Sensitivity study of aftershock occurrence for a Wellington Fault earthquake

A. Christophersen & D. A. Rhoades

GNS Science, Lower Hutt, New Zealand.

S. Hainzl

GFZ, Potsdam, Germany.

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ABSTRACT: We investigate the sensitivity of the number of large aftershocks (N_{AS}) for three aftershock models to changes in model parameters. The three models are the Short-Term Earthquake Probability (STEP), for which we calculated the average N_{AS} for a set of parameters, and two implementations of the Epidemic Type Aftershock (ETAS) model, for which we run a number of simulations. The model parameters are the *p*-value that controls the temporal decay of aftershock activity, the *b*-value of the magnitude-frequency distribution of earthquakes, the productivity constant that is formulated in different ways for each of the three models, and $m_{\max}$, the maximum allowed magnitude for aftershocks. For all models N_{AS} is sensitive to the parameter changes that cover ranges well within observations for New Zealand aftershock sequences. The sensitivity to parameter variations is much stronger for the ETAS model than for the STEP model. Further, the range of N_{AS} for the ETAS model with the medium set of parameters is larger than the range of N_{AS} for the STEP model with varying parameters. The results of this sensitivity analysis contributes to two on-going larger projects: (1) the task of the 'It's Our Fault Project' to investigate whether there are any potential deficiencies in the currently used probabilistic Wellington earthquake design spectra due to the exclusion of aftershock, and (2) an EQC funded project to investigate the likelihood of late and large aftershocks from global earthquake catalogues.

1 INTRODUCTION

The recent Canterbury earthquakes have shown the devastating effects that aftershocks can have on a region. Yet, the New Zealand National Seismic Hazard Model (NSHM) (Stirling, 2012) follows the standard approach of probabilistic seismic hazard modelling and explicitly excludes aftershocks from the hazard estimations. One task of the 'It's Our Fault Project' recently investigated whether there are any potential deficiencies in the currently used probabilistic Wellington earthquake design spectra due to the exclusion of aftershocks (Rhoades, et al., 2012). The results suggest that not allowing for aftershocks may potentially result in the underestimation of the earthquake hazard in Wellington. The study used the Short-Term Earthquake Probability (STEP) model to estimate aftershock occurrence for a period of 50 years following scenario mainshocks of magnitude 7.3, 7.5 and 7.7 rupturing the Wellington-Hutt Valley segment. We now investigate the sensitivity of N_{AS}, the number of large aftershocks ($M \geq 5.0$) in a 50-year time period following a magnitude 7.5 mainshock of the Wellington Fault to variation in a range of model parameters for three different aftershock models. The three models are the STEP model and two implementations of the Epidemic Type Aftershock (ETAS) model. All models have a spatial component and a method to distribute the expected number of aftershocks in space. The spatial distribution of aftershocks is important for earthquake design spectra at selected locations. However, for the purpose of this study, we focus on the effects of the parameters variations on the number of large aftershocks.

In the next section we briefly introduce the general behaviour of aftershocks, i.e. the decay of aftershock rate with time, the magnitude-frequency distribution of aftershocks, and the increase of aftershock number with mainshock magnitude. We then introduce the three models that we apply to forecast aftershocks for a hypothetical Wellington Fault event. We discuss the parameter variation used in the modelling and show the effect on N_{AS} as well as the variation of aftershock number between simulations with the same parameter values. Finally, we discuss the practical implications of the results.

2 AFTERSHOCK BEHAVIOUR

2.1 The Omori-Utsu law for aftershock decay

The Omori law is the oldest empirical relationship in seismology and originally described how the number of felt aftershocks per day decayed with time t as $1/t$ for the 1891 Nobi, Japan earthquake (Omori, 1894). The relationship was found to still be ongoing 100 years after the Nobi earthquake (Utsu, et al., 1995) which can be partly attributed to Nobi being located in an otherwise relatively seismically quiet area in Japan. In contrast, our current and ongoing analysis shows that the aftershocks of the March 2011 M9.3 Tōhoku earthquake are already nearly indistinguishable from the background seismicity, i.e. the current seismicity levels are no longer much above the long-term average.

Equation 1 shows the modified Omori-Utsu law, which allows the parameter p to control the temporal decay. The parameter K represents the productivity, and while the meaning of the constant c is still being debated, its presence is necessary to avoid a singularity in the modelled number of aftershocks at time $t = 0$.

$$\dot{n} = \frac{K}{(t + c)^{p}} \tag{1}$$

Integration of Equation 1 over time provides the expected number of aftershocks as illustrated in Figure 1 with New Zealand generic parameters (see STEP model below), and for 30 ETAS simulations using the Hainzl implementation (introduced below).

2.2 The Gutenberg-Richter relation for the magnitude-frequency distribution

The Gutenberg-Richer relation describes the magnitude-frequency distribution of earthquakes, where the number of earthquake of magnitude M decreases exponentially with increasing magnitude $N(M)\sim 10^{-bM}$ (Gutenberg & Richter, 1944; Ishimoto & Iida, 1939). The relationship also holds for the distribution of aftershock magnitudes (Utsu, 1969). The b-value describes the relationship between small and large earthquakes and is included in our sensitivity analysis.

2.3 The abundance law for aftershock productivity

The number of expected aftershocks increases exponentially with mainshock magnitude M_m as $N(M_m) \sim 10^{\alpha M_m}$. The growth parameter α is often assumed to be identical to the b-value in the Gutenberg-Richter relation as in STEP and one of the ETAS models implementations.

3 AFTERSHOCK MODELLING

3.1 The STEP model

The Short-Term Earthquake Probability (STEP) model is based on the idea of superimposed Omori sequences (M. Gerstenberger, 2003; M. C. Gerstenberger, et al., 2005), where every earthquake triggers its own aftershock decay according to Equation 1. The model is usually applied for short-term earthquake forecasting for active earthquake sequences where it is up-dated as new data become available. There is computer code available (STEP Java) that we used to calculate expected number of aftershocks for a mainshock. The model has two components: 1) a background seismicity model; and 2) a time-dependent clustering model. However, in the version of the model applied here the background component plays no part as we only study the number of aftershocks. The clustering model has three components: 1) generic; 2) sequence specific and 3) locally varying within a sequence. The mixture of the components depends on data availability for parameter fitting and comparison of the performance of each of the parts. In our study of applying the model to only one large mainshock, we use the generic component, for which the productivity parameter K in Equation 1 is replaced by a mainshock magnitude dependent equation (Reasenberg & Jones, 1989)

$$\dot{n} = \frac{10^{a+b(M_m - M_{\min})}}{(t+c)^p} \tag{2}$$

Two studies investigate the parameter values for Equation 2 in New Zealand (Eberhart-Phillips, 1998; Pollock, 2007). Here we use the values by Pollock as given in Table 1.

3.2 The ETAS model (Hainzl implementation)

The ETAS model is a stochastic point-process model that is also based on overlapping Omori sequences according to Equation 1 (Ogata, 1988). In the Hainzl implementation each earthquake has a magnitude M dependent ability to trigger aftershocks according to $k_0 10^{\alpha(M - M_{\min})}$, where $M_{\min}$ is the lower cut-off magnitude of earthquakes of interest (Hainzl, et al., 2008). For the purpose of our parameter sensitivity analysis we set the ETAS model parameters equivalent to the generic New Zealand parameters (see Table 1). Thus we need to calculate k_0 to be equivalent to 10^a in Equation 2. We use the equations below for the branching parameter n_{br}, i.e. the number of direct aftershocks per earthquake averaged over all magnitudes in the magnitude interval [$m_{\min}$, $m_{\max}$](Sornette & Werner, 2005),

$$n_{br} = \frac{k_0 b \ln(10)(m_{max} - m_{min})}{1 - 10^{-b(m_{\max} - m_{\min})}} = k_0 f(b, m_{\min}, m_{\max}) \text{, for } \alpha = b \tag{3}$$

$$n_{br} = \frac{k_0 b}{b - \alpha}\left(\frac{1 - 10^{-(b-\alpha)(m_{max} - m_{min})}}{1 - 10^{-b(m_{\max} - m_{\min})}}\right) = k_0 f(b, \alpha, m_{\min}, m_{\max}) \text{, for } \alpha \neq b \tag{4}$$

For n_{br} <1, the total number of direct and indirect aftershocks of the initial mainshock is given by,

$$N_{total} = \frac{N_0}{1 - n_{br}} \tag{5}$$

(Sornette & Werner, 2005). We can deduct,

$$k_0 = \frac{10^a}{1 + 10^a f(b, \alpha, m_{\min}, m_{\max})} \tag{6}$$

This is for an unlimited time period. To adjust k_0 for the simulation over a limited time period T, we also need to add a factor for the integration of Equation 1 with respect to time

$$f_T = (c^{(1-p)} - (c+T)^{(1-p)})/(p-1)\,. \tag{7}$$

Figure 1 includes 30 simulations of the ETAS-Hainzl model with the medium parameters as shown in Table 1. The productivity of the model was calculated so that the mean number of aftershocks would match the STEP model after 50 years. The figure shows that the ETAS model covers a range of realisations far beyond the 95% confidence interval of the STEP model. Each jump in the cumulative curve corresponds to a large aftershock that triggered its own aftershock sequence similar to the February 2011 Christchurch earthquake (Kaiser, 2012). For the analysis of the mean number of large aftershocks with varying parameters we use 1,000 simulations of the model.

3.3 The ETAS model (Rhoades implementation)

The Rhoades ETAS model has been fitted to the New Zealand earthquake catalogue with magnitude M > 2.95, targeting the forecast of earthquakes with M > 3.95 over the period 1987-2006, and is implemented in the New Zealand earthquake testing centre (M.C. Gerstenberger & Rhoades, 2010).

The model parameters are given in Table 1. The key difference to the Hainzl model implementation are: α and b are always assumed to be the same. The equations for aftershock decay magnitude frequency distribution are implemented as probability density functions, and thus the productivity parameter is about an order of magnitude larger than for the Hainzl model as can be seen in Table 1. The productivity parameter is estimated from the New Zealand data. Estimating ETAS from earthquake challenges as illustrated nicely recently for New Zealand (Harte, 2013).

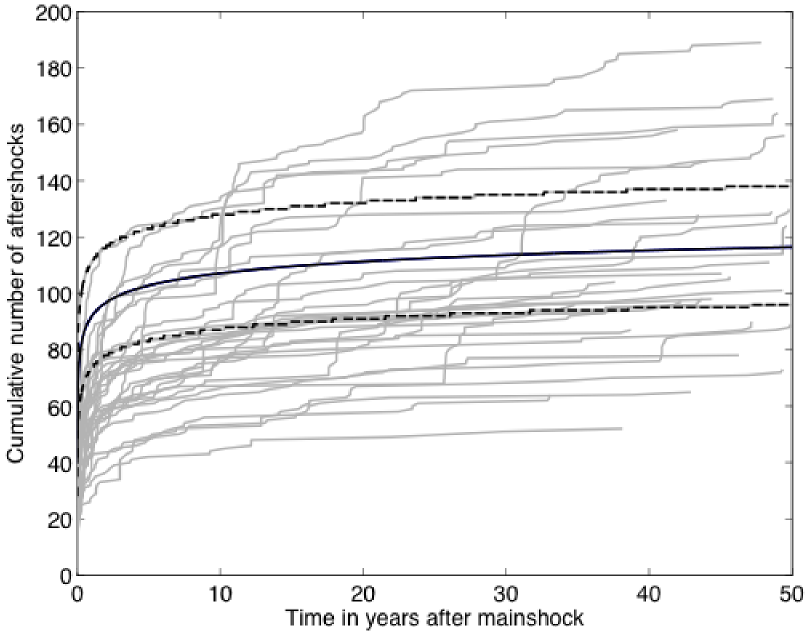

Figure 1: The cumulative number of aftershocks $M \geq 5.0$ for a 50 year time period. The solid black line represents the STEP model with generic parameters for New Zealand. The dashed lines are 95% confidence lines of the model assuming a Poisson distribution. The light grey lines show 30 simulations for the ETAS Hainzl model with parameters equivalent to the STEP model. The magnitude range for the simulations is [3, 7.5].

4 EFFECT OF PARAMETER VARIATION ON NUMBERS OF AFTERSHOCKS

4.1 Variation of model parameters

The purpose of this study is to investigate the sensitivity of the number of large aftershock N_{AS} to model parameter variation. We decided to vary the p-value of Omori's law and the b-value of the magnitude-frequency relation by plus and minus 0.1 as per Table 1, which is well within the range of observed values for New Zealand (Pollock, 2007). In the Hainzl model we left the α-value unchanged when changing the b-value. The other two models intrinsically assume $\alpha = b$. We also varied m_{max}, the maximum magnitude for aftershocks for the two ETAS models, with the medium value allowing the largest aftershock to be the size of the mainshock. Table 1 shows the medium values for each model parameter and the change.

Table 1. Variation of model parameters.

Model	c	p	b	Productivity	m_{max}
STEP	0.04	1.07±0.10	1.03±0.10	$10^{-1.59}$±20%	n/a
ETAS Hainzl	0.04	1.07±0.10	1.03±0.10	0.0065±20%	6.9, 7.5, 8.0
ETAS Rhoades	0.03	1.11±0.10	1.16±0.10	0.072±20%	6.95, 7.55, 8.05

4.2 The mean number of large aftershocks for different model parameters

Table 2 shows N_{AS} for the three different models and the range of model parameters. The STEP Java code with generic parameters produces 103 aftershocks of $M \geq 5.0$, which is slightly less than 116 resulting from integrating Equation 4 over 50 years. This small difference is probably caused by cutting in space. The ETAS Hainzl implementation is very similar to the STEP model as it was set up to be. The ETAS Rhoades model forecast on average about 50% more aftershocks than the STEP model. Figure 2 illustrates the results for the four sets of parameter values that were varied. Below we discuss the observation for the different parameters.

Table 2. Mean number of aftershocks $M \geq 5.0$ for different model parameters. The bomb symbolises that the number of aftershocks increased explosively with the respective parameter setting.

Model	Ave	*p*+	*p*-	*b*+	b-	Prod +	Prod-	m_{max} +	m_{max} -
STEP	103	85	140	183	58	124	82	n/a	n/a
ETAS Hainzl	109	68	524	38	💣	275	57	153	81
ETAS Rhoades	153	466	44	341	71	274	85	190	125

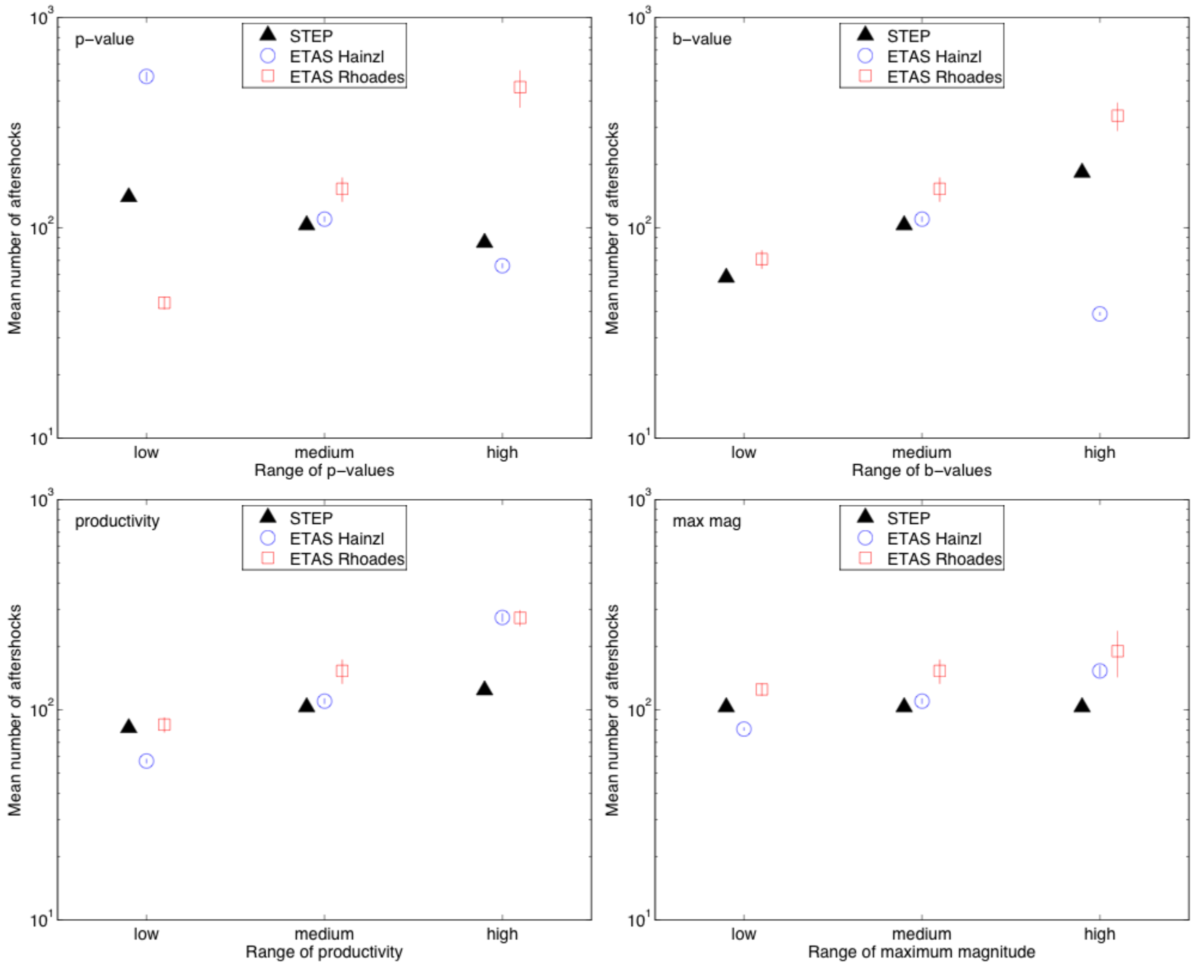

Figure 2. The comparison of the mean number of aftershocks for variation of the model parameters *p*-value, *b*-value, productivity and maximum magnitude for the STEP and the two ETAS model implementations. The error bar for the ETAS models shows two standard deviation of the mean.

4.2.1 *p-value variation*

An increase in *p*-value corresponds to faster decay of aftershock activity, and thus more large aftershocks would be expected in the first 50 years. Similarly, a decrease in *p*-value leads to slower aftershock decay and fewer aftershocks would be expected in the first 50 years. This trend can only be observed for the Rhoades model, for which the temporal decay is implemented as a proper probability density function. For the other two models, the omission to integrate over all times according to Equation 9, keeps the productivity value 10^a or k_0 higher that it should be and therefore the opposite trend is observed. Decreasing the *p*-value by 0.1 in the Hainzl model leads to a near five times increase in N_{AS} the largest observed variation for all parameters and models.

4.2.2 *b-value variation*

A decrease of the *b*-value should lead to an increase in large aftershocks; and an increase of the *b*-value should lead to a decrease in larger aftershocks. This trend can only be observed for the Hainzl model, which kept the growth parameter α of the productivity unchanged when varying the *b*-value. Decreasing the *b*-value by 0.1 with constant α led to an explosion of aftershocks as indicated by the bomb in Table 2 and 3. The other two models implicitly assume that $\alpha = b$ and for increasing α the average number is expected to go up.

4.2.3 *Variation of the productivity parameter*

A decrease in the productivity parameter should lead to a decrease in aftershock numbers and vice versa, an increase in the productivity parameter should lead to an increase in aftershock numbers. This trend can be observed for all three aftershock models. For the STEP model a 20% change in the productivity results in about a 20% change in the number of aftershocks. The change for the two ETAS models is much larger, possibly due to the epidemic nature of the model set-up, the effect of the parameter variation is compounded over the generations.

4.2.4 *Variation of the maximum magnitude*

The maximum magnitude of aftershocks is not a model parameter for the STEP model and thus the mean number of large aftershocks remains unchanged. For the ETAS model, a larger maximum magnitude leads to an increase in N_{AS} if all other parameters are unchanged. To keep the average number of large aftershocks the same for the Hainzl model, the productivity k_0 would have to be decreased from 0.0065 to 0.0060 for an increase of $m_{\max}$ from 7.5 to 8.0 according to Equation 6.

4.3 **The variation of number of large aftershocks for different model parameters**

The range of N_{AS} for 30 simulations in Figure 1 exceeds the range of N_{AS} for all parameter changes in the STEP model. Figure 3 shows the frequency of occurrence of N_{AS} for 1000 simulations of the ETAS Hainzl model for the medium model parameters. To compare the variation between different parameters we calculate the coefficient of variation, i.e. the standard deviation divided by the mean as presented in Table 3. The coefficient of variation of N_{AS} is largest when $m_{\max}$ is increased and smallest when $m_{\max}$ is decreased. This makes intuitive sense as the magnitude of the aftershocks determines the size of further generations of aftershocks.

Table 3. The coefficient of variation of N_{AS} for the two ETAS model implementations. The bomb symbolises that the number of aftershocks increased explosively with the respective parameter setting.

Model	Ave	*p*+	*p*-	*b*+	*b*-	Prod +	Prod -	$m_{\max}$ +	$m_{\max}$ -
ETAS Hainzl	0.50	0.44	0.81	0.32	💣	0.75	0.40	0.98	0.28
ETAS Rhoades	0.45	0.71	0.19	0.53	0.32	0.30	0.27	0.88	0.19

5 SUMMARY OF RESULTS

We employ three models to analyse the sensitivity of the number of large aftershock N_{AS} to parameter changes. All models are sensitive to all the parameter changes, except for the STEP model that is not affected by the maximum magnitude $m_{\max}$. Varying a specific parameter does not always have the expected effect, e.g. the *b*-value in the STEP and ETAS-Rhoades model, or the *p*-value in the STEP and ETAS-Hainzl model. The ETAS models show larger sensitivity in N_{AS} than the STEP model. The range of N_{AS} for the ETAS-Hainzl model with medium parameters exceeds the range of N_{AS} for the STEP model for the parameter changes. Also changing any one parameter has a much larger effect on the mean N_{AS} for the ETAS models compared to the STEP model. This is partly caused by the compounding effect of different generations of aftershocks in the ETAS simulation and partly by the correlation of parameters as shown in Equations 3-7.

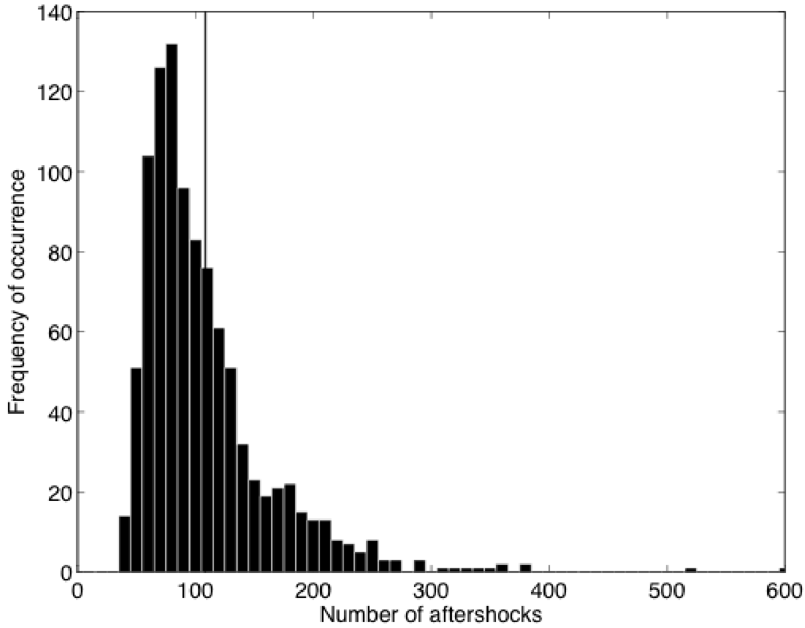

Figure 3: The frequency of occurrence of $M \geq 5.0$ aftershocks within 50 years for 1000 simulations of the ETAS Hainzl model for the medium model parameters. The bin width is 10.

6 DISCUSSION AND FUTURE OUTLOOK

In simulating aftershock sequences, it is important to appreciate the possible range of numbers of large aftershocks N_{AS} for a set of parameter, as well as the sensitivity of N_{AS} to parameter changes. Work is still on going to fine-tune the model parameters for realistic representation of aftershocks following a large Wellington Fault earthquake and other earthquakes in New Zealand. An EQC project is close to completion to investigate the Canterbury earthquake sequence within the context of global earthquake statistics. The focus of this work is on the likelihood of large and damaging late aftershocks. Figure 1 of the cumulative number of large aftershocks with time shows bursts of activity on a time-scale of decades. Similar bursts of activity have been observed in Canterbury on a time-scale of 18 months. One might wonder whether it is realistic to model aftershock sequences for decades. Recalling that the Nobi aftershock sequence was still observed to be on-going a century after the mainshock, and that each aftershock can trigger its own cascade of aftershocks, which follow the Gutenberg-Richter relation, it is possible that any of the late aftershocks has a magnitude large enough to be of concern. The 1968, M_w 7.2 Inangahua Earthquake occurred within 30 km of the 1929, M_w 7.3 Buller Earthquake. The 1962, M_w 5.7 Westport and the 1991, M_w 5.9 Hawks Crag earthquakes also occurred within the aftershock zone of the 1929 Buller Earthquake. Are they all part of the same 60 years-plus lasting sequence (Litchfield & Berryman, 2012)? Up-dates to the New Zealand National Seismic Hazard Model that will no doubt attempt to accommodate aftershock hazard, will need to consider these issues.

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REFERENCES

Eberhart-Phillips, D. 1998. Aftershock sequence parameters in New Zealand. *Bull. Seism. Soc. Am.,* 88, 1095-1097.

Gerstenberger, M. 2003. Earthquake clustering and time-dependent probabilistic hazard analysis for California (Ph.D. thesis).

Gerstenberger, M. C., & Rhoades, D. A. 2010. New Zealand earthquake forecast testing centre. *Pure and Applied Geophysics,* 167, 877-892.

Gerstenberger, M. C., Wiemer, S., Jones, L. M., & Reasenberg, P. A. 2005. Real-time forecasts of tomorrow's earthquakes in California. *Nature,* 435(7040), 328-331.

Gutenberg, R., & Richter, C. F. (1944). Frequency of earthquakes in California. *Bulletin of the Seismological Society of America,* 34, 185-188.

Hainzl, S., Christophersen, A., & Enescu, B. 2008. Impact of earthquake rupture extensions on parameter estimations of point-process models. *Bull. Seism. Soc. Am.,* 98(4), 2066-2072.

Harte, D. S. 2013. Bias in fitting the ETAS model: a case study based on New Zealand seismicity. *Geophysical Journal International,* 192(1), 390-412.

Ishimoto, M., & Iida, K. (1939). Observations of earthquakes registered with the microseismograph constructed recently. *Bull. Earthq. Res. Inst.,* 17, 443-478.

Kaiser, A., Holden, C., Beavan J, Beetham, D. , Benites, R., Celentano,A., Collett, D., Cousins,J., Cubrinovski, M., Dellow, G., Denys, P., Fielding, E., Fry, B., Gerstenberger, M., Langridge, R., Massey, C., Motagh, M., Pondard, N., McVerry, G., Ristau, J., Stirling, M, Thomas, J., SR Uma & Zhao, J. 2012. The Mw 6.2 Christchurch earthquake of February 2011: preliminary report. *New Zealand Journal of Geology and Geophysics,* 55(1), 67-90.

Litchfield, N. J., & Berryman, K. R. 2012. *Times and distances between events from selected global earthquake sequences; comparison with the 2010-2012 Canterbury earthquake sequence*: GNS Science Report 2012/43.

Ogata, Y. 1988. Statistical models for earthquake occurrences and residual analysis for point processes. *Journal of American Statistical Association,* 83(401), 9-27.

Omori, F. 1894. On aftershocks. *Report of Imperial Earthquake Investigation Committee,* 2, 103-109.

Pollock, D. 2007. *Aspects of short-term and long-term seismic hazard assessment in New Zealand.* Unpublished Master Thesis, ETH Zurich.

Reasenberg, P. A., & Jones, L. M. 1989. Earthquake hazard after a mainshock in California. *Science,* 243, 1173-1176.

Rhoades, D. A., Stirling, M., Gerstenberger, M. C., McVerry, G. H., & Van Dissen, R. J. 2012. *It's Our Fault - Contribution of Wellington Fault rupture aftershocks to the long-term earthquake hazard: A Progress Report*: GNS Science Consultancy Report 2012/193.

Sornette, D., & Werner, M. J. 2005. Constraints on the size of the smallest triggering earthquake from the epidemic-type aftershock sequence model, Bath's law, and observed aftershock sequences. *Journal of Geophysical Research-Solid Earth,* 110(B8).

Stirling, M., McVerry, G., Gerstenberger, M., Litchfield, N., Van Dissen, R., Berryman, K., Barnes, P., Wallace, L., Villamor, P., Langridge, R., Lamarche, G., Nodder, S., Reyners, M., Bradley, B., Rhoades, D., Smith, W., Nicol, A., Pettinga, J., Clark, K., Jacobs, K. 2012. National seismic hazard model for New Zealand: 2010 update. *Bulletin of the Seismological Society of America,* 102(4), 1514-1542.

Utsu, T. 1969. Aftershocks and earthquake statistics (I) Source parameters which characterize an aftershock sequence and their interrelations. *J.Fac. Sci., Hokkaido Univ., Ser.,* VII(3), 129-195.

Utsu, T., Ogata, Y., & Matsu'ura, R. S. 1995. The centenary of the Omori formula for a decay law of aftershock activity. *Journal of the Physics of the Earth,* 43, 1-33.