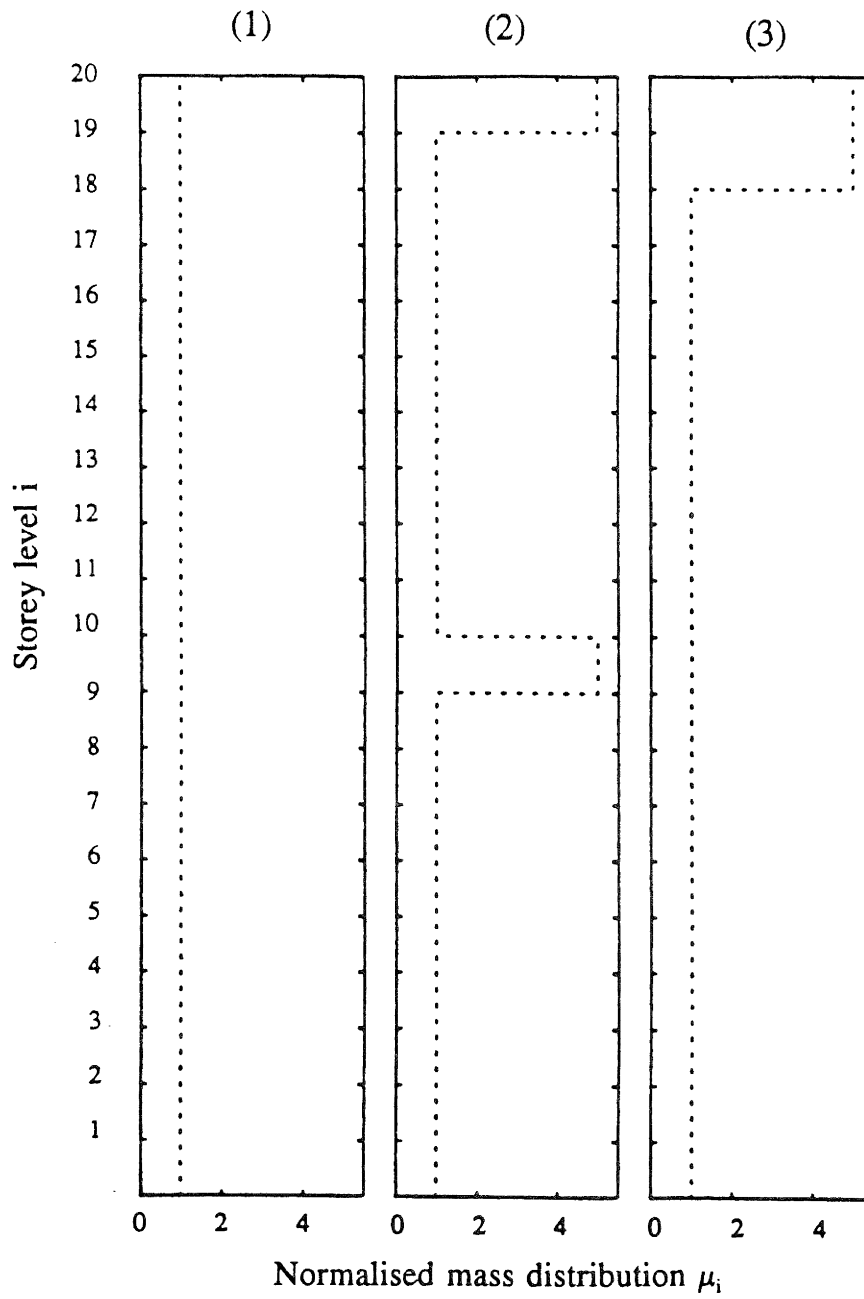


Figure 2 Vertical distributions of mass for a 20-storey building;
(1) uniform
(2) heavy floors at the 10th and 20th levels,
and
(3) heavy floors at the 19th and 20th levels.

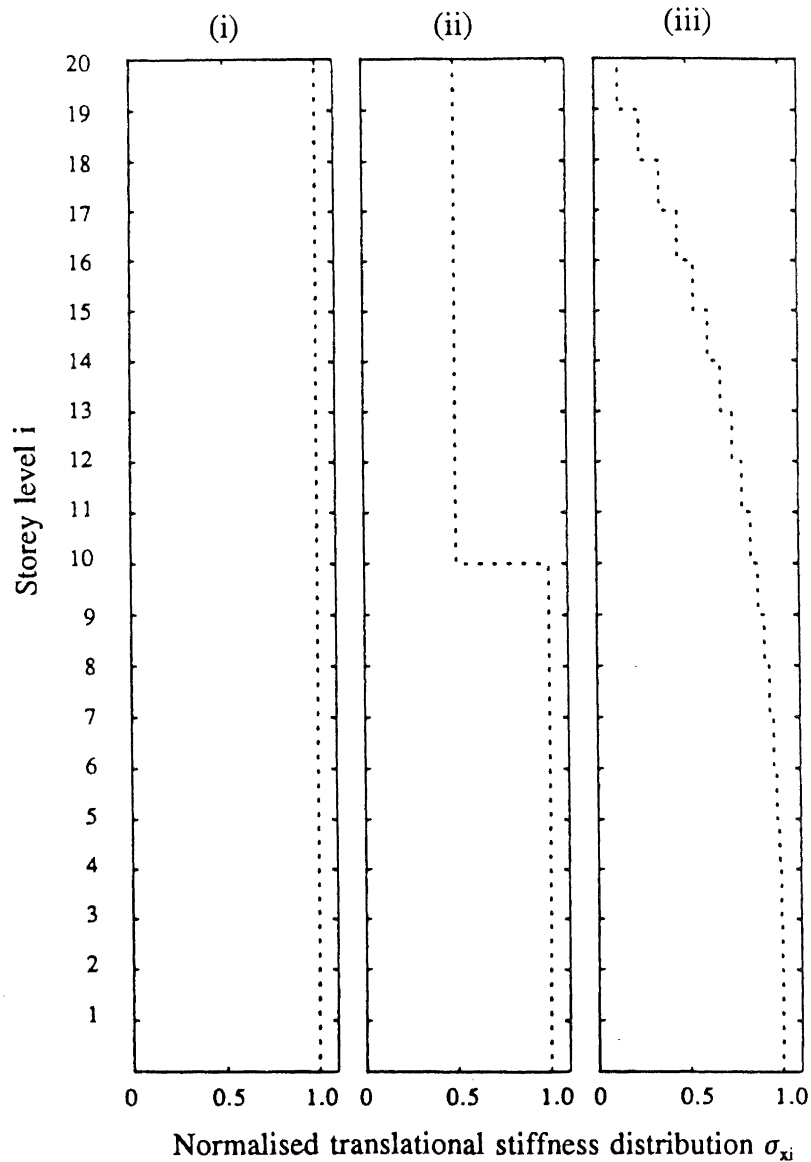


Figure 3 Vertical distributions of translational stiffness for a 20-storey building;
 (i) uniform
 (ii) one-step stiffness reduction
 (iii) gradually tapered

Results

The normalised storey stiffnesses, σ_{xi} , for distribution pattern (iii) are estimated by consideration of the conditions of static equilibrium for the uncoupled building model. Using the vertical distribution recommended by the Uniform Building Code, 1985 version [13] for the equivalent static seismic forces, and imposing a condition of equal inter-storey drifts, the vertical distribution of translational stiffness can be expressed [7] in terms of the codified distribution coefficients for the design lateral forces acting at the various floor levels.

The variation of the dynamic amplification of eccentricity with storey level for the three mass distribution patterns is shown in Fig.4(a). The results are compared with the values obtained from analysis of the equivalent single storey building model, having the same values of e_r , R_n and damping. The uncoupled translational frequency of the equivalent single storey model is taken to be f_{x1} , as defined above. Fig.4(b) shows the amplification factors expressed as percentages of the values obtained for the uniform distribution.

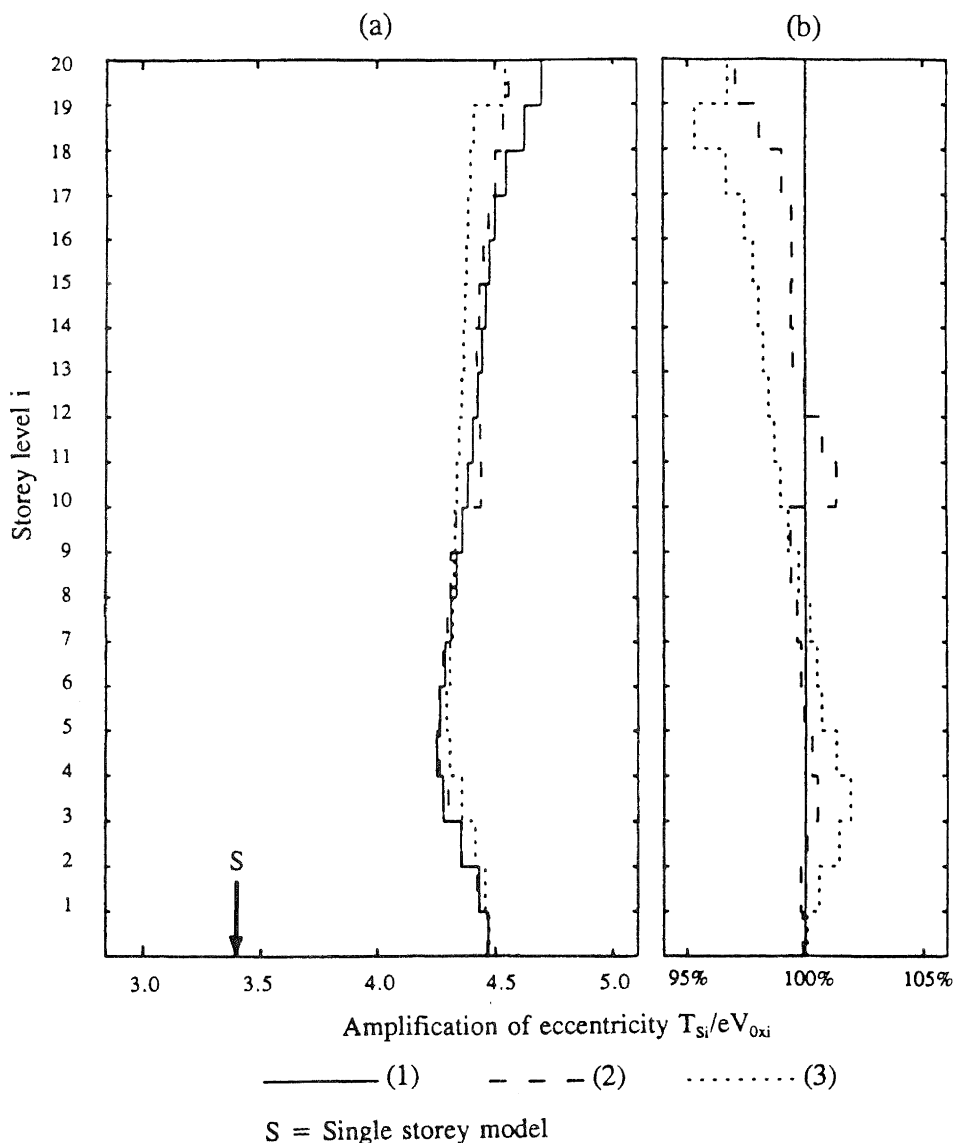


Figure 4 Variation of the dynamic amplification of eccentricity with storey level, given as
 (a) actual values for the three mass distributions and for the equivalent single storey model,
 (b) a percentage of the values for the uniform distribution (1)

Fig.4(a) indicates that the dynamic eccentricity distribution patterns are essentially the same for the three cases. The maximum dynamic eccentricity amplification for the uniform mass distribution (at the top floor) is greater than the maxima for the two irregular distributions. The accuracy of the single storey approximation varies with floor level. Clearly this approximation significantly underestimates the dynamic eccentricity amplification for this representative multi-storey building. This aspect is further examined in reference [3]. Fig.4(b) shows that the percentage difference of the results for mass distributions (2) and (3) are within +2% to -5% of those for the uniform distribution (1).

For all three stiffness distributions the maximum dynamic eccentricity amplification (Fig.5(a)) occurs at the top floor, with the gradually tapered distribution giving the largest maximum value. Again, the single storey approximation significantly underestimates the eccentricity amplification. Fig.5(b) shows that the dynamic eccentricity amplification for distributions (ii) and (iii) is within +2% to -2% of that for distribution (i), throughout the height of the building.

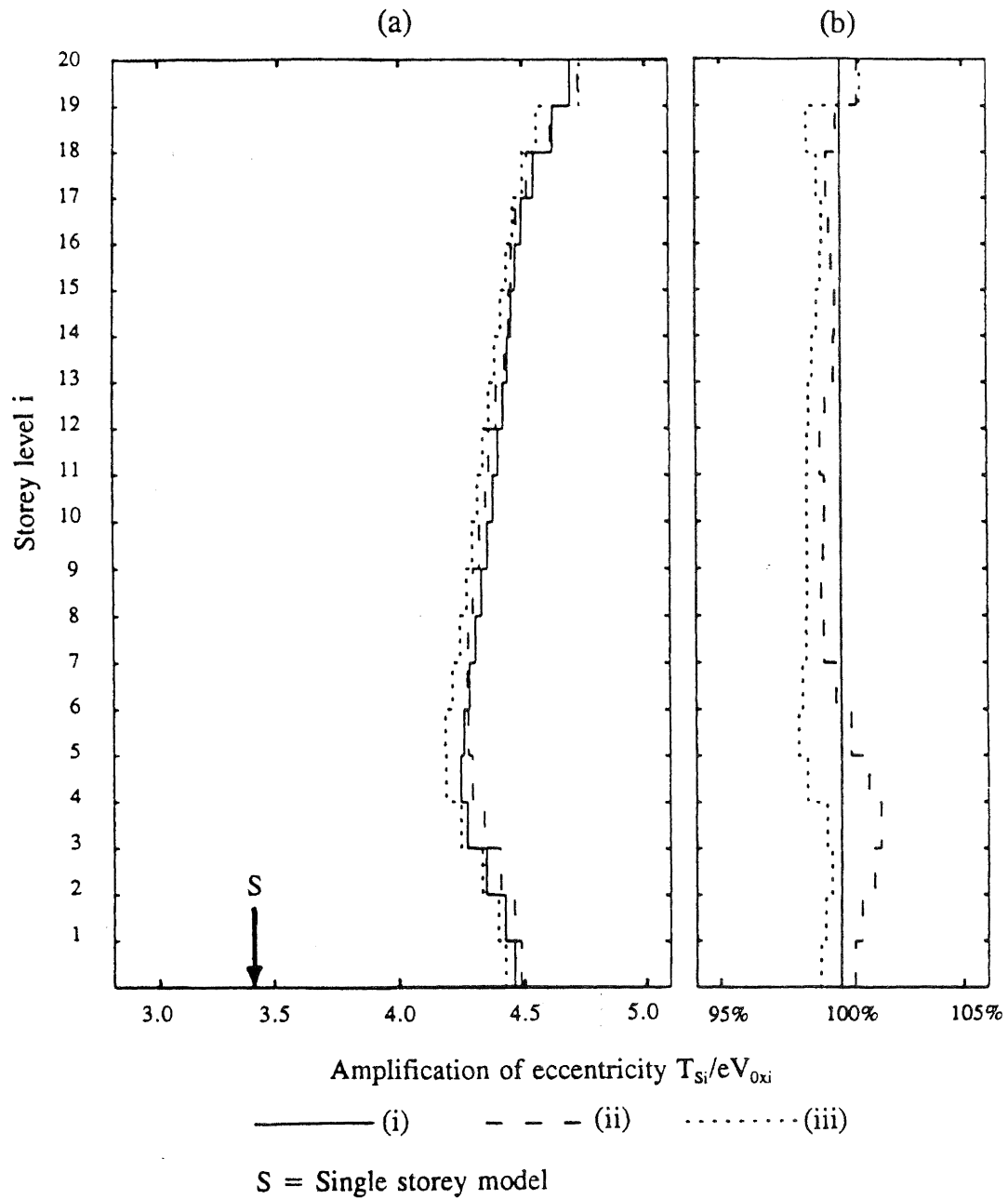


Figure 5 Variation of the dynamic amplification of eccentricity with storey level, given as:

- (a) actual values for the three translational stiffness distributions and for the equivalent single storey model,
- (b) a percentage of values for the uniform distribution (i)

It is evident that the variations in the normalised vertical distributions of mass and translational stiffness (which influence the spacing of the higher translational and torsional natural frequencies [7] do not have a significant effect on the amplification of the torsional response. It is therefore reasonable for the purpose of further parametric studies [3,4] to adopt the uniform mass and the gradually tapered stiffness distributions as a combination which is realistic for many code-designed buildings.

CONCLUSIONS

- 1) The similarities between the seismic torsional response of the special class of multi-storey building and the corresponding single storey models are only qualitative. The contribution of the higher translational and torsional modes for the multi-storey model can, in some cases, lead to significant numerical differences.
- 2) The distribution of torsional coupling effects is not uniform over the height of the building. The coupling effects in a given storey are dependent (amongst other factors) on its position in the building, the top floor exhibiting the most pronounced effects.
- 3) It is reasonable for the purpose of parametric studies of torsional coupling in multi-storey buildings to adopt the combination of a uniform vertical mass distribution and a gradually tapered vertical stiffness distribution.

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