SYNOPSIS

This article reviews, at an elementary level, the ways in which information from strong-motion earthquake records may be presented. The various methods of presentation are illustrated with reference to the strong-motion records obtained at Pacoima Dam, in the San Fernando earthquake of 1971. As acceleration response spectra from the basis of most codes for the design of earthquake resistant structures, the historical development of response spectra is traced from the initial concept. Simplification of presentation by the use of 'pseudo' response spectra, and the use of spectra to define earthquake intensity are outlined.

STRONG-MOTION RECORDS

It was after the 1906 San Francisco earthquake that engineers became fully aware of the necessity to design buildings to resist earthquake forces. It was recognised that strong-motion records of actual earthquakes were required as design information. At that time, seismologists were making teleseismic records, but these gave no indication of the nature of the motion in the epicentral area.

Strong-motion devices were set up, mostly in California, but it was not until 1933 that the first useful record was made, at Vernon. Because of delay in the starting device, and over-sensitivity, the initial part of the record was unreliable. It was, however, an encouraging start. Since then, many strong-motion records have been made, the best known being at El Centro, 1940, which recorded the strongest acceleration then measured (0.32g). This has since been exceeded at several sites. A record at Pacoima Dam in the San Fernando earthquake of 9 February 1971 which exceeded 1g will now be considered as an example. The instrument was sited on a rock spine adjacent to the dam abutment. Its location may possibly have influenced the recorded motion.

Figure 1, after Trifunac and Hudson, shows the acceleration records for the three orthogonal components at this site. In theory, these do not fully describe the motion at the site for two reasons; rotational components about the three axes are not measured; and the initial part of the record is missing as some motion is required to trigger the recorder. These are minor matters, however, and all the information of importance is embodied in these three traces.

The maximum acceleration in any direction is the parameter most readily obtained from an acceleration record. The maximum horizontal acceleration is likely to have occurred in a direction other than those in which components were measured. The resultant may be evaluated to find the true maximum. By integration, the components of velocity and displacement of the ground surface at the instrument site may be readily determined. These are shown in Figure 2 for the S16°E component of the Pacoima record. If this is done without any corrections being made, it is found that the ground surface velocity does not return to zero at the end of the earthquake record. A correction has to be made, just one of many which must be applied to recorded accelerograms. Correction techniques have become more refined over recent years, and the methods used at the California Institute of Technology provide an example of current practice.

From inspection of the velocity and displacement histories it is clearly apparent that the predominant frequency for velocity is much lower than for acceleration, while that for displacement is lower still.

THE RESPONSE CONCEPT

In the 1930's earthquake-resistant design was introduced into New Zealand following the Napier earthquake and buildings were designed to resist a horizontal acceleration, applied uniformly over the height of the structure. It is probable that ideas of dynamic action of structures had not been widely accepted by engineers at that time, and it may have been considered that the structure should be designed to simply resist a certain maximum horizontal ground acceleration, independent of building period. This approach would be theoretically valid, of course, for a perfectly rigid structure. From that viewpoint, the only information of value obtainable from a strong-motion record was the maximum horizontal acceleration.

As it became more generally accepted that a structure behaves under dynamic loading as a complex multi-degree-of-freedom system, the designer's problem became more difficult. He now wishes to know how each part of the structure will...
respond to the ground-motion from a certain earthquake. The information required is the maximum value of bending moment, shear and axial force at many sections of the structure. Only recently, since the advent of fast digital computers, has it been possible to analyse the problem in this way and such methods are justified only for unusually large or important structures.

The seismologist Benioff\(^{(4)}\) in 1934 considered the response to earthquake motion of very simple systems each consisting of a spring and a mass. The concept no doubt arose from his use of seismographs which basically take this form, with some damping, however.

In his own words, "Suppose we substitute for the engineering structure a series of undamped pendulum seismometers having frequencies ranging from the lowest fundamental frequency of engineering structures to the highest significant overtones. During one earthquake each component seismometer would write a characteristic seismogram. Plotting the maximum recorded deflection of each pendulum against its frequency we obtain a curve which may be termed the undamped pendular spectrum of the earthquake".

In present-day terminology this would be called a displacement spectrum for zero damping. He then suggested that the area beneath this curve could be used as a measure of the destructiveness of the earthquake.

The initial concept was soon extended to include velocity and acceleration response spectra, in which some damping of the single-degree-of-freedom oscillators could be included. By mathematical analysis, or by the use of a mechanical or electrical model, the time history of displacement and velocity of the mass relative to the base (which moves with the ground surface) and of the absolute acceleration of the mass may be obtained. These quantities are independent of the actual values of mass \((m)\), spring stiffness \((k)\) and the constant \((c)\) of the viscous damping device.

They are dependent only on -

1. the natural period \(T = 2\pi \sqrt{\frac{m}{k}}\) and
2. the damping ratio \(\zeta = \frac{c}{2\sqrt{km}}\)

It is only the maximum values which are of interest from these time histories. The analysis must be repeated for a considerable number of periods within the required range (say 0.1 to 10 seconds). If \(x\) denotes the displacement of the mass relative to the base, and \(y\) the absolute displacement of the ground surface, then -

- the relative displacement \((S_d = \text{max}|x| \text{ vs. } T)\),
- the relative velocity \((S_v = \text{max}|\dot{x}| \text{ vs. } T)\) and
- the absolute acceleration \((S_a = |\ddot{x} + \ddot{y}| \text{ vs. } T)\)

may be plotted, for a given damping ratio (sometimes referred to as the spectral damping ratio) in each of the single-degree-of-freedom oscillators.

Response spectra thus provide one method of conversion from the "time domain" (as in Figure 2) to the "frequency domain" and indicate the frequency (or period) content of the original record. Some information is lost, however. Response spectra give no indication of the duration of strong motion, nor the number of cycles (above a certain level) that one oscillator, of a particular period, would experience.

It is of interest to note the limiting values of these quantities at very low periods (that is, for a very rigid structure) and at high period values (corresponding to a very flexible structure):

As \(T \to 0\), \(S_d \to 0\)
\(S_v \to 0\)
\(S_a \to \text{maximum ground acceleration}\)

As \(T \to \infty\), \(S_d \to \text{maximum ground displacement}\)
\(S_v \to \text{maximum ground velocity}\)
\(S_a \to 0\)

METHODS OF ANALYSIS

The methods by which these analyses were made are interesting historically. Before the advent of computers, mechanical models were used. Biot\(^{(5)}\) in 1941 described a torsion pendulum used for this purpose which produced a voltage proportional to the spectral value of acceleration for the period of the system. The experiment had to be repeated many times to obtain a complete spectrum.

The next development was to use an electrical analogue. The acceleration record was used to make a function generator which produced a voltage proportional to the spectral acceleration. This was fed into an inductance-capacitance circuit exactly analogous to the single-degree-of-freedom oscillator. Maximum values of electrical measurements gave points on the response spectrum. This was the standard method about 1953\(^{(1)}\).

By 1962 the digital computer had become competitive. Hudson\(^{(6)}\) then reported that 200 points could be computed in seven hours, using a Burroughs 220 machine. The increase in speed and capacity of computers has been so great in the last decade that a Burroughs 6700 (for example, that at the University of Auckland) now computes the same number of points in 2-3 minutes. As
part of the computer analysis, results may be automatically drawn using a graph plotter or, with reasonable accuracy, using the line printer.

**PACOIMA DAM RECORD**

Response spectra for the S16°E component of the San Fernando earthquake of 9 February, 1971 will now be considered in more detail. The computed spectral response values may be plotted against frequency or, as is more commonly done, against period. Figure 4 shows the velocity spectra, for 0, 5% and 20% spectral damping, plotted in both forms. When plotted against period, the graph gives an accumulation of oscillations in the low period range (less than one second) too numerous to plot. To avoid this, some authors prefer to plot against frequency(7). Structural engineers are, however, more accustomed to working in terms of period rather than frequency. The present author believes that the use of a logarithmic scale for period provides a reasonable compromise and further spectra will be presented in this form.

It will be noted from Figure 4 that increased spectral damping gives lower response values, as might be expected, and smoother curves. For a general indication of the frequency content of ground motion, 5% spectral damping is commonly used (that is, the heavy lines in Figure 4).

Figure 5 shows (for the Pacoima S16°E component) relative displacement, relative velocity and absolute acceleration response curves with 5% spectral damping (full lines). Now the difficulty in determining the "predominant period" of the earthquake motion becomes apparent. This term appears frequently in the literature without adequate definition. For maximum relative displacement, it is about 6 seconds; for maximum relative velocity, 1.2 seconds; and for maximum absolute acceleration, 0.4 seconds for the record analysed. It is the author's opinion that the term "predominant period" should always be qualified by the phrase "with respect to acceleration" (or velocity, or displacement as the case may be).

**USE OF RESPONSE SPECTRA**

The most direct application of this information is to evaluate the response of a single-degree-of-freedom structure. An elevated water tank on a single supporting column might approximate this. (we must suppose the tank to be enclosed and full so that water movement does not complicate the problem). If the structure has a natural period of 1 second, with damping 5% of critical and is constructed so that it remains elastic throughout this disturbance, the tank would be displaced 0.3m relative to the base, would attain a maximum velocity of 1.2m/s, and its maximum absolute acceleration would be 12 m/s² (1.2g) in the direction S16°B. Such a structure is unlikely to be found in practice.

Seismic coefficients used in the design of earthquake resisting structures were based initially on acceleration response spectra. Assuming a value of structural damping, averaging the spectra for several earthquake records (all of much lower intensity than the Pacoima record) then dividing the acceleration values by a ductility factor of about 4 gave the design accelerations. These curves have subsequently been modified in the light of experience.

Structural engineers are familiar with the use of seismic coefficients, in the "equivalent static analysis" method, (as it is termed in the revised N.Z. Code of Practice) wherein horizontal forces are distributed over the height of the structure in a manner which approximates the effects of dynamic loading. For modal analysis, the effects of the first few modes (say three) may be separately evaluated, then combined in a suitable way.

**OTHER RELATIONSHIPS**

Housner(8) sought to provide a quantitative measure of intensity of earthquakes, as a more accurate means of comparison than is provided by the commonly used (but subjective) scales such as the Modified Mercalli (MM). While Benioff had suggested the use of the displacement of the structure, Housner used the velocity spectrum, and defined the intensity as the area beneath the velocity spectrum (plotted against period on a linear scale, and for a stated value of spectral damping) between 0.1 and 2.5 seconds. This range includes the fundamental periods of most buildings. The area is shown shaded in Figure 4 for 20% damping. The intensity so obtained, being the product of velocity and time, has the dimension of length. Other writers(9) also have suggested that the damage sustained by structures (or the potential damage in a postulated earthquake) may be more closely related to the velocity spectrum than to the acceleration spectrum of the earthquake. This question remains a topic for discussion(10).

Having determined the displacement response spectrum for an earthquake record, approximations can be used for the relative velocity and absolute acceleration spectra. If S_d is the displacement response value, these approximations are:

- the pseudo relative velocity = \( \omega S_d = S_v \)
- the pseudo absolute acceleration = \( \omega^2 S_d = S_a \)

where \( \omega \) is the angular frequency (\( \omega = 2\pi/T \)). The pseudo relative velocity is shown by a dotted line in Figure 5. The approximation is close for acceleration, and reasonably good except in the long period range for the velocity. If the structural damping of 5% is reduced to 2% this is because \( \omega S_d \to 0 \) while \( S_v \to (\text{maximum ground velocity}) = T^2 \). It is generally agreed that these spectra are close enough to the true velocity and acceleration spectra for all practical purposes(6). Their use has the advantage that it enables all three response spectra to be plotted on a single tripartite graph. The values of \( \omega S_d \) labelled V in Figure 6 are plotted vertically to a logarithmic scale against period, (also to a logarithmic scale) in the normal way. Two diagonal scales, for relative displacement (labelled D) and for pseudo absolute acceleration (labelled A) can be added, using the relationships given above. This single graph...
diagram thus incorporates the three types of spectra, and is drawn here for five different values of spectral damping. The logarithmic scales tend, however, to mask features which stand out clearly in a more conventional diagram. This form of presentation is becoming more commonly used, and several examples may be found in Newmark and Rosenblueth (11).

UNITS
A great variety of units is in current use. Velocity may be in inches/second, feet/second or cm./second, while acceleration is traditionally quoted in terms of gravity but may be in other units.

It is to be hoped that as S.I. units come into general use, they will be accepted as standard for response spectra. Displacement should be quoted in metres, velocity in metres/second (m/s) and acceleration in metres/(second)^2 (m/s^2). While some engineers would prefer to retain gravity as the "unit" for acceleration, there is little inconvenience incurred in using m/s^2. Division by 10 gives, within 2%, the values in terms of gravity.

ACKNOWLEDGEMENT
The author wishes to thank the Earthquake Engineering Research Laboratory, California Institute of Technology for providing the data cards of the Pacoima record, and for the use of diagrams (2) from which Figures 1 and 2 were prepared.

REFERENCES
FIGURE 1: ACCELERATION RECORDS FOR THE THREE COMPONENTS AT PACOIMA DAM, SAN FERNANDO EARTHQUAKE, 9 FEBRUARY, 1971.
FIGURE 3: BIOT'S TORSION PENDULUM.

FIGURE 4: VELOCITY SPECTRA, S16°E COMPONENT, PACOIMA DAM, SAN FERNANDO EARTHQUAKE, 1971. SPECTRAL DAMPING 0, 5% AND 20%.
FIGURE 5: DISPLACEMENT, VELOCITY AND ACCELERATION RESPONSE SPECTRA, S16°E COMPONENT, PACOIMA DAM, SAN FERNANDO EARTHQUAKE, 1971. SPECTRAL DAMPING 5%.
FIGURE 6: TRIPARTITE GRAPHS OF RESPONSE SPECTRA OF DISPLACEMENT, PSEUDO VELOCITY AND PSEUDO ACCELERATION FOR S16°E COMPONENT, PACOIMA DAM, SAN FERNANDO EARTHQUAKE, 1971. SPECTRAL DAMPING 0, 2%, 5%, 10%, 20%.