ABSTRACT: The estimation of shallow foundation bearing strength is a standard operation in geotechnical design. The earliest formulation was concerned with the foundation subject to vertical load only. Subsequent developments enabled the effects of moment and shear loading to be included, but a difficulty arises because the initial formulation was based on the assumption that only vertical actions would be sustained by the foundation. Different thinking is required when moment and shear are applied to the foundation; this can be clarified by recognising that countless combinations of vertical load, horizontal shear, and moment can induce bearing failure; the locus of all these combinations forms a bearing strength surface. The benefit of such surfaces is that they assist one to envisage what will happen when combinations of vertical load, shear and moment are applied to a shallow foundation. Next there is the question of the effect of earthquake loading on foundation bearing strength. Several researchers have investigated this from the point of the vertical capacity of the foundation. Not surprisingly, all show that the bearing strength is reduced by the inertia generated in the soil beneath the foundation. Somewhat surprisingly, these results are not directly relevant to the earthquake bearing strength of shallow foundations. The paper shows, using the bearing strength surfaces given in Part V of Eurocode 8, how it is that shallow foundation bearing strength for cohesive soils is not sensitive to the earthquake horizontal acceleration but that in the case of dry cohesionless soil it may be significant.

1 INTRODUCTION

In this paper the way in which earthquake acceleration in the ground beneath a shallow foundation affects bearing strength is discussed. The purpose of the paper is to demonstrate that for shallow foundations on cohesive soils any effect of earthquake accelerations is minimal and that generally static bearing strength evaluations are adequate for design. In the case of foundations in cohesionless soils the situation is more complex. The main tool used in the paper is the pair of bearing strength surfaces given in Part V of Eurocode 8 (CEN 2003).

The paper is a companion to one dealing with shallow foundation static bearing strength (Pender 2015). In that paper the point is made that the classic Terzaghi bearing strength formulation is based on vertical actions being applied to the foundation; in other words the vertical load that will cause failure of the foundation is evaluated and the designer ensures that the actual vertical loads applied to the foundation are considerably less than this failure load, so ensuring an adequate reserve of bearing strength. However, shallow foundations are often required to sustain horizontal shear and moment as well as vertical loading. This greatly complicates the evaluation of bearing strength as there are a myriad of action paths (involving vertical load, horizontal shear and moment) that can lead to bearing failure. Understanding what might happen is greatly facilitated by using a visual aid provided by the bearing strength surface. This is the locus of all combinations of vertical load, horizontal shear and moment that will lead to failure. Surfaces of this type, for strip foundations on the surface of a layer or saturated clay and dry cohesionless soil, are presented in Part 5 of Eurocode 8. The details of the derivation of these surfaces are given by Salençon and Pecker (1994a and 1994b). The surfaces enable static bearing strength calculations which can be compared with the estimates obtained for
conventional static bearing strength (Pender 2016). More importantly, though, these surfaces provide a means for assessing the effect of earthquake induced inertia forces in the soil on the bearing strength of the shallow foundation.

Prior to the work of Salencon and Pecker others (see for example: Sarma and Iossifelis 1990, Budhu and Al-Karni 1993, Richards, Elms and Budhu 1993) looked at the effect of earthquake actions on shallow foundation bearing strength. They came to the conclusion that the effect of the earthquake is to reduce that bearing strength substantially. They do this by plotting the bearing strength factors $N_c$, $N_q$ and $N_γ$, as functions of the earthquake horizontal acceleration which reveals that there is a substantial decrease in these parameters with increasing horizontal acceleration. Dormieux and Pecker (1995) pointed out that the impression given by these curves can be misleading as the effect of the shear force applied at the soil-foundation interface is included as well the effect of the horizontal acceleration in the ground beneath the foundation.

In this paper the important role of the static actions sustained by the foundation prior to the earthquake is emphasised.

## 2 EUROCODE 8 BEARING STRENGTH SURFACES FOR STRIP FOUNDATIONS AT THE GROUND SURFACE

These surfaces are expressed in terms of three dimensionless parameters:

$$V_n = \frac{V}{V_{uo}}, \quad H_n = \frac{H}{V_{uo}}, \quad M_n = \frac{M}{V_{uo}B}$$  \hspace{1cm} (1)

where: $B$ is the width of the foundation, $V_{uo}$ is the ultimate vertical load capacity of the foundation in the absence of shear and moment, $V$, $H$ and $M$ are a combination of applied foundation actions, $V_n$, $H_n$ and $M_n$ are the normalized actions equivalent to $V$, $H$ and $M$.

The equation for the static bearing strength surface in Eurocode 8, Part 5 (a reduced version of the actual surface specified in EC8), in terms of the dimensionless parameters above, is:

$$f_{static}^8(V_n, H_n, M_n) = \left(\frac{\beta[H_n]}{V_n[1-V_n]^{c_T}}\right)^{c_T} + \left(\frac{\chi[M_n]}{V_n[1-V_n]^{c_M}}\right)^{c_M} - 1 = 0$$  \hspace{1cm} (2)

where values for the eight parameters in this equation, $a – d$, $c_T$, $c_M$, $\beta$ and $\chi$, are given in Appendix I; there are two sets of parameters, one for undrained conditions and the other for drained behaviour of the soil. (Note that equation 2 employs slightly different notation for actions from that in EC8, Part 5.) Figure 1 shows a view of the upper part of the EC8 surface for cohesive soils, Figure 2 the same for cohesionless soils.

Figures 1 and 2 are drawn to the same scale to aid comparison and to indicate that the drained surface is so much “skinnier” than the undrained surface. Note that the vertical load is restricted to positive values whereas the moment and shear can be positive or negative (only positive moments are shown as the upper half, not the complete surface, is plotted). Note also that for very small vertical loads or vertical loads near $V_{uo}$ only small values for shear and moment are possible, whereas the largest values of shear and moment are possible when $V_n$ is about 0.5. Consequently, the role of the vertical force changes when $V_n$ is less than about 0.5. In this situation the shear and moment are the actions driving instability whilst the vertical load has a stabilising effect.

It is of interest to compare the surfaces specified in equation 1, and shown in Figures 1 and 2, with those implied in conventional bearing strength calculations, this reveals that the EC8 undrained surface has more shear capacity at low values $V_n$ and that the drained surfaces have very similar shapes, (Pender 2015).

To achieve the real purpose of the EC8 surface we need two more dimensionless parameters and more complex version of equation 2. The additional dimensionless parameters are:
Figure 1 EC8 undrained bearing strength surface for a ground surface strip foundation subject to static actions.

Figure 2 EC8 drained bearing strength surface for a ground surface strip foundation subject to static actions.
where: $F_{\text{undrained}}$ and $F_{\text{drained}}$ are the dimensionless parameters for undrained and drained loading,
- $a_g$ is the earthquake horizontal acceleration,
- $\phi$ is the angle of shearing resistance of the soil,
- $\rho$ is the density of the soil,
- $g$ is the gravitational acceleration.

The full form of the EC8 bearing strength surface is then:

$$f^{8}\text{seismic} \left( V_n, H_n, M_n, F \right) = \frac{(1 - eF)^{\gamma} \left( \beta |H_n| \right)^{\gamma}}{V_n^{\gamma} \left( 1 - mF^{\delta} \right)^{\delta} - V_n^{\beta}} + \frac{(1 - F)^{\gamma} \left( \chi |M_n| \right)^{\gamma}}{V_n^{\gamma} \left( 1 - mF^{\delta} \right)^{\delta} - V_n^{\beta}} - 1 = 0$$

where: $F$ is $F_{\text{undrained}}$ or $F_{\text{drained}}$ as appropriate.

Numerical values for the 14 parameters are given in Appendix I. Design examples using this equation are given in Chapter 10 of Fardis et al (2005).

At first sight this is a formidable equation. However, one can gain an appreciation of the significance of the surface by plotting sections and noting how the shapes are affected by the values of the $F_{\text{drained}}$ and $F_{\text{undrained}}$ parameters.

The reduction of $V_n$ at the end of the surface with increasing $F$ is plotted in Figure 3, the left hand side for the undrained case and the right hand side for the drained case. In the diagram $V_{\text{end}}$ is the value to which $V_{\text{uo}}$, under zero shear and moment, is reduced by the effect of the horizontal acceleration in the ground beneath the foundation. The figure shows first that the range of values for $F_{\text{undrained}}$ is quite different from those for $F_{\text{drained}}$. The limiting condition when $V_{\text{end}}$ becomes zero gives the maximum possible value for $F$; the maximum value for $F_{\text{undrained}}$ is 3.60 (Fig. 3a) and for $F_{\text{drained}}$ is 1.04 (Fig. 3b). Figure 3b also shows that for the drained case there is a rapid decline in $V_{\text{end}}$ once $F$ goes beyond about 0.8, this might be a manifestation of the “fluidisation” effect noted by Richards et al (1993) when discussing the effect of horizontal acceleration on the bearing strength of shallow foundations in sand. However, it is important to note that this fluidisation has nothing to do with liquefaction; deterioration of foundation bearing strength due to liquefaction is an additional effect.

Figure 4 shows zero shear sections through the undrained surface with various values of $F_{\text{undrained}}$ from 0.0 (static conditions) to 1.5. Figure 5 zero shear sections through the drained surface with various values of $F_{\text{drained}}$ from 0.0 (static conditions) to 0.8. These two figures indicate that the undrained response of cohesive soils is very different from the drained response of cohesionless soils. For cohesive soils the effect of horizontal acceleration is significant only when $V_n$ is greater than about 0.6, whereas for the cohesionless soils Figure 5 shows the effect of horizontal acceleration is significant over a wider range of $V_n$ values.

Figures 3, 4 and 5 were originally presented by Pender (2007).

The intersection of the contours in Figures 4 and 5 with the $V_n$ axis can be interpreted as the effect of horizontal acceleration on the bearing strength parameter $N_c$ in Figure 4 and on $N_f$ in Figure 5.

At this point it is necessary to consider the static actions on the foundation prior to the earthquake. Typically the value of $V_n$ will be about 0.3 (a common design approach would be for the foundation to mobilise about 30% of the bearing strength ($V_{\text{uo}}$) under long term static loading, but conservative design decisions for soil property values often mean that the actual value will be less).

Figure 4 shows for shallow foundations on cohesive soil that the bearing strength surface is hardly affected by the earthquake horizontal acceleration when $V_n$ lies in the range 0.0 to about 0.4. This comment is supported by the observations of Auvenit et al (1996).
Figure 3 Relationship between the seismic parameters $F_{\text{undrained}}$ and $F_{\text{drained}}$ and the dimensionless vertical load at the apex of bearing strength surface: (a) $\phi = 0$, (b) $\phi > 0$.

Figure 4 Undrained sections of the EC8 bearing strength surface with increasing seismic acceleration.

Figure 5 Drained sections of the EC8 bearing strength surface with increasing seismic acceleration.

From the definition given in equation 3 we need to note that the foundation width also contributes to $F_{\text{undrained}}$. For clay with an undrained shear strength of 100 kPa and density of 1.8 tonnes/m$^3$, the $F_{\text{undrained}}$ value of 1.5 in Figure 4 corresponds to a value of 83 for the product $a_g B$. Thus for a foundation 20 m wide an earthquake acceleration of 0.42g would be required, but for a 40m wide
foundation an earthquake acceleration of 0.21g would be required to give $F_{\text{undrained}}$ of 1.5. On the other hand the undrained shear strength of cohesive soil is larger at rapid rates of shearing (Ahmed-Zeki et al 1999) and not affected by modest numbers of cycles (Andersen et al 1980).

Figure 5 shows for shallow foundations on cohesionless soil that, for $V_n$ values of about 0.3 and less, the bearing strength surface is reduced in size by the earthquake acceleration. For a cohesionless soil with an angle of shearing resistance of 35 degrees, a value of $F_{\text{drained}}$ 0.8 would be produced by a horizontal acceleration of 1.14g, for a value of 0.5 by a horizontal acceleration of 0.71g, and for a value of 0.2 by a horizontal acceleration of 0.29g. In other words, this effect plus the narrow shape of the EC8 bearing strength surface for cohesionless soil (Figure 2) means that there is less scope for moment and shear to be sustained by a shallow foundation on cohesionless soil under earthquake than one on cohesive soil.

One needs to take account of the comparative values of $V_{uo}$ for shallow foundations when making comparisons between foundations on clay and sand. Consider a strip foundation 5 m wide at the ground surface: $V_{uo}$ for a clay with $s_u = 100$ kPa ($N_c = 5.14$) is about 2570 kN/m ($=(5.14\times100)\times5$); for a saturated sand with $\phi' = 35$ degrees ($N_r \approx 50$) and $\gamma' = 8$ kN/m$^3$ it is about 5000 kN/m ($=(0.5\times5\times8\times50)\times5$). This means that if we plot cross-sections (rather than longitudinal sections as in Figures 4 and 5) of the two surfaces at $V_n = 0.3$ the cross-sections span very approximately similar ranges of shear and moment when the normalised parameters are converted back to actual foundation actions.

3 APPLICATION TO SHALLOW FOUNDATION DESIGN

There are two likely configurations of shallow foundation for a multi-storey building: a raft foundation or separate footings (or strip foundations) supporting the individual columns. In the case of the raft foundation the total vertical load will remain constant during the earthquake (apart from the effect of vertical upward and downward accelerations generated during the earthquake). Possible action paths for this situation, with constant or near constant vertical load, are shown in Figure 6 for the undrained case. A similar diagram would cover the drained case.

For a frame structure in which the columns are supported on individual footings the response during the earthquake is such that the outside footings are subject to cyclic vertical forces whereas there will be negligible change in vertical load on the interior footings (Wotherspoon and Pender 2010). Possible action paths for this situation are also shown in Figure 6.

Figure 6 then illustrates how one could use the plotted longitudinal sections of the EC8 bearing strength surface to assess the sensitivity of a shallow foundations to seismic excitation. If necessary sections could be calculated for specific values of the $F_{\text{undrained}}$ or $F_{\text{drained}}$ parameters, this is illustrated in Pender (2016).

The discussion to this point assumes an LRFD (load and resistance factored design) ultimate limit state approach. There are other possibilities. For example a shallow raft foundation could be permitted to rock which would lead to a substantial reduction in the rotational stiffness of the foundation and reduced foundation actions (Pender et al 2013).

Another approach would be to permit brief instances of bearing failure of the foundations which would have a similar effect to the rocking discussed above. An example of such a design approach is given by Anastasopoulos et al (2015).

The discussion to this point assumes an LRFD (load and resistance factored design) ultimate limit state approach. There are other possibilities. For example a shallow raft foundation could be permitted to rock which would lead to a substantial reduction in the rotational stiffness of the foundation and reduced foundation actions (Pender et al 2013).

Another approach would be to permit brief instances of bearing failure of the foundations which would have a similar effect to the rocking discussed above. An example of such a design approach is given by Anastasopoulos et al (2015).
In both these “yielding” methods sections of the EC8 bearing strength surface would provide valuable insight into what earthquake excitation would be required to produce brief instances of shallow foundation bearing “failure” during the course of the earthquake.

4 CONCLUSIONS

The following conclusions are reached:

• There are in the literature a number of investigations that indicate that the bearing strength parameters $N_c$, $N_q$ and $N_\gamma$ are decreased because of EQ generated inertia in the soil beneath a shallow foundation; this may be misleading as these analyses include the effect of the horizontal shear at the soil-foundation interface, an effect considered in standard static bearing strength estimates.

• Considering the pre-earthquake loading on shallow foundations, in conjunction with the Eurocode 8 bearing strength surface, indicates that the effects of earthquake generated inertia are limited in the region of the applied static loading.

• The bearing strength surface provided in Eurocode 8 shows that shallow foundations on clay, which have an adequate static reserve of bearing strength, are unlikely to be affected by earthquake acceleration.

• A similar conclusion is reached using the EC8 bearing strength surface for shallow foundations on cohesionless soils, although these are more sensitive to inertia effects in the soil beneath the foundation.

• The lack of sensitivity of shallow foundations on clay is confirmed by post-earthquake reconnaissance observations.

• For buildings supported on raft foundations the vertical load carried by the foundation is changed only if vertical acceleration is applied by the earthquake; this means that earthquake induced changes in foundation actions are confined to the region of the BSS at or near the static vertical load.

• For framed buildings the outer foundations will experience cyclic vertical loads which may mean alternate uplifting and then a large increases in vertical load which may induce additional settlement.

• The EC8 bearing strength surfaces provide useful insight about the likelihood of brief instances of bearing strength failure during an earthquake.
5 REFERENCES


**APPENDIX 1: PARAMETER VALUES FOR THE EC8 BEARING STRENGTH SURFACES**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Undrained</th>
<th>Drained</th>
<th>Parameter</th>
<th>Undrained</th>
<th>Drained</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>0.70</td>
<td>0.92</td>
<td>k</td>
<td>1.22</td>
<td>1.00</td>
</tr>
<tr>
<td>b</td>
<td>1.29</td>
<td>1.25</td>
<td>k'</td>
<td>1.00</td>
<td>0.39</td>
</tr>
<tr>
<td>c</td>
<td>2.14</td>
<td>0.92</td>
<td>c_T</td>
<td>2.00</td>
<td>1.14</td>
</tr>
<tr>
<td>d</td>
<td>1.81</td>
<td>1.25</td>
<td>c_M</td>
<td>2.00</td>
<td>1.01</td>
</tr>
<tr>
<td>e</td>
<td>0.21</td>
<td>0.41</td>
<td>c'_M</td>
<td>1.00</td>
<td>1.01</td>
</tr>
<tr>
<td>f</td>
<td>0.44</td>
<td>0.32</td>
<td>β</td>
<td>2.57</td>
<td>2.90</td>
</tr>
<tr>
<td>m</td>
<td>0.21</td>
<td>0.96</td>
<td>χ</td>
<td>1.85</td>
<td>2.80</td>
</tr>
</tbody>
</table>