A “retrofit” solution for Force-Based Design: eliminating the need for iteration and initial period estimation

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ABSTRACT: The limitations and inconsistencies of current force based design procedures, as used worldwide by most practicing engineers, have been recently highlighted and acknowledged. Some of the most significant drawbacks recognized include the uncertainty in the determination of the structure’s initial period and the incorrect assumption that such initial period/stiffness remains regardless of the structure’s design strength, not known at the beginning of design. As noted in displacement based design methodology, a structure’s strength and stiffness are proportional for a given geometric arrangement and such compatibility must be accounted for to obtain a feasible design solution. The actual secant-to-yielding period of the equivalent elasto-plastic system (behind any equal displacement or equal velocity rule in elastic spectra-based design approaches) may differ considerably from the initial design assumption, leading to weak control of the design. The current force-based design methodology may be corrected following an iterative procedure to guarantee that the initial period is equal to the secant-to-yielding period of the elasto-plastic equivalent system. In this paper a closed-form ‘retrofit’ solution to the iterative procedure is proposed to allow a force-based design to be conducted respecting the aforementioned strength-stiffness compatibility. The closed-form design method provides a set of feasible design solutions, composed of multiple pairs of design base shears and structural ductilities, which may be obtained without the need of an initial period estimation and following iterations. A design example is provided with reference to a single-degree-of-freedom bridge pier, designed following the common-practice force-based design (no iteration), the proposed corrected force-based design (iterative or closed-form), and a direct displacement based design methodology. The designs from the different methodologies are then verified and compared using non-linear time history analysis.

1 INTRODUCTION

Force-based design procedures have been used in many design codes as the most conventional method for determining a structure’s seismic design actions. The basic procedure utilizes an elastic design spectrum, assumed reduction from ductility, and an estimated elastic period in order to determine the design level base shear. As discussed by Priestley (1998) and Priestley et al. (2007), many inconsistencies have been uncovered in recent studies involving the development of Direct Displacement Based Design (DDBD). Some of these inconsistencies, however, are a result of not completing design checks to verify the structure’s initial assumptions and determine feasible design solutions. These design checks may involve multiple iterations and, possibly, the need to conduct multiple non-linear static (push-over) analyses in order to verify the actual structural response.

Employing the basic principles outlined within a DDBD approach, there has recently been much more information, namely empirical equations, on the relationship between a structure’s strength and stiffness, or similarly on the yielding curvature, rotations, and displacements of a section, a member, and the overall structure. This information can be implemented into a force based design in order to create a (displacement-based) ‘retrofitted’ force-based design procedure. This procedure corrects two
of the main inconsistencies with the common-practice procedure by eliminating the need to estimate the structure’s initial period and providing feasible design solutions without the need for iteration.

The DDBD procedure, as mentioned previously, has become a powerful alternative approach to the current force-based design procedures (Priestley 1998-2007). This approach has been developed as a simple method of designing a structure to achieve a set of target displacement limits by characterizing the structure’s effective (secant to the target displacement) stiffness, instead of the initial stiffness as estimated in force-based procedures. This method therefore does not require the estimation of the structure’s initial period, however requires the assumption of the structure’s displaced shape at yield and of the design deflection envelope.

In this paper, a common-practice force-based design (no iteration to check initial assumptions), a proposed “corrected” force-based design (iterative procedure or “retrofitted” closed-form procedure), and a direct displacement based design method are considered. The procedures for each of these methods are described and a single-degree-of-freedom case study structure is designed following each procedure. The results of the designs are then compared following an inelastic time history analysis of the structure using the program Ruaumoko (Carr, 2008).

2 COMMON-PRACTICE FORCE-BASED DESIGN APPROACH

2.1 Method

The common-practice Force-Based Design (FBD) procedure uses a reduced elastic design spectrum to obtain the seismic base shear. The sequence of operations is summarized in Figure 1, and the steps described in more detail below.

1. The structure’s geometry and element sizes are estimated, which initially may be based on architectural and non-seismic load considerations. The seismic weight and seismic mass of the structure must be determined based on the preliminary structure geometry.

2. The elastic base shear, or simply the spectral acceleration of the elastic structure, is reduced by a reduction factor that is determined as a function of the structure’s assigned ductility. The ductility factor, μ, (or reduction factor, R, or behaviour factor, q) is specified by the appropriate design or material code. Traditionally, force-based design follows the equal-displacement rule approximation for structures with medium and long range periods, and the equal-energy rule approximation for structures with short periods.

3. The elastic period of the structure, T, is estimated based on its initially determined geometry. Many methods are available for estimating the period and are available in most design codes (NZS1170.5, 2004).

4. Entering the reduced spectral acceleration plot at the estimated elastic period, a design
acceleration, $Sa(T)$, may be found for the assumed inelastic response. The elastic base shear of the structure may then be simply determined from Equation 1 below.

$$V_b = S_{a,\text{reduced}} \cdot g \cdot m$$

(1)

5. Following the equal displacement approximation, the yield displacement of the structure will coincide with the inelastic response and therefore can be calculated through the relationship between spectral acceleration ($Sa(T)$) and spectral displacement, $Sd(T)$ or $\Delta$. The maximum displacement demand, $\Delta_m$, can then be determined based on the ductility. Therefore, the design displacements may be found as shown in Equations 2 and 3.

$$\Delta_y = S_{a,\text{reduced}} \cdot g \cdot (T/2\pi)^2$$

(2)

$$\Delta_m = \Delta_y \cdot \mu$$

(3)

6. The base shear is then distributed throughout the building and the internal design forces are determined. At this stage the design is deemed complete and a check of the assumed initial period and thus actual ductility-demand are usually not carried out.

2.2 Inconsistencies

In the above FBD procedure it is apparent that the design lateral forces are directly related to the estimation of the fundamental period of the structure. Slight differences in the assumption of the period can cause large differences in the design base shear and design deflections. Although the design method is very sensitive to the fundamental period estimation, the equations provided by design codes to estimate such initial period give a wide range of results.

Another major inconsistency arises with using an estimation of the structure’s fundamental period regardless of the structure’s strength. It has been shown (Priestley et al., 1998) that one of the main fallacies with common-practice force based design is the assumption that the stiffness, and hence period, of the structure may be kept constant. It should be noted that if a constant initial period and stiffness are used, the designer is assuming that the stiffness of the structure is independent of the structure’s strength. This implies that the yield curvature is directly proportional to strength, as shown in Figure 2(a) below. Based on detailed analysis and experimental evidence, this assumption has been proven invalid for concrete structures (Priestley, 2003). Realistically, the yield curvature is essentially independent of strength and the “initial”, or better yet secant to yielding, stiffness of the structure varies based on strength, as shown in Figure 2(b).

Following the more realistic constant yield curvature assumption, it is apparent that the initial stiffness of the structure will decrease as the base shear decreases. However, in common-practice the fundamental period and stiffness are kept constant and are not influenced by a reduction in strength due to ductility. Such an approach might lead to either very unconservative results, with either the target displacement thus damage level being substantially underestimated, or vice versa, the required
base shear being overestimated, leading to excessive and thus unnecessarily expensive overstrength demand into the foundation/soil system as well as excessively stiff structures with associated higher floor accelerations and internal force demand.

3 ITERATING TO DETERMINE FEASIBLE DESIGN SOLUTIONS

3.1 Strength and Stiffness Compatibility

Compatibility between the structure’s strength and stiffness must be checked to determine if the assumed design is feasible. In a compatible system, assuming elastic perfectly plastic behaviour, the yield strength and the initial stiffness are related by the yield deflection, as shown in the following relationship.

\[ V_b = \Delta_y \cdot K_i \]  \hspace{1cm} (4)

Making the realistic assumption that yield curvature is constant, the yield deflection of the structure can also be assumed to be (approximately) constant. Therefore, by rearranging Equation 4, the actual stiffness of the structure may be calculated. This actual stiffness can then be compared with the originally assumed stiffness, which may be calculated from the assumed elastic period. In order for the design solution to be “feasible”, the assumed and actual stiffness values must match.

\[ K_{i,\text{actual}} = \frac{V_b}{\Delta_y} \]  \hspace{1cm} (5)

\[ K_{i,\text{assumed}} = \left(4 \cdot \frac{\pi^2}{T^2}\right) \cdot m \] \hspace{1cm} (6)

If the actual and assumed stiffness do not match the initial period, hence stiffness, estimation must be re-estimated and an iteration is required. This will result in the spectral ordinates Sa(T), thus design base shear, which will again change the actual stiffness of the equivalent SDOF elasto-plastic structure. The iterative process must be repeated until the actual and assumed stiffness match. In some cases, this will not be possible and the stiffness iteration will never converge. This would signify that the level of assumed ductility is not feasible (too high) and must be reduced.

3.2 Yield Deflection

As can be seen above, the yield deflection is necessary in order to check the initially assumed conditions and determine if the design solution is feasible. This can be calculated using moment-curvature analysis software, spreadsheets, or a structural analysis program. Analyses have also been conducted that have demonstrated that the yielding curvature of a section may be determined primarily as a function of the section geometry and reinforcement yield strain (type/grade of steel). The yielding curvature for concrete structural members can be approximately estimated from Equation 7 (Priestley 1998-2007).

\[ \phi_y = k \cdot \frac{\varepsilon_y}{D} \] \hspace{1cm} (7)

where \( \phi_y \) is the yield curvature, \( \varepsilon_y \) is the yielding strain of the reinforcing steel, \( D \) is the depth of the section perpendicular to the axis of bending, and \( k \) is a constant varying with the section geometry. The constant \( k \) has been defined as 2.25 for circular columns, 2.10 for rectangular columns, and 2.00 for rectangular walls. For a structure assuming a linear distribution of curvature, the yield deflection may be calculated from the moment-area theorem (Priestley, 2003). The yield deflection for a single-degree-of-freedom system is given below.

\[ \Delta_y = \phi_y \cdot \frac{H^2}{3} \] \hspace{1cm} (8)

where \( \Delta_y \) is the yield deflection, and \( H \) is the height of the single-degree-of-freedom system.

3.3 Sequence of Design Steps

The method for determining feasible solutions through iteration is simply an addition, itself implicitly
required but most likely not followed, to the common-practice force-based design procedure. The common-practice FBD procedure should be followed with a design check, which usually would result in the need for iteration on the originally assumed "initial period" until there is an agreement between assumed structural period/stiffness and actual structural period/stiffness. The sequence of steps is summarized in Figure 3 below.

![Figure 3: Sequence of Design Steps for FBD Iterative Procedure](image)

4 A CLOSED-FORM ‘RETROFIT’ SOLUTION FOR FORCE-BASED DESIGN

4.1 Strength-Stiffness Compatibility Domain Curves

The proposed closed-form approach takes advantage of the fact that the yield displacement of the structure is approximately constant and easy to obtain for most structures. Combining Equations 1 and 5 and setting them equal to Equation 6, a relationship can be formed between spectral acceleration and period. A similar method has been described by Smith and Tso (2002).

\[ S_a(T) = \frac{4\pi^2}{T^2} \Delta_y \]  \hspace{1cm} (9)

Plotting this relationship directly on the design spectrum provides locations of feasible design solutions where the structure will have compatibility between its design strength and stiffness, which will be referred to as the compatibility domain curve. The intersection of the compatibility domain curve and a reduced spectral acceleration curve will give the design spectral acceleration and period for the assigned ductility. If the lines do not intersect the reduced spectra curve, there is no feasible design solution for that ductility. Figure 4 below shows compatibility domain curves plotted for a range of yield deflections on both acceleration response spectra and acceleration-displacement response spectra.
Determine Feasible Design Solutions

Plotting spectra for a range of targeted ductilities, the designer may determine a suitable and “feasible” (or compatible) design options earlier in the design process. An example is shown in Figure 5 below. The preferred design option may be selected out of the feasible design solutions for a given yield displacement (e.g., given structural geometry). These alternative design solutions are labelled as a, b, c, and d in Figure 5. Although all of the design solutions may be theoretically feasible, some may be uneconomical, as they might require an inappropriate or not allowed amount of reinforcement, or exceed set drift and ductility limits. Table 1 shows the possible variation between the different design solutions for this example, where each solution is feasible according to strength and stiffness compatibility.

![Image](image_url)

**Figure 5**: Design Solutions from ‘Retrofit’ Force Based Design; Left: Acceleration Spectrum; Right: Acceleration Displacement Spectrum for Capacity Method Approach

<table>
<thead>
<tr>
<th>Design Solution</th>
<th>Period</th>
<th>Acceleration</th>
<th>Expected Displacement</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>(T_1)</td>
<td>(S_{a1})</td>
<td>(1.0 \cdot \Delta_y)</td>
</tr>
<tr>
<td>b</td>
<td>137% (T_1)</td>
<td>53% (S_{a1})</td>
<td>(1.5 \cdot \Delta_y)</td>
</tr>
<tr>
<td>c</td>
<td>173% (T_1)</td>
<td>33% (S_{a1})</td>
<td>(2.0 \cdot \Delta_y)</td>
</tr>
<tr>
<td>d</td>
<td>206% (T_1)</td>
<td>23% (S_{a1})</td>
<td>(2.5 \cdot \Delta_y)</td>
</tr>
</tbody>
</table>
4.3 Sequence of Design Steps

The procedure for the closed-form “retrofitted” force-based design arrives at the same feasible design solutions as the iterative method in a much more efficient method. More so, the possibility to clearly visualize, in a synoptic manner, the alternative design options can provide the designer with a better overview and control of his/her design choice. The sequence of operations is summarized in Figure 6 below.

5 DIRECT DISPLACEMENT BASED DESIGN

Direct displacement based design has been developed as a simple method for designing to achieve a set displacement limit. The procedure involves characterizing the structure by an effective stiffness to the design displacement and a level of equivalent elastic damping, which combines the effects of elastic and hysteretic damping. The general procedure for direct displacement based design is shown in Figure 7 below. A more detailed description of the method can be found in many other references in literature and thus will not be repeated here (Priestley et al., 2007).

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**Figure 6: Sequence of Operations for “Retrofit” Force Based Design**

**Figure 7: Sequence of Operations for Direct Displacement Based Design (modified from Priestley 2007)**
6 DESIGN EXAMPLE USING A CASE-STUDY STRUCTURE

A simple bridge pier is to be designed for construction on Class D soil as classified by NZS1170.5, considering a peak ground acceleration of 0.3g and a return period factor of 1.3. The pier is to be considered as a single-degree-of-freedom structure with no interaction between the adjacent piers through the bridge deck. A 2.5% maximum drift is considered for the design limit state.

The pier height is 10 meters to the centre of the superstructure mass, with a total seismic mass calculated to be 500 tonnes. The tributary weight is determined from the seismic mass, therefore the axial load on the column is 4905 kN. From preliminary design considerations, a 1.5x1.5 meter square column has been selected. The schematic geometrical and mechanical characteristics of the pier, as well as the DDBD design values, are shown in Figure 8 below.

![Figure 8: Case Study Structure and Design Values](image)

Note: Shear reinforcement not shown for clarity. Drawings not to scale.

<table>
<thead>
<tr>
<th>$\theta_d$</th>
<th>D</th>
<th>H</th>
<th>m</th>
<th>N</th>
<th>$F_y$</th>
<th>$E_s$</th>
<th>$F'_c$</th>
<th>$E_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(%)</td>
<td>(m)</td>
<td>(m)</td>
<td>(t)</td>
<td>(kN)</td>
<td>(Gpa)</td>
<td>(Gpa)</td>
<td>(Mpa)</td>
<td>(Gpa)</td>
</tr>
<tr>
<td>2.5</td>
<td>1.5</td>
<td>10</td>
<td>500</td>
<td>4905</td>
<td>450</td>
<td>200</td>
<td>45</td>
<td>30</td>
</tr>
</tbody>
</table>

Figure 8: Case Study Structure and Design Values

6.1 Comparison of Design Methodologies

Seven acceleration time histories recorded from real earthquake events were selected. All seven records were matched with the design spectrum described in the previous section. SeismoMatch (Seismosoft Ltd., 2012) was used to match the records with the design spectrum for the period range of 0.5 to 2.0 seconds, adjusting the earthquake accelerograms to match the period range using the wavelets algorithm. Figure 9 shows the design spectrum as defined by NZS1170.5, the matched response spectra from the earthquake time histories, and the mean spectrum from the matched response spectra. Table 2 shows the details of the selected earthquakes.

![Figure 9: Design Spectrum and Matched Records](image)

Table 2: Details of Selected Earthquakes

<table>
<thead>
<tr>
<th>Event</th>
<th>Station</th>
<th>Year</th>
<th>Mw</th>
<th>D (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Superstition</td>
<td>Brawley</td>
<td>1987</td>
<td>6.7</td>
<td>18.2</td>
</tr>
<tr>
<td>Hills</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Northridge</td>
<td>Canoga Park – Topanga Clan</td>
<td>1994</td>
<td>6.7</td>
<td>15.8</td>
</tr>
<tr>
<td>Northridge</td>
<td>N Hollywood – Coldwater Clan</td>
<td>1994</td>
<td>6.7</td>
<td>14.6</td>
</tr>
<tr>
<td>Loma Prieta</td>
<td>Capitola</td>
<td>1989</td>
<td>6.9</td>
<td>14.5</td>
</tr>
<tr>
<td>Landers</td>
<td>Desert Hot Springs</td>
<td>1992</td>
<td>7.3</td>
<td>13.3</td>
</tr>
<tr>
<td>Landers</td>
<td>Yemo Fire St.</td>
<td>1992</td>
<td>7.3</td>
<td>24.9</td>
</tr>
</tbody>
</table>
6.2 Comparison of Design Methodologies

The case study structure was designed according to the three alternative procedures described above in order to provide a simple comparison of the resulting design values. Shear reinforcement was not calculated as the pier was considered to be designed to fail in flexure. The common-practice force-based design method used an initial period estimation from the structural analysis software ETABS (Computes and Structures, Inc. 2012) and an assumed ductility of 6, as allowed by New Zealand Design codes (NZS3101 2006). These results show the design as it would most likely be completed in a design office, where no strength and stiffness compatibility was checked. The corrected force based design utilizes the ‘retrofit’ closed-form design method, which provides the same results as the iterative method to arrive at a feasible design solution. The ductility value was selected based on the maximum drift of the structure. The structural performance factor ($S_p$) was assumed to be 1.0 for both force-based design procedures. The Direct Displacement Based Design was conducted incorporating strain penetration of the rebar and following methods as described in (Priestley 2007). Table 3 below shows all of the values resulting from the three different design procedures.

<table>
<thead>
<tr>
<th>Design Method</th>
<th>Initial Period (s)</th>
<th>Effective Period (s)</th>
<th>Assumed Ductility</th>
<th>Design Base Shear (kN)</th>
<th>Design Moment (kNm)</th>
<th>Yield Deflection (mm)</th>
<th>Expected Maximum Deflection (mm)</th>
<th>Section Design</th>
</tr>
</thead>
<tbody>
<tr>
<td>Common-Practice Force Based Design</td>
<td>0.94</td>
<td>-</td>
<td>6</td>
<td>500</td>
<td>5000</td>
<td>22</td>
<td>132</td>
<td>1.5x1.5 m ρ = 0.3%</td>
</tr>
<tr>
<td>Corrected Force Based Design</td>
<td>1.52</td>
<td>-</td>
<td>2.3</td>
<td>901</td>
<td>9010</td>
<td>105</td>
<td>243</td>
<td>1.5x1.5 m ρ = 1.0%</td>
</tr>
<tr>
<td>Direct Displacement Based Design</td>
<td>-</td>
<td>2.37</td>
<td>2.29</td>
<td>910</td>
<td>9100</td>
<td>105</td>
<td>258</td>
<td>1.5x1.5 m ρ = 1.0%</td>
</tr>
</tbody>
</table>

It is important to notice the difference between the initial period used in common-practice force based design as compared with the proposed corrected force based design. The low estimation of initial period corresponds to low estimations of expected deflections, according to the equal displacement rule approximation. This leads to a very high assumption of design ductility and a very low design base shear when compared to a corrected force-based design procedure and a displacement based design procedure, where the period/stiffness of the equivalent single degree of freedom structure is determined based on characteristic deflections and the assumed ductility is found to be much lower.

6.3 Inelastic Time History Analysis

Each of the designed piers summarized in the previous section was modelled in Ruaumoko (Carr 2007,2008), using a lumped plasticity model, and were subjected to the scaled accelerograms of the events listed in Table 2. The ratio of the resulting maximum base shear to the design base shear was calculated and averaged for the seven ground motions. The same calculations were completed for the resulting maximum deflection at the top of the pier. The average ratios and standard deviations, given as percentages, are shown in Table 4 below.

<table>
<thead>
<tr>
<th>Design Method</th>
<th>Actual Maximum Base Shear (% of Expected Design Value)</th>
<th>Actual Maximum Deflection (% of Expected Design Value)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Avg</td>
<td>StdDev</td>
</tr>
<tr>
<td>Common-Practice Force Based Design</td>
<td>101.1%</td>
<td>0.6%</td>
</tr>
<tr>
<td>Corrected Force Based Design</td>
<td>99.8%</td>
<td>0.6%</td>
</tr>
<tr>
<td>Direct Displacement Based Design</td>
<td>98.9%</td>
<td>0.6%</td>
</tr>
</tbody>
</table>
From the results, it is evident that the common-practice force based design severely underestimates the maximum deflection. Due to the very low assumed value of base shear (apparently more cost-effective and appealing to the designer) and the excessively high ductility or reduction factor assumed. As a result, the bridge pier might experience higher than expected damage if not suffering additional second-order effects (P-Δ, herein not included for simplicity) which could compromise the overall structural stability. The corrected or “retrofitted” force-based design and the direct displacement based design both provide accurate predictions of the maximum displacement of the case study structure.

7 CONCLUSIONS

Many observations can be made from the results of this study and the design example and time-history analysis of one Single-Degree of Freedom case study structure.

- Although the results of the corrected/retrofitted force-based design and displacement based design are essentially equal, this may not always be the case, as already anticipated. The force-based design correction confirms strength and stiffness compatibility of the structure, however does not change the original assumptions and fundamental steps of the method. For example, a force-based design reduces the elastic spectra by a reduction factor based on the structure type and ductility, while a displacement based design reduces the displacement spectra from an equivalent viscous damping. The differences between the two design methods will vary based on the design ductility level. As an additional modification, the acceleration spectra ordinates might be reduced, also in a FBD method, as a function of the damping.
- Assuming that the initial stiffness of a structure is constant may result in an inaccurate estimation of the actual deflections of the system. In most cases, the design deflection will be much less than the actual deflection.
- Correcting force-based design with an iterative procedure ensures that the design complies with stiffness-strength compatibility. The ‘retrofit’ closed-form force-based design method provides a very efficient method to finding the same result and would be easy to implement to provide more accurate force based design calculations in practice.

REFERENCES


