Rocking behaviour of a rigid foundation with an arbitrary embedment

2013 NZSEE Conference

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ABSTRACT: Proper determination of dynamic foundation behaviour including subsoil is a very significant factor for dynamic analysis of structures in earthquakes. In recent investigations it has been confirmed that rockable structures can have a beneficial effect in earthquakes. This paper addresses the rocking vibration of a rigid foundation embedded in anisotropic half-space. The rocking behaviour has been formulated analytically. Hankel transform has been utilized, and the mixed boundary-value problem arisen is formulated via concept of dual integral equations, which in turn could be reduced to Fredholm integral equation of the second kind. To evaluate integral equations a robust numerical procedure has been developed for different anisotropic materials. The impedance or compliance functions for foundation could then be determined as a dimensionless function. The accuracy of the developed procedure is validated by comparing the results with those published in the literature for isotropic media.

1 INTRODUCTION

In the area of soil-structure interaction (SSI) the dynamic behaviour of a foundation embedded in a half-space plays a fundamental role in determining the response of structures to dynamic loadings, such as seismic excitation or machine vibration. Rocking vibration occurs when foundations are subjected to incoming seismic waves and the coupled behaviour of soil-foundation-structure system will determine then the impact of the earthquake on the structure.

Most existing analytical solutions are derived based on the assumption that the elastic parameters are spatially homogeneous in the soil medium. Works on the dynamic interaction of a rigid footing with an elastic medium have been performed for several decades, e.g. the works by Robertson (1966), Gladwell (1968), Luco & Westmann (1971).Pak & Gobert (1991)have considered vertical vibrations of a rigid footing embedded at a depth below the surface of an isotropic half-space, formulated its relaxed treatment and made an in-depth investigation of the problem. Research on axial, torsional, horizontal and rocking response of a surface footing can be found e.g. in Luco & Westmann (1971). Similar work by Veletsos & Wei (1971**)** focused on rocking and horizontal vibrations. Pak & Saphores (1991) provided an analytical formulation for the general rocking problem of a rigid footing embedded in an isotropic half-space.

For soils whose behaviour in different horizontal directions are almost the same but differs mainly in the vertical direction, the soil may be treated as one with transverse isotropy. Because of the differences between isotropic and anisotropic media, wave propagation in these two media is significantly different.

However, an analytical consideration of the problem of vibrations of a circular footing associated with an anisotropic medium is left unsolved, mainly because of the more complicated nature of its constitutive properties. Selvadurai (1980) studied a rigid footing embedded in a transversely isotropic full-space for different boundary conditions. Eskandari-Ghadiet al.(2010) considered vertical vibration of a rigid footing embedded at an arbitrary depth of transversely isotropic half-space.Recently, Moghaddasi et al. (2012) presented an analytical solution for lateral interaction of a rigid footing embedded at a transversely isotropic half-space for static case.

The main objective of the present work is finding a solution for rocking vibration of a rigid circular footing embedded at an arbitrary depth in a transversely isotropic half-space.

The coupled partial differential equations are uncoupled by using potential functions introduced by Eskandari-Ghadiet al.(2005). Khojasteh et al. (2008) obtained fundamental Green's functions for a transversely isotropic elastic half-space subjected to arbitrary, harmonic, finite and embedded vibration source. By utilizing the Fourier series and the Hankel integral transforms, the relaxed treatment of mixed boundary-value problem formulated in this paper is transformed into a coupled dual integral equation, which itself can be written into the form of Fredholm integral equation of the second kind. General solution for the Fredholm integral equation is numerically obtained.

The dynamic compliance functions are evaluated numerically and it is shown that the impedance functions for a surface rigid footing obtained for the isotropic domain coincide with the solution given by Luco &Westmann (1971)for all frequencies considered. Numerical evaluations for various transversely isotropic materials are presented in the following sections in the form of compliance functions to reveal the effect of different material anisotropy.

2 MATHEMATICAL FORMULATION

A rigid and massless footing of a radius a with an embedment h in a homogeneous transversely isotropic, elastic half-space is considered. The form of soil anisotropy is assumed to be deposit soil representing common cases like a transversely isotropic medium, i.e. each layer of the deposit has approximately the same properties in plane but different with the depth (Figure 1). A prescribed harmonic rotation excitation, $\Omega\ e^{i\omega t}$, is considered. Ω and $\omega$are the amplitude and circular frequency of the loading of the footing, respectively.

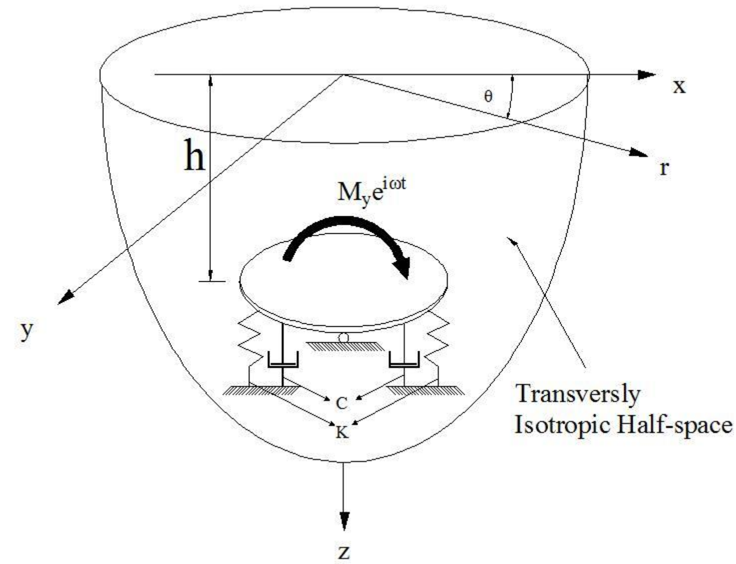

Figure1. Model of a rigid foundation under rocking excitation embedded in transversely isotropic half-space with representing lumped parameters of the surrounding soil

A relaxed treatment of this mixed boundary-value problem can be stated in terms of components of the displacement vector u and the Cauchy stress tensor σ in cylindrical coordinate (r,θ,z) as follows

$$u_z(r,\theta,h) = \Omega r cos(\theta) e^{i\omega t} r < a, \quad (1)$$

$$u_i(r,\theta,h^-) = u_i(r,\theta,h^+), \sigma_{zi}(r,\theta,0) = 0, \qquad i = r,\theta,z r \geq 0, \quad (2)$$

$$\sigma_{zi}(r,\theta,h^-) = \sigma_{zi}(r,\theta,h^+), \qquad i = r,\theta r > 0, \quad (3)$$

$$\sigma_{zz}(r,\theta,h^-) = \sigma_{zz}(r,\theta,h^+), \qquad r \geq a, \quad (4)$$

$$\sigma_{zz}(r,\theta,h^-) - \sigma_{zz}(r,\theta,h^+) = R(r,\theta;h) r < a \quad (5)$$

where R(r, θ,h) denotes the unknown resultant normal contact stress distribution acting on the footingat z=h, and h^+ and h^-could be defined as $h \pm \varepsilon$in which ε is a small number. It was noticed that all the above equations hold for $0 < \theta < 2\pi$ .For a half-space, the foregoing requirements must be appended by the regularity condition at infinity that $\sigma \rightarrow 0$ as $\sqrt{r^2 + z^2} \rightarrow \infty$. The equations of motion for a homogeneous transversely isotropic elastic solid in terms of displacements and in the absence of body forces can be found in Khojasteh et al. (2008).In order to uncouple these equations, a set of complete potential functions F and χ introduced by Eskandari-Ghadi (2005) is used. With the help of F, χ and condition (5) together with the continuity of displacements across the plane z=h and the traction free conditions at the surface, equations required for the solution of the vertical displacement u_z of the entire soil in terms of the transform of the Fourier components $R_m$of the contact-load distribution can be derived. In particularly, one may verify that the vertical displacement and stresscan, in general, be expressed in terms of inverse Hankel Transform(Khojasteh et al. 2008)as

$$u_{z_m} = \int_0^\infty \xi\, \Omega_2(\xi, z; h) \frac{Z_m}{c_{44}} J_m(r\xi) d\xi \qquad (6)$$

$$\sigma_{zz_m} = \int_0^\infty \xi \{ c_{33} \frac{d\Omega_2(\xi, z; h)}{dz} - c_{13}\xi\gamma_3(\xi, z; h)\} \frac{Z_m}{c_{44}} J_m(r\xi) d\xi$$

where the kernel functions$\Omega_2(\xi, z; h)$and $\gamma_3(\xi, z; h)$can be obtained from Khojasteh et al. (2008), c_{ij} is the elasticity constants of the soil medium and $J_m(r\xi)$ is Bessel function of the first kind and m^{th} order. This leads to

$$Z_m = \tilde{R}_m^m(\xi)$$

In the above equations, superscript and subscript *m* denoting the m^{th} order Hankel Transform and m^{th} component of Fourier expansion, respectively. On the account of Equation (1) and the orthogonality of $\{e^{im\theta}\}$, it can be shown that

$$Z_1 = Z_{-1}, \qquad Z_m = 0 \qquad m \neq \pm 1 \qquad (7)$$

By recourse to Equation (6),the remaining two conditions (1) and (4) of the mixed boundary-value problem can thus be reduced to

$$\int_0^\infty \Omega_2(\xi, h; h)\, Z(\xi) J_1(r\xi) d\xi = \frac{\Omega r}{2} \qquad r < a \qquad (8)$$

$$\int_0^\infty Z(\xi) J_1(r\xi) d\xi = 0 \qquad r > a \qquad (9)$$

, where

$$Z(\xi) = \frac{Z_1 \xi}{c_{44}}$$

$\Omega_2(\xi, h; h)$has the property that

$$l = \lim_{\xi\rightarrow\infty} \xi\Omega_2(\xi, h; h) = \frac{c_{44} + c_{33}s_1s_2}{2c_{33}(s_1 + s_2)} \qquad (10)$$

In the above equations, s_1 and s_2 are the roots of the following equation, which in view of the positive definiteness of the strain energy, are not zero or pure imaginary numbers

$$c_{33}c_{44}s^4 + (c_{13}^2 + 2c_{13}c_{44} - c_{11}c_{33})s^2 + c_{11}c_{44} = 0 \tag{11}$$

3 REDUCTION OF SYSTEM OF DUAL INTEGRAL EQUATIONS

With the aid of Sonine's integrals (Noble, 1963), the system of coupled dual integral equations in Equations (8) and (9) can alternatively be expressed as

$$\int_0^\infty \frac{1}{\sqrt{\xi}}(1 + H(\xi, h; h))\, Z(\xi) J_{\frac{1}{2}}(r\xi) d\xi = \frac{\Omega}{l}\sqrt{\frac{2r}{\pi}} \qquad r < a \tag{12}$$

$$\int_0^\infty \frac{1}{\sqrt{\xi}} Z(\xi)\, J_{\frac{1}{2}}(r\xi) d\xi = 0 \qquad r > a \tag{13}$$

where

$$H\,(\xi, h; h) = \frac{\xi\, \Omega_{\,2}(\xi, h; h)}{l} - 1$$

For further reduction, it is useful to define a function$\theta(r)$ as

$$\theta(r) = \begin{cases} \sqrt{\frac{\pi r}{2}} \int_0^\infty \frac{1}{\sqrt{\xi}} Z(\xi)\, J_{\frac{1}{2}}(r\xi) d\xi & r < a \\ 0 & r > a \end{cases} \tag{14}$$

With Equation (14) the governing system of coupled dual integral equations can be reduced to a pair of Fredholm integral equations of the second kind

$$\theta(r) + \int_0^a K(r, \rho; h)\, \theta(\rho) d\rho = \frac{\Omega r}{l} \tag{15}$$

where

$$K(r, \rho; h) = \sqrt{r\rho} \int_0^\infty \xi\, H\,(\xi, h; h) J_{\frac{1}{2}}(r\xi) J_{\frac{1}{2}}(\rho\xi) d\xi$$

By utilizing similar relations provided by Pak and Saphores (1991) the contact-load distribution in the vertical direction can be evaluated directly in terms of the solution of the Fredholm equation as

$$R(r, \theta; h) = \frac{4c_{44}\cos(\theta)}{\pi}\left[\frac{a\theta(a)}{r\sqrt{a^2 - r^2}} - \int_r^a \frac{\rho\theta'(\rho)}{r\sqrt{\rho^2 - r^2}} d\rho\right] \tag{16}$$

The moment, M_y, required to sustain the rotation, Ω, can be simplified as

$$M_y = 8\, c_{44} \int_0^a r\theta(r) dr \tag{17}$$

It can be rewritten in terms of the rocking impedance

$$K_{MM} = \frac{M_y}{c_{44} a^3 \Omega} \tag{18}$$

It may also express into the dynamic rocking compliance, which is the ratio of Ω to M_y.

4 NUMERICAL RESULTS AND DISCUSSION

In the previous sections, the Fredholm integral equations were expressed in terms of θ. It is not easy to deal with the dual integral equations analytically. For this reason, numerical solutions of the integral equation can be obtained by standard quadrature methods. On tackling Equation (15) some special considerations are needed due to the presence of singularities within the range of integration including branch points. In addition, some functions in $\Omega_2(\xi, z)$ yields pole at ξ_R which corresponds to Rayleigh wave number. The numerical results presented here are dimensionless by using a dimensionless frequency$\omega_0 = a\omega\sqrt{\rho_s/c_{44}}$. In the forgoing equations, ρ_s stands for soil density. To understand the effect of anisotropy of the materials on the interaction between the two media, several types of isotropic (Mat 1) and transversely isotropic materials (Mats2–5) are considered. The material properties are given in Table 1, where E and E′ are the Young’s modulus in the plane of isotropy and perpendicular to it; ν is Poisson’s ratio that characterizes the effect of horizontal strain on the complementary vertical strain; ν′ is the Poisson’s ratio which characterizes the effect of vertical strain on the horizontal one; and G' is the shear modulus for the plane normal to the plane of isotropy. In defining these materials, the positive-definiteness of strain energy that observe the following constraints for material constants c_{ij}, have been evaluated (see e.g. Payton, 1983).

$$c_{11} > |c_{12}|, \qquad (c_{11} + c_{12})c_{33} > 2c_{13}{}^2, \qquad c_{44} > 0 \tag{19}$$

Table 1 Properties of synthetic materials

[1]Mat	E	E'	G	G'	ν and ν'	c_{11}	c_{12}	c_{13}	c_{33}	c_{44}	c_{66}
1*	5	5	2	2	0.25	6	2	2	6	2	2
2T	5	10	2	2	0.25	5.6	1.6	1.8	10.9	2	2
3T	5	15	2	2	0.25	5.5	1.5	1.8	15.9	2	2
4T	10	5	4	2	0.25	14	6	5	7.5	2	4
5T	15	5	6	2	0.25	26	14	10	10	2	6

[1]The unit of all quantities are GPa except the Poisson’s ratio of medium which is dimensionless.
*Isotropic [T]Transversely isotropic

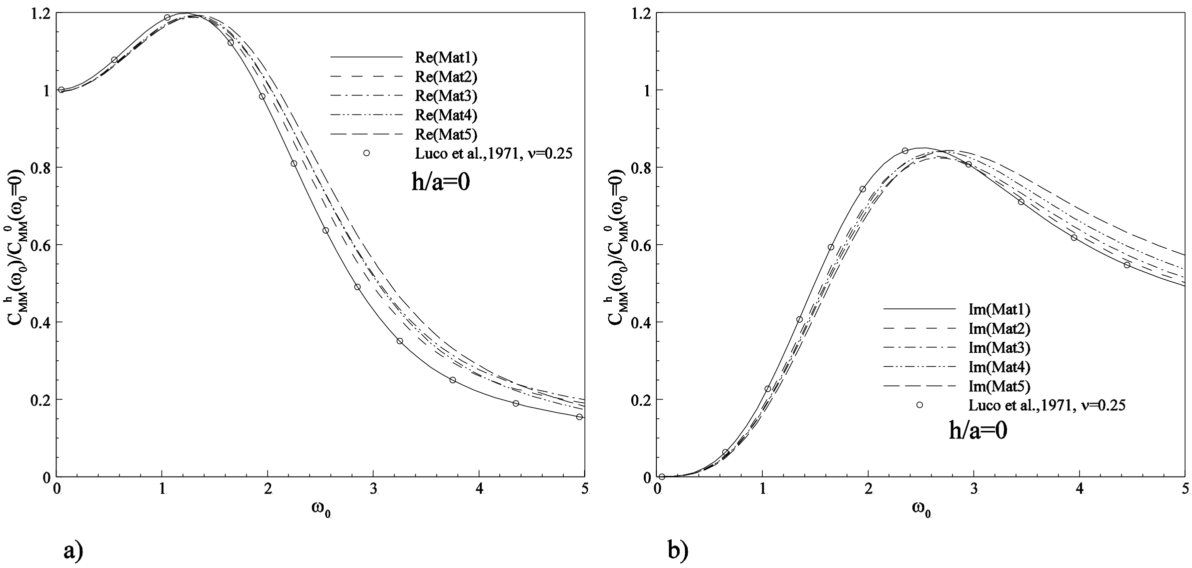

Figure 2. Effect of transversely isotropic materials on compliance function in terms of dimensionless frequency ω_0 for a surface footing (h/a = 0) in comparison with the result by Luco &Westmann (1971) for isotropic material. (a) Real and (b) imaginary part

Figure 2 shows the real and imaginary parts of the rocking compliance function for h/a=0 obtained from the present study and the respective results from Luco &Westmann(1971) for a range of dimensionless frequency for one isotropic material. The result from this study is in an excellent agreement with the result published by Luco and Westmann. This demonstrates the accuracy of the numerical evaluation for all frequencies considered. In the same figure the rocking compliance function for rest of transversely isotropic materials considered is displayed. Although the effect of material anisotropy has already been observed, however, for a better demonstration of the material anisotropy effect, an embedment of the foundation is considered. Figure 3 shows the rocking compliance function for different transversely isotropic materials for h/a=1.It was worth mentioning that dynamic compliances calculated in the present study are in the dimensionless form $C_{MM}(\omega_0)/C_{MM}(\omega_0=0)$ where $C_{MM}(\omega_0=0)$ is the rocking compliance of a rigid circular footing on a transversely isotopic half-spacewhen the frequency was hold zero.

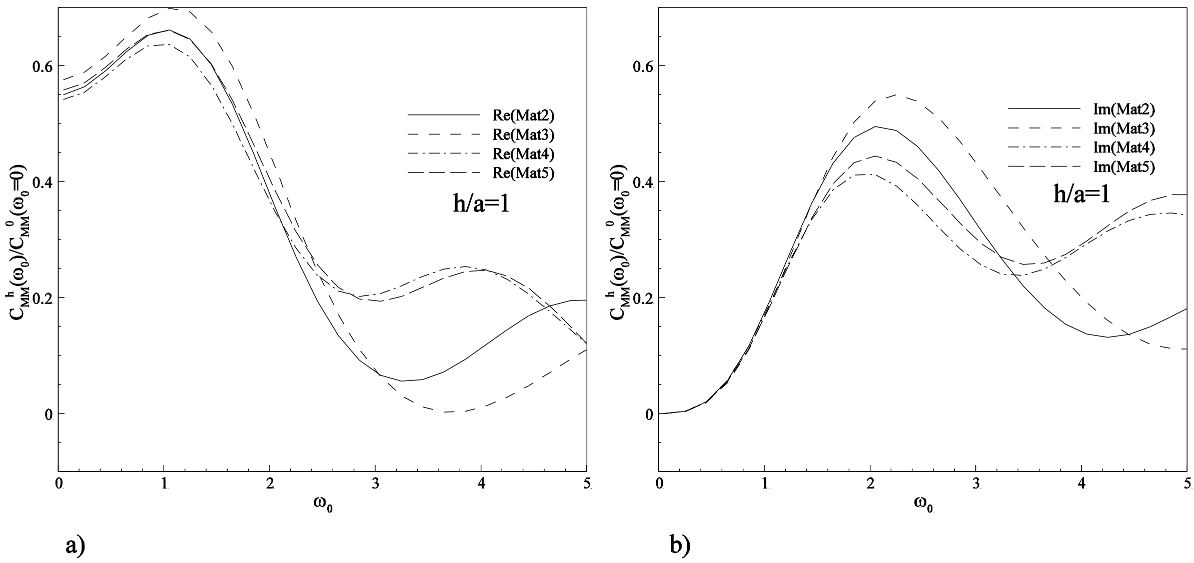

Figure 3.Compliance function in terms of dimensionless frequency ω_o for different transversely isotropic materials for h/a = 1. (a) Real and (b) imaginary part

As the frequency increases, successive oscillations in both the real and imaginary parts of the compliance function can be observed. This behaviour has not been observed in rocking vibration of a surface foundation. It could be originated from the fact that standing waves emerged between the free surface and the embedded foundation that practically vanish for the case of a surface foundation. Besides, by increasing E value (Mats4 and 5) the results become similar to each other which indicate that E value has only a minor effect on development of the compliance function with the frequency.

5 CONCLUSIONS

A mathematical formulation for the rocking behaviour of a rigid foundation embedded at an arbitrary depth has been presented in the frequency domain. In the past decades, there are demands for more accurate modelling of behaviour of footing supported by natural geological deposits, composites and engineered materials which could be treated as transversely isotropic medium. To facilitate a direct engineering applications e.g. in soil-foundation-structure problems, the dynamic rocking compliance for different transversely isotropic materials are investigated. It was evident that between material coefficients in transversely isotropic media, the modulus of elasticity perpendicular to the plane on isotropy has the most significant influence on rocking behaviour. It is also observed that the material anisotropy could have a significant effect when the foundation has a finite depth embedment.

The soil flexibility (compliance) obtained can be used in analysis of soil-foundation-structure system under an earthquake loading with a proper consideration of the radiation damping due to wave propagation from a vibrating footing.

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