Soil-foundation-structure-interaction: A unified approach and applicability of constitutive relations for sandy soils

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ABSTRACT: It is widely recognized that soil-foundation-structure-interaction (SFSI) can be a major factor in the seismic response of structures built on soft and deformable soils, such as was observed in the Canterbury Earthquakes. However, difficult numerical procedures involved in SFSI studies and a general lack of understanding of constitutive models of soils are the main obstacles in conducting SFSI analysis. With the vision of encouraging practising engineers to undertake SFSI analysis for important structures, the paper presents a fundamental, unified and pragmatic view of SFSI problems, both for shallow and deep foundations. The paper presents a method for determining lumped stiffness and viscous damping coefficients for deep foundation systems with distributed springs and dashpots which can considerably reduce computational and design efforts. Subsequently, discussion is given to applicability of constitutive models in non-linear SFSI problems: from the simplest discrete spring approach based on deformation theory, to complicated elasto-plastic models based on rate independent plastic flow theory.

1 INTRODUCTION

The central philosophy of structural design is to transmit loads to soil through a foundation system. Though the choice and adequacy of a foundation system is governed by deformation of soil under such loads, it is often assumed that such deformation is sufficiently small to produce any recognisable effect on the response of a structure. This assumption is valid where structures are built on stiff soil and foundation movement is of negligible order. But when structures are built on soft soil, foundation movement can have a profound effect on response of structures, particularly during earthquakes. Soft soil conditions not only cause a shift in natural vibration period towards the flexible end of spectrum and reduce the radiation damping effect, but also large deformations can produce unacceptable levels of total and differential settlements as was experienced during the recent Darfield and Christchurch earthquakes. (Cubrinovski et al., 2011a,b).

The interaction between superstructure and foundation can be captured by an integrated analytical framework referred to as Soil-Foundation-Structure-Interaction (SFSI) analysis. However, SFSI analysis entails rigorous computation, a good understanding of complicated cyclic stress-strain behaviour of soil, and is usually carried out for response history analysis (RHA) as opposed to the response spectrum analysis, which is a practical method for design. In this context, the paper attempts to present a theoretical background and simplified procedures for SFSI analysis which can be practised both in conventional design using the response spectrum method, and in RHA for more rigorous studies. The paper also presents a simple overview of the principal constitutive relations of sandy soils.

2 SOIL-FOUNDATION-STRUCTURE-INTERACTION: AN ANALYTICAL FRAMEWORK

In structural dynamics, the underlying concept is every component has three mechanical properties: mass, stiffness and damping, which respectively give rise to inertia, restoring and damping forces. SFSI analysis can be simply viewed as an extended structural analysis where all the three mechanical properties of soil are incorporated in the form of equivalent structural elements. Since soil is a continuum, an intuitive choice to idealise soil will be a continuum finite element, in which case, the mechanical properties are implicitly incorporated through the finite element formulation. However, the mechanical properties can be also incorporated by much simpler discrete uniaxial springs and dashpots with lumped mass. Although finite element modelling enables more accurate representation of physical behaviour, it is often found to be practically infeasible. In contrast, the latter approach offers computational ease and flexibility and is therefore, often adopted in conventional engineering analysis and design.

2.1 Impedance: condensing dynamical property to kinematic property

The first step in the simplified SFSI is to idealise the three mechanical properties of a continuum by a discrete spring and dashpot, which are collectively termed as the impedance. Using impedances, soil behaviour can be incorporated in an analysis without continuum finite elements. The formulas for impedances are available in literatures (e.g. Gazetas, 1991; Wolf and Deeks, 2004; ATC-40, 1996) for various foundation shapes, embedment, soil profiles and depth of soil boundary, which cover almost all practical ranges of concern. When a particular problem does not fit into those cases, a numerical procedure is required to derive the impedance as described below.

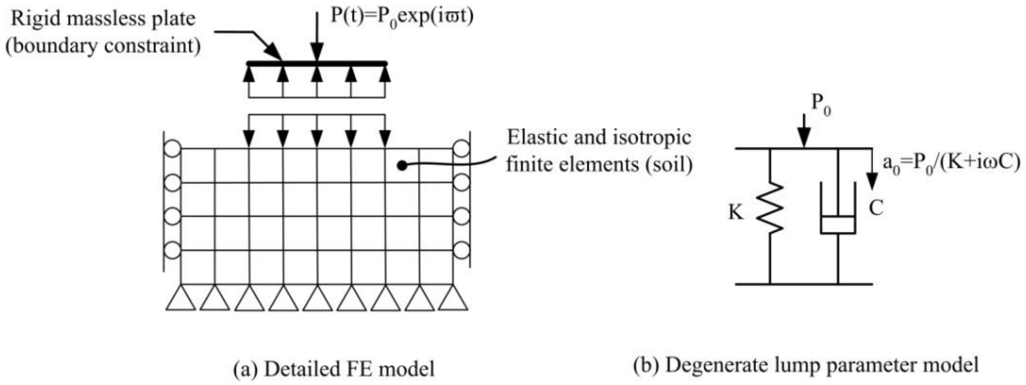

Figure 1 A finite element model to derive impedance

Consider a rigid massless plate overlying a discretized bounded volume of elastic and isotropic soil (also referred to as an elastic half-space) as shown in Figure 1. The plate is subjected to a transverse harmonic load of $P(t) = P_0 \exp(i\varpi t)$ with frequency ϖ. The plate being rigid will not undergo any flexural deformation and displacement at nodes underneath the plate will be equal. In order to model the soil with semi-infinite lateral and vertical extents, supports or boundary conditions are assigned at a sufficient distance away from the plate so that its displacement is not significantly affected. The governing equation of motion is given by Equation 1 below. The vector $\{p\}$ is the consistent nodal load vector obtained from uniform stress σ_0 where, $\sigma_0 = P_0/A$; A = area of the plate. It is to be noted that rigidity of the plate allows consideration of uniform stress σ_0.

$$[m]\{\ddot{x}\} + [c]\{\dot{x}\} + [k]\{x\} = \{p\}\exp(i\varpi t) \qquad (1)$$

where, $[m]$ = mass matrix; $[c]$ = viscous damping matrix; $[k]$ = stiffness matrix; and $\{\ddot{x}\}, \{\dot{x}\}, \{x\}$ = vectors of nodal acceleration, velocity and displacement, respectively. The steady state solution of Equation 1 is given as: $\{x\} = \{a\}\exp(i\varpi t)$. Now, Equation 1 can be rewritten as Equation 2 after substituting $\{\ddot{x}\}, \{\dot{x}\}, \{x\}$ in Equation 1.

$$[K]\{a\} = \{p\} \qquad (2)$$

where, $[K] = [k] - \varpi^2[m] + i\varpi[c]$ is also known as the impedance matrix, or simply, the impedance. The amplitude $\{a\}$ will be complex in nature. If the amplitude at the centre of the plate can be expressed as $a_0 = G_R + iG_I$ (G_R and G_I are real and imaginary coefficients), then a scalar impedance can be defined as $P_0/a_0 = \bar{k} + i\varpi\bar{c}$, where $\bar{k}$ is the stiffness of a lumped or discrete spring and $\bar{c}$ is the viscous damping coefficient of a discrete dashpot; the spring and dashpot are also collectively termed as lumped parameters. Hence, following the above procedure, dynamical properties of a continuum are condensed in two quasi kinematic properties. Both $\bar{k}$ and $\bar{c}$ depend on the size and shape of the plate, soil properties and the excitation frequency ϖ. For numerical analysis, solution of Equation 1 under sinusoidal loading $P(t) = P_0 \sin(\varpi t)$ can be considered. The solution is of the form $x = a_0 \sin(\varpi t - \varphi)$ where $G_R = a_0 \cos\varphi$; $G_I = a_0 \sin\varphi$. Table 1 provides a set of formulas for obtaining impedances of a square pad footing on a homogeneous half-space, taken from Gazetas (1991) for vertical, horizontal and rocking modes of vibration. The dimensionless parameter $\varpi B/V_s$ controls the functions f_{kv}, f_{kh}, f_{kr} and f_{cv}, f_{ch}, f_{cr} , which are the dynamic modification factors of the respective terms; the dynamical response increases with this parameter (since zero value is equivalent to static behaviour as $\varpi = 0$ and the solution of Equation 1 is simply $[k]^{-1}\{p\}$). Once the impedances are derived either numerically or from the available analytical solutions, SSI analysis is immensely simplified as exemplified in the next section.

Table 1: Formulas for impedances of a square pad foundation on a homogeneous half-space, after Gazetas (1991)

List of symbols:
A_b= foundation contact area; B = foundation width; G = shear modulus of soil; I_b = foundation contact second moment of area; L = foundation length; V_{La} = Lysmer's analog wave velocity; V_s = shear wave velocity of soil; c_h, c_v, c_r = viscous damping coefficients corresponding to horizontal, vertical and rocking mode of vibrations denoted by the respective subscripts; ${c_h}^R, {c_v}^R, {c_r}^R =$ radiation damping coefficients; f_{kh}, f_{kv}, f_{kr} = dynamic stiffness coefficient functions available in chart form; f_{ch}, f_{cv}, f_{cr} = radiation damping coefficient functions, also available in chart form; $k_h, k_v, k_r =$ dynamic stiffness; ρ = bulk density of soil; υ = Poisson's ratio; ϖ = excitation frequency; $\xi_h =$ hysteretic damping coefficient as a fraction of critical damping.

$$G = \rho {V_s}^2 \qquad V_{La} = \frac{3.4}{\pi(1-\nu)} V_s$$

$$k_v = \frac{4.54GB}{1-\nu} f_{kv}\left(\frac{L}{B}, \nu, \frac{\varpi B}{V_s}\right) \qquad k_h = \frac{9GB}{2-\nu} f_{kh}\left(\frac{L}{B}, \nu, \frac{\varpi B}{V_s}\right) \qquad k_r = \frac{3.6GB^3}{1-\nu} f_{kr}\left(\frac{L}{B}, \nu, \frac{\varpi B}{V_s}\right)$$

$${c_v}^R = \rho V_{La} A_b f_{cv}\left(\frac{L}{B}, \nu, \frac{\varpi B}{V_s}\right) \qquad {c_h}^R = \rho V_s A_b f_{ch}\left(\frac{L}{B}, \nu, \frac{\varpi B}{V_s}\right) \qquad {c_r}^R = \rho V_{La} I f_{cr}\left(\frac{L}{B}, \nu, \frac{\varpi B}{V_s}\right)$$

$$c_v = {c_v}^R + \frac{2k_v}{\varpi}\xi_h \qquad c_h = {c_h}^R + \frac{2k_h}{\varpi}\xi_h \qquad c_r = {c_r}^R + \frac{2k_r}{\varpi}\xi_h$$

2.2 A simplest case of SFSI: a single storey building with shallow foundation

In absence of SFSI, mass of a single degree of freedom (SDOF) system subjected to support acceleration experiences a total acceleration of $\ddot{u_g} + \ddot{x_1}$ ($\ddot{u_g}$ is the support acceleration and $\ddot{x_1}$ is the acceleration relative to the support) as shown in Figure 2c. This gives rise to the inertia force of $m(\ddot{u_g} + \ddot{x_1})$ acting at center of mass at height h above the foundation level. Hence, the foundation is subjected to a horizontal force of $m(\ddot{u_g} + \ddot{x_1})$ and an overturning moment of $m(\ddot{u_g} + \ddot{x_1})h$. If the soil is considered to be deformable, then it will undergo translation, x_2 and rotation θ which become additional unknowns. Hence, all SFSI models are in essence multi-degree-of-freedom (MDOF) systems. The mass of the superstructure now undergoes a total translation of $u_g + x_1 + x_2 + \theta h$. As shown in the figure, the semi-infinite soil medium is represented by two uncoupled translational (k_2, c_2) and rotational (k_3, c_3) impedances which can be obtained from Table 1. The equation of motion incorporating SSI is given by Equation 3 below in terms of variables $y_1 = x_1 + x_2 + \theta h$ and $y_2 = x_2$. This transformation renders the mass matrix diagonal for numerical convenience.

$$\begin{aligned} &\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & J_1 + J_2 \end{bmatrix} \begin{Bmatrix} \ddot{y_1} \\ \ddot{y_2} \\ \ddot{\theta} \end{Bmatrix} + \begin{bmatrix} c_1 & -c_1 & -c_1 h \\ -c_1 & c_1 + c_2 & c_1 h \\ -c_1 h & c_1 h & c_3 + c_1 h^2 \end{bmatrix} \begin{Bmatrix} \dot{y_1} \\ \dot{y_2} \\ \dot{\theta} \end{Bmatrix} + \begin{bmatrix} k_1 & -k_1 & -k_1 h \\ -k_1 & k_1 + k_2 & k_1 h \\ -k_1 h & k_1 h & k_3 + k_1 h^2 \end{bmatrix} \begin{Bmatrix} y_1 \\ y_2 \\ \theta \end{Bmatrix} \\ &= -\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & J_1 + J_2 \end{bmatrix} \{r\} \ddot{u_g} \end{aligned} \tag{3}$$

where, J is the mass moment of inertia; for a rectangular plate of dimensions l (along x-axis) and b (along y-axis), J about y-axis is $m\,l^2/12$ and about x-axis is $m\,b^2/12$, where m is the mass of the plate; and $\{r\}$=[1   1   0]^T is the influence vector of ground acceleration. It can be observed from Equation 3 that mode shapes are coupled and the damping matrix is non-proportional.

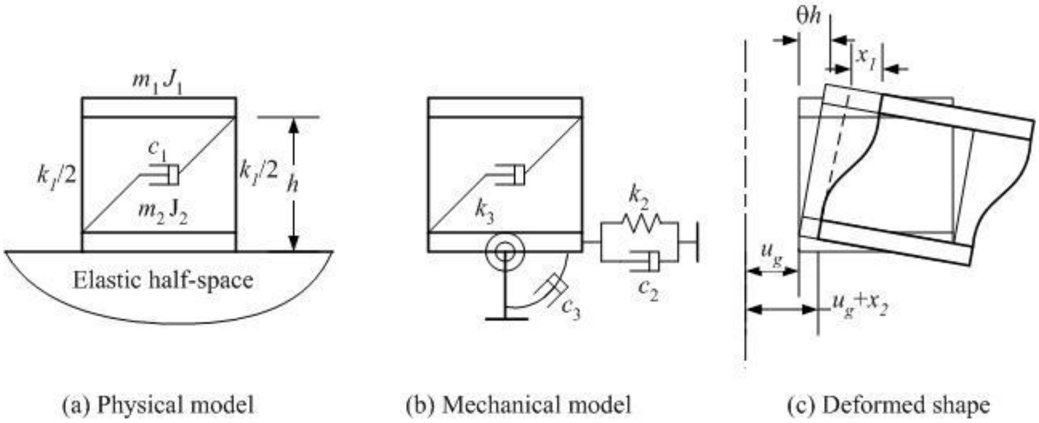

Figure 2 A simplest SSI model: a single storey building with shallow foundation

In the mode superposition method, such as the response spectrum method, off-diagonal terms of the damping matrix are neglected. The approximate fundamental frequency can be determined by the Dunkerley's method as shown in Equation 4 below in absence of a free vibration analysis.

$$\frac{1}{\omega_{eff}^2} = \frac{m_1}{k_1} + \frac{m_2}{k_1 + k_2} + \frac{J_1 + J_2}{k_3 + k_1 h^2} \tag{4}$$

This equation is a useful approximation of the fundamental frequency of the system. If it is compared with that of the fixed base system, then the additional flexibility incurred owing to the soil deformability in terms of a shift in frequency can be readily assessed. Subsequently, by judging the shift in frequency, a decision can be made whether to carry out a more rigorous SSI analysis. This equation is a useful approximation of the fundamental frequency of the system. If it is compared with that of the fixed base system, then the additional flexibility incurred owing to the soil deformability in terms of a shift in frequency can be readily assessed. Subsequently, by judging the shift in frequency, a decision can be made whether to carry out a more rigorous SSI analysis.

For use in response spectrum analysis, the equivalent modal damping ratio (ξ_n) is given by Equation 5 below.

$$\xi_n = \{\phi\}_n{}^T [C_s] \{\phi\}_n / 2\omega_n + \xi_h \tag{5}$$

where, $\{\phi\}_n$ is the mass normalised mode shape vector of mode n; $[C_s]$ is the diagonal matrix comprising radiation and hysteretic damping coefficients of soil; ξ_h is the constant hysteretic damping ratio of the structure which is generally considered as either 5% or 2% of the critical for reinforced concrete or steel structures, respectively. In the given example $[C_s]$ can be derived by putting $c_1 = 0$ in the damping matrix of Equation 3. Equation 5 shows that the effective damping depends on the shape of vibration which must be considered while evaluating effective damping; for example, the radiation damping is not as effective as it appears to be in the rocking mode of vibration (Gazetas, 1991).

As generally, $m_1 \ddot{y}_1 \gg m_2 \dot{y}_2, (J_1 + J_2)\ddot{\theta}$ only the inertia force due to m_1 can be considered, which allows the effective stiffness (k_{eff}) to be derived as below, following the static condensation procedure:

$$\frac{1}{k_{eff}} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{h^2}{k_3} \tag{6}$$

Hence the effective stiffness of the system becomes less than the minimum of the stiffnesses of the individual springs when soil flexibility is considered. This has an important implication, as the flexibility increases with the square of the height of the structure, in design of tall and slender displacement sensitive structures like elevated water tanks, bridge piers, stacks and chimneys, not only under earthquake but also under wind induced vibrations.

2.3 Governing equation of motion for generalised system

Although the simple SFSI model of Figure 2a reveals some critical design issues, but is not adequate for idealisation of an embedded foundation system. A general arrangement of a MDOF system with deep foundation is shown in Figure 3. Soil impedance is represented by independent springs and dashpots. A basic difference in SFSI analysis between surficial or shallow foundation and embedded or deep foundation is the spatial variation of ground acceleration along the depth of foundation, which renders the latter as a multiple support excitation problem. In Figure 3a, non-uniformity in ground acceleration is shown by far field response of soil column in a shear beam deformation mode under a vertically propagating horizontal shear (SH) wave. The only change that takes place in the governing equation of motion is the left hand side terms of Equation 3. In Figure 3, the superstructure is designated by a DOF and the foundation component excluding the impedances is designated by b DOF. It is assumed *a-priori* that the far-field ground acceleration is known at b DOF via a separate ground response analysis, which is represented by a vector term $\{\ddot{u_g}\}_{b,1}$ instead of the scalar term $\ddot{u_g}$ of Equation 3. The vector $\{r\}$ in Equation 3 expands to a matrix quantity $[r]_{a+b,b}$ which conceptually describes the influence of foundation displacements on the global structure. The complete expression of $[r]$ is available in Clough and Penzien (1993).

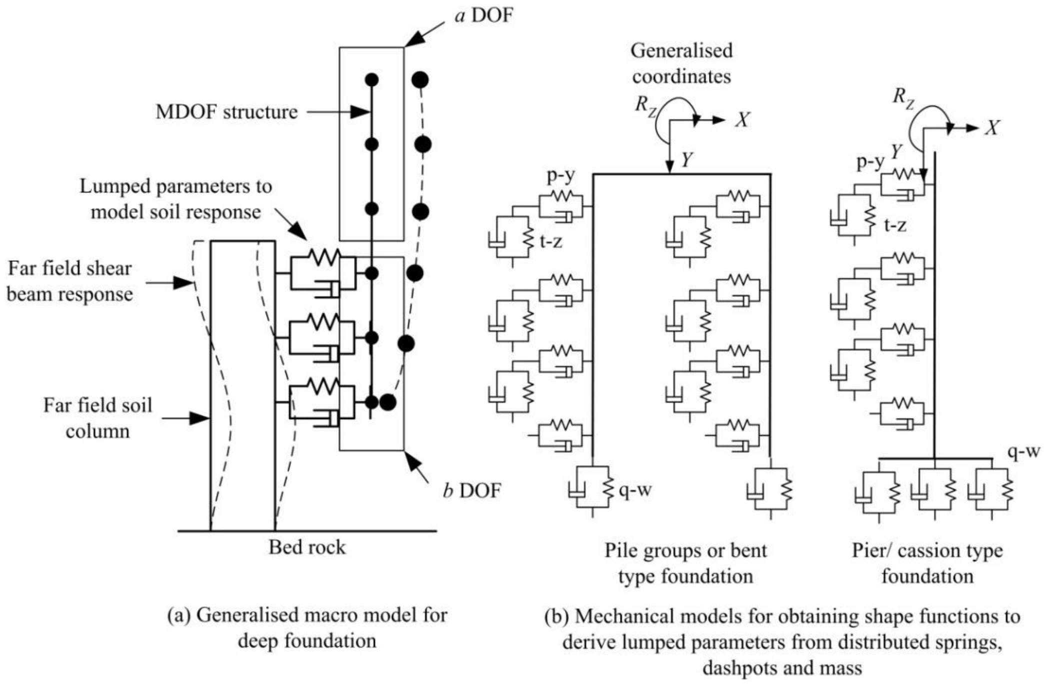

Figure 3 Lumped SFSI model for deep foundation

Pending a rigorous ground response analysis, it is often assumed that the ground acceleration is spatially uniform to the depth of interest. The spatial variation can approach uniformity only when the wavelengths of the prinicipal components of a vertically propagating SH wave is much greater than the depth of the foundation. When the ground acceleration is considered uniform, then it implies a rigid body displacement of the structure, and thus, every DOF experiences equal acceleration. This is equivalent to having a vector $\{r\}$ of unity as in Equation 3. Thus, with this assumption ground acceleration can be imposed in the sameway as that of the shallow foundations.

2.4 A procedure for lumping distributed impedances of deep foundations

It is computationally advantageous if the distributed impedance of a deep foundation can be lumped like a shallow foundation. Such a procedure is described below for both the pile and the pier (cassion) type foundations. It is well established that the vertical strength is derived from skin friction and end bearing mechanisms, indicated by *t-z* and *q-w* springs, respectively in Figure 3b; and lateral strength is generated by a mechanism represented as *p-y* springs. The force-displacement relation of these springs is non-linear and the methods for deriving these non-linear functions are available, for example, in the text by Reese et al. (2006). The stiffness thus derived is the static stiffness; the dynamic effect can be included by reducing the stiffness using some frequency dependent factors available in Gazetas and Dobry (1984a). However, such factors are usually close to unity for a wide range of frequency and can be neglected for practical purposes. It is more important to reduce the stiffness for group effect in case of pile foundation. Simplified closed form expressions of radiation damping coefficients corresponding to each of the three deformation mechanisms in Figure 3 are available in Gazetas and Dobry (1984b). Like stiffness, the radiation damping coefficients should be also reduced to accommodate group effect. The procedure described here is based on linear static analysis. Hence, instead of nonlinear springs with variable stiffness at different displacements, a constant stiffness should be used. Therefore, the nonlinear force-deformation curve should be idealised as elastic and perfectly plastic by assuming the area under the actual curve is same as the idealised one. First, a convenient generalised DOF should be located, which is usually chosen at the centre of a pile group or a pier foundation, as shown in Figure 3b. The quasi-static equilibrium condition of the foundation system with respect to the generalised DOF can be expressed by Equation 7 below.

$$\begin{bmatrix} k_{yy} & k_{yx} & k_{y\theta} \\ k_{xy} & k_{xx} & k_{x\theta} \\ k_{\theta y} & k_{\theta x} & k_{\theta\theta} \end{bmatrix} \begin{Bmatrix} y \\ x \\ \theta \end{Bmatrix} = \begin{Bmatrix} F_y \\ F_x \\ F_\theta \end{Bmatrix} \tag{7}$$

where, x, y, θ are the displacements; F_x, F_y, F_θ are the forces applied along the directions indicated by the subscripts; and $k_{(.)}$ represent the stiffness coefficients. The stiffness matrix can be determined by applying arbitrary forces F_x, F_y, F_θ in turn while restraining the DOF at the other two directions by assigning supports. The corresponding deflected shapes $(\phi_x, \phi_y, \phi_\theta)$ will be also known if the nodal displacements are divided by the displacement at the generalised DOF. Now the generalised mass matrix and radiation damping matrix can be calculated using well known formulas, as shown in Equation 8.

$$m_{ij} = \int \bar{m}\, \phi_i \phi_j dl; \quad c_{ij} = \int \bar{c}\, \phi_i \phi_j dl \tag{8}$$

where, m_{ij} and c_{ij} are the elements of the mass and damping matrix; subscripts i, j corresponds to directions x, y, θ; l is the length of the foundation; $\bar{m}$ is the mass of the foundation per unit length; $\bar{c}$ is the coefficient of radiation and hysteretic damping divided by the average length of the foundation elements on either side of a node. Thus, a deep foundation system with distributed springs and dashpots is lumped into a coupled mass, spring and dashpot system. Such lumped and coupled impedance for pile foundation is available in the text by Poulos and Davis (1980) which may relieve this computation. The procedure described here is general that can be applied for various deep foundation systems with different geometry and also for stratified soil deposits. Once the displacements at the generalised DOF are known from a global analysis, the foundation forces can be determined by applying these displacements in the analysis model of the foundation that was used to generate the lumped impedance. If desired, nonlinear static analysis based on foundation forces can be performed to check the serviceability criterion which more than often governs the design of foundations under lateral loads.

3 CONSTITUTIVE MODELLING

The methodology discussed in the previous sections is based on linear behaviour of soils and structure, but is equally applicable for nonlinear RHA given that the governing equation of motion in structural dynamics is expressed in incremental (differential) form. The differential form of the governing equation of motion in Equation 1 can be written as Equation 9, where the term $[k]\{x\}$ is written as $\{f_s(x, \dot{x})\}$ and differentiated using the method of partial differentiation.

$$[m]\{d\ddot{x}\} + [c]\{d\dot{x}\} + \left\{\frac{\partial f_s}{\partial x} dx + \frac{\partial f_s}{\partial \dot{x}} d\dot{x}\right\} = \{dp\} \tag{9}$$

In nonlinear RHA, equilibrium is enforced in 'incremental' sense rather than in 'total' sense. The restoring force $\{f_s\}$ is given as a nonlinear function of displacement and velocity in general sense. The framing of the nonlinear function $f_s(x, \dot{x})$ is referred to as so called 'constitutive modelling', and the function, as the so called 'constitutive model'. In a majority of constitutive models, other than the visco-plasticity models, the restoring force is considered to be independent of velocity; hence, $df_s = \partial f_s/\partial x\, dx$. In nonlinear analysis, $\partial f_s/\partial x$ is a function of x; if $\partial f_s/\partial x$ is a constant, then $f_s(x)$ is a linear function of x, and hence the terms 'linear' and 'nonlinear' arise.

There are two approaches to constitutive modelling. In the first approach, the restoring force is expressed by closed form analytical functions of displacement satisfying some experimentally observed key events in the plastic behaviour of the material concerned. This approach is based on the deformation or total strain theory of plasticity, where a direct nonlinear stress-strain relation can be framed because no distinction is made between total and elastic strains or displacements. In the second approach, which is based on the plastic flow theory, the stress-strain relation can be only expressed in non-integrable differential form, but such differential form depicts merely a change in the state of model in an infinitesimal time interval and does not depict rate over time. Another distinction is that the total strain comprises elastic and plastic strains, and the stress increment is linearly related to the elastic strain increment.

3.1 Deformation theory

It is well known that both strength and stiffness of sandy soils depend on effective pressure. While the strength is governed by Mohr-Coulomb's failure criterion, stiffness is governed by the power law: $G = G_r(p/p_r)^n$, where G is the initial shear modulus at effective pressure p; G_r is the initial shear modulus known at some reference pressure p_r; n is usually between 0.5 and 1. A stress (τ)-strain (γ) relation can be thus written as $\tau = f(\gamma, p)$, and the tangent modulus (G_t) is expressed as $G_t = \partial\tau/\partial\gamma$ i.e. by considering p = constant. Drained (p = constant) shear tests, such as triaxial, on different specimens with different void ratios have commonly shown monotonic τ-γ response with negative curvature; G_t approaching zero, and τ approaching a maximum threshold value (τ_{max}), both occurring at a large strain. This is the critical state for the sandy soils. Mathematical restrictions are thus: $G_t > 0$ and $G_t' < 0$ (where the prime denotes derivative with respect to γ). Several continuous functions can be framed with these governing differential equations, where the boundary or the critical state conditions are: $\lim_{\gamma\to\infty} G_t = 0$ and $\lim_{\gamma\to\infty} \tau = \tau_{max}$; and the initial condition is $G_t = G_{max}$. A simple and intuitive choice of function for G_t can be an exponentially decaying function: $G_t = G_{max}\exp(-\gamma/\gamma_0)$. Some examples of such functions are given from Equation 10 to 13 below. Equation 10 is obtained by integrating the aforementioned exponential function; Equation 11 is obtained from Boulanger et al. (2003), which is due to Prager (Chakrabarty, 2006) and also provides an excellent methodology for deep foundation analysis in liquefiable strata; Equation 12 is adopted from Ahn and Gould (1992); and Equation 13 is adopted from Hardin and Drnevich (1972).

$$\frac{\tau}{\tau_{max}} = 1 - \exp\left(-\frac{\gamma}{\gamma_0}\right) \tag{10}$$

$$\frac{\tau}{\tau_{max}} = \tanh\left(\frac{\gamma}{\gamma_0}\right) \tag{11}$$

$$\frac{\tau}{\tau_{max}} = \frac{\frac{\gamma}{\gamma_0}}{\left\{1 + \left|\frac{\gamma}{\gamma_0}\right|^n\right\}^{1/n}} \tag{12}$$

$$\frac{\tau}{\tau_{max}} = \frac{\frac{\gamma}{\gamma_0}}{1 + \left|\frac{\gamma}{\gamma_0}\right|} \tag{13}$$

where, $\gamma_0 = \tau_{max}/G_{max}$ and is often referred to as the reference strain. This list is not complete and there are several such functions, (see Stewart et al., 2008 for a comprehensive review), but whatever be the form of function, it is always within the aforementioned mathematical restrictions.

3.2 **Rate-independent plastic flow theory**

The deformation theory discussed in the previous section is only applicable for uniaxial cases. In a three dimensional stress problem, there are six independent stresses and strains, and the plastic flow theory attempts to derive a unique relation among their incremental quantities and entails several assumptions. A review of plastic flow theory based models is beyond the scope of the paper. Some successful models based on this theory are those of Bardet (1985); Wang et al. (1990); Cubrinovski and Ishihara (1998a); Yang et al. (2003); Dafalias and Manzari (2004); Yang and Elgamal (2008); Zhang and Wang (2012), amongst others. However, almost all the plastic flow theories for sandy soils require mathematical description (constitutive relations) of three main physical phenomena which are discussed as follows.

The first constitutive relation is a monotonically increasing shear stress – plastic shear strain relation which satisfies the same mathematical restrictions of a stress-strain relation of the deformation theory. Such a mathematical expression is often referred to as a 'hardening' function because $G_t = d\tau/d\gamma \geq 0$ and thus $d\tau \geq 0$ as long as $d\gamma \geq 0$ (here, γ is the plastic strain). Hence, shear stress continues to increase or the response 'hardens' with accumulation of plastic strain. This curve is experimentally derived and a convenient equation is chosen by trial and error that gives the closest fit. As the stress-strain relation of sand depends on the effective pressure, hardening functions are usually described for a constant effective pressure.

In the deformation theory, it is assumed that effective pressure remains constant. However, soils have a tendency to contract or expand even when directly sheared and consequently, there is a change in the effective pressure. Let the total volumetric strain be denoted as ϵ and shear induced volumetric strain (the plastic volumetric strain) as ϵ_d. The relation between the shear induced volumetric strain and the shear strain is written simply by using the rate formula: $d\epsilon_d = (\partial\epsilon_d/|\partial\gamma|)d\gamma$, or, $d\epsilon_d = Dd\gamma$ where $D = \partial\epsilon_d/|\partial\gamma|$ is known as the dilatancy, which is a function of stress ratio τ/p. The expression of dilatancy for sandy soils is usually Rowe's formula (Equation 14)

$$D = \sin\psi = \frac{\sin\phi_c - \sin\phi}{1 - \sin\phi_c \sin\phi} \tag{14}$$

where, ψ is the angle of dilation; $\sin\phi = \tau/p$; ϕ_c corresponds to the stress ratio when $D = 0$; from Equation 14, one can note that $|D| \leq 1$. Now, the change in effective pressure p can be written as $dp = -K_t(Dd\gamma - d\epsilon)$, where K_t is the elastic bulk modulus. As a result of the dilatancy, the effective pressure and the void ratio (hence, the relative density) change with the shear stress, which subsequently modify the elastic modulus and the hardening function. Consequently, the stress-dilatancy behaviour has the most significant impact on model performance and is least understood, particularly under cyclic loading. The majority of the models differ in this aspect and further research is required in this area.

In drained tests which are slow tests, specimens are allowed to consolidate at a constant pressure so that $d\epsilon = Dd\gamma$. However, earthquakes are extremely short duration phenomena and do not allow sufficient time for soils to consolidate. Hence, typically $d\epsilon \sim 0$ and the effective pressure reduces as proportional to the dilatancy. This is equivalent to a change in insitu state of soil to a relatively loose state, which is uniquely defined by its current void ratio and effective pressure. When sandy soils are sheared to a point of failure at a large strain, they attain unique combinations of void ratio and effective pressure. This unique function of void ratio and effective pressure is known as the Steady State Line (SSL) and is usually expressed in the form $e_{cr} = f(p)$(e_{cr} is the void ratio along the SSL). The relative state of the sand with respect to the SSL is known as the state parameter. The simplest definition of a state parameter is: $e_{cr} - e$ (Been and Jefferies, 1985); there are also other measures of the state parameter (e.g. Cubrinovski and Ishihara, 1998b; Wang et al., 2002). The elastic modulus, the parameters of the hardening function and the dilatancy depend on the state parameter. Thus, the dependence of the overall constitutive behaviour of sands on the current confining pressures and void ratios (hence, the relative densities) can be mathematically modelled via the state parameter. Some examples of models where the state parameter is incorporated are those of Cubrinovski and Ishihara (1998a) and Dafalias and Manzari (2004), amongst others.

4 CONCLUSION

The paper demonstrates the importance of soil-foundation-structure-interaction (SFSI) for tall and slender structures, and presents practical SFSI analysis procedures using impedance functions for shallow and deep foundations. A detailed SFSI analysis using finite continuum elements with plasticity based constitutive models is the most rigorous approach, but the complexity becomes a deterrent for practical application. SFSI analysis of structures of realistic proportions and complexity is often still based on discrete springs and dashpots, which produces results within an acceptable range of accuracy required for design. Simplified SSI is practically suitable and more convenient than a detailed finite element model to carry out several parametric variations for optimising a design.

REFERENCES

Ahn K. J., and Gould P. L. 1992. Interactive base-isolation foundation system: I. Finite element formulation. *Journal of Engineering Mechanics, ASCE,* 118(10), 2048-2058.

Applied Technology Council (ATC 40). 1996. *Seismic evaluation and retrofit of concrete buildings Volume 1,* California, USA.

Been K., and Jefferies M. J. 1985. A state parameter for sands. *Geotechnique*, 35(2), 99-112.

Bardet, J.P. 1985. Bounding surface plasticity model for sands. *Journal of Eng. Mech., ASCE,* 112(11), 1198-1217

Boulanger R. W., Kutter B. L., Brandenberg S. J., Singh P., and Chang D. 2003. Pile foundations in liquefied and laterally spreading ground earthquakes: Centrifuge experiments and analysis. *UCD/CGM-03/01,* University of California at Davis.

Chakrabarty J. 2006. *Theory of plasticity,* Elsevier Butterworth-Heinemann.

Clough R. W., and Penzien J. 1993. *Dynamics of structures*, McGraw-Hill.

Cubrinovski, M., Bradley, B., Wotherspoon, L., Russell, G., Bray, J., Wood, C., Pender, M., Allen, J., Bradshaw, A., Rix, G., Taylor, M., Robinson, K., Henderson, D., Giorgini, S., Winkley, A., Zupan, J., Rourke, T. O., DePascale, G., and Wells, D. (2011a). "Geotechnical aspects of the 22 February 2011 Christchurch earthquake." Bulletin of the New Zealand Society for Earthquake Engineering, 44(4), 205-226.Cubrinovski, M., and Ishihara K. 1998a. State concept and modified elastoplasticity for sand modelling. *Soils and Foundations*, 38(4), 213-225.

Cubrinovski, M., Bray, D. J., Taylor, M., Giorgini, S., Bradley, B., Wotherspoon, L., and Zupan, J. (2011b). "Soil liquefaction effects in the Central Business District during the February 2011 Christchurch earthquake." Seismological Research Letters, 82(6), 893-904.

Cubrinovski M., and Ishihara K. 1998b. Modelling of sand behaviour based on state concept. *Soils and Foundations*, 38(3), 115-127.

Dafalias Y. F., and Manzari M. T. 2004. Simple plasticity sand model accounting for fabric change effects. *Journal of Engineering Mechanics, ASCE,* 130(6), 622-634.

Gazetas G. 1991. Formulas and charts for impedances of surface and embedded foundations. *Journal of Geotechnical Engineering, ASCE,* 113(5), 458-475.

Gazetas G., and Dobry R. 1984a. Horizontal response of piles in layered soils. *Journal of Geotechnical Engineering, ASCE,* 110(1), 20-40.

Gazetas G., and Dobry R. 1984b. Simple radiation damping model for piles and footings. *Journal of Engineering Mechanics, ASCE*, 110(6), 937-956.

Hardin B. O., and Drnevich V. P. 1972. Shear modulus and damping in soils: Design equations and curves. *Journal of Soil Mechanics and Foundations Division, ASCE,* 98(6), 603-624.

Poulos H. G., and E. H. Davis. 1980. *Pile foundation analysis and design,* John Wiley & Sons.

Reese L. C., Isenhower W. M., and Wang S. T. 2006. *Analysis and design of shallow and deep foundations,* John Wiley & Sons.

Stewart P. J., Kwok A. O. L., Hashash Y. M. A., Matasovic, M., Pyke R., Wang Z., and Yang Z. 2008. Benchmarking of nonlinear geotechnical ground response analysis procedures. *PEER Report 2008/04,* University of California at Berkley.

Wang Z. L., Dafalias Y. F., Li X. S., and Makdisi, F. I. 2002. State pressure index for modeling sand behaviour. *Journal of Geotechnical and Geoenvironmental Engineering, ASCE*, 128(6), 511-519.

Wang Z. L., Dafalias Y. F., and Shen, C. K. 1990. Bounding surface hypoplasticity model for sand. *Journal of Eng. Mech., ASCE*, 116(5), 983-1001.

Wolf J. P., and Deeks A. J. 2004. *Foundation vibration analysis: A strength-of-materials approach*, Elsevier.

Yang, Z., and Elgamal, A. 2008. Multi-surface cyclic plasticity sand model with Lode angle effect. *Geotechnical and Geological Engineering*, 26(3), 335-348.

Yang Z., Elgamal A., and Parra E. 2003. Computational model for cyclic mobility and associated shear deformation. *Journal of Geotechnical and Geoenvironmental Engineering, ASCE*, 129(12), 1119-1127.

Zhang, J.M. and Wang, G. 2012. Large post-liquefaction deformation of sand, part I: physical mechanism, constitutive description and numerical algorithm. *Acta Geotechnica*, 7(2), 69-113