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Comparison of different global optimization algorithms for model updating with an application to a full-scale bridge structure

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ABSTRACT: Due to inherent simplifying assumptions in the finite element (FE) models, the actual behaviour of full scale structures often differs resulting in incorrect detection of the dynamic response under seismic conditions. On site measurements may reveal important differences between measured data and predictions from an FE model. In model updating, dynamic measurements such as natural frequencies, mode shapes and damping ratios are correlated with their FE model counterparts to calibrate the FE model. In this paper, different optimization techniques for model updating have been investigated. Different global optimization algorithms (GOAs), including particle swarm optimization (PSO), genetic algorithms (GAs) and simulated annealing (SA), were used for model updating. The results are compared in terms of accuracy of the updated frequencies. The first part of the paper gives the details of the modal testing of a full scale cable stayed footbridge. The bridge is composed of two spans with composite steel concrete deck, a centrally located steel pylon and six pairs of stays. The bridge was excited using three dynamically synchronized shakers. A dense array of sensors was employed to measure the response. The second part describes model updating of the bridge FE model. The aforementioned GOAs were used to calibrate the FE model with the experimental results. The paper concludes with a discussion on the efficacy of using the different GOAs to obtain a representative FE model.

1 INTRODUCTION

Bridges are important components for the modern infrastructural development. The construction of cable stayed bridges is increasing worldwide due to their distinctive styles. These unique and modern styles are adopted due to various aesthetical and structural reasons but can add difficulties in assessing the performance of the final built structure. Especially, the slender design of cable stayed bridge structures often renders serviceability as governing criteria and can pose problems in accurate prediction of the dynamic system response.

It has been reported in literature that there are often significant differences in the finite element (FE) model and built structure due to inherent simplifying assumptions in the FE models (Friswell and Mottershed 1995). On site measurements reveal important differences between original structure and its FE model. These differences can be attributed not only to modelling errors associated with simplification of a complicated structure but also from parametric errors in the estimation of geometry, materials properties, and boundary conditions. Model updating is a branch of structural optimization which calibrates the FE model by comparing the modal properties of the built structure with these of the FE predictions.

The success of model updating is dependent on an appropriate numerical model, correct identification of experimental parameters, choice of physically significant updating parameters, meaningful error function and efficient global optimization algorithm (Brownjohn and Xia 2000; Zárate and Caicedo 2008). As the number of measurements available is usually much smaller than the number of uncertain parameters, and, consequently, not all uncertain parameters are selected for model updating, different local minima may exist in the solution space for a specific error function. The use of different global optimization algorithms (GOAs) to solve this problem has gained interest in the last decade (Perera et al. 2009; Tu and Lu 2008). This study attempts to investigate the effectiveness of several popular

GOAs when applied to model updating of dynamic systems.

In this paper, model updating of a full scale cable stayed footbridge is carried out using GOAs. In the sections to follow, a description of the full scale bridge is given and initial FE model assumptions are explained. Then experimental testing and system identification of the bridge is explained. Finally, FE model is updated by application of different GOAs.

2 DESCRIPTION OF THE BRIDGE AND INITIAL FINITE ELEMENT MODELLING

The full scale bridge under study is a 60 m long cable stayed footbridge with two spans of 30 m each separated by a central A-shaped pylon over a 4 lane motorway as shown in Figure 1. The bridge deck is composed of a trapezoidal steel girder with a cast in situ concrete slab of thickness 130 mm (Figure 2). The deck is continuous over the entire span and is supported by a total of 12 pre-tensioned cables with 8 m centre to centre distances. Different pre-tension forces have been applied to the cables ranging from 55 kN to 95 kN. The cables are connected to the top of the centre pylon, which is composed of two steel I-sections joined with cross bracing. The bridge deck is supported on the cross bracing at the middle of the bridge deck. The size of the pylon I-section is 400WC328 and the diameter of the cables used is 32 mm.

Figure 1. Full scale cable stayed footbridge

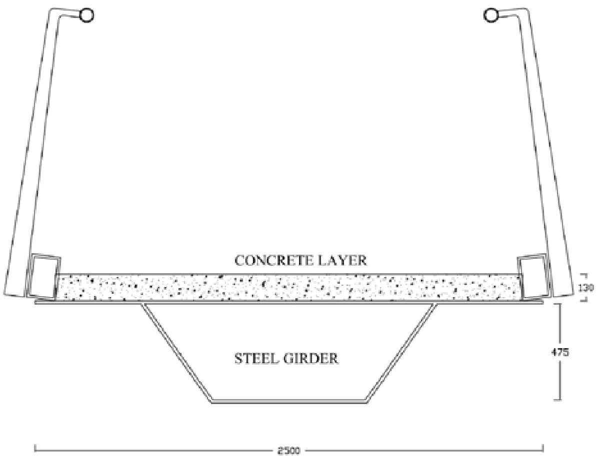

Figure 2. Trapezoidal steel-concrete composite deck

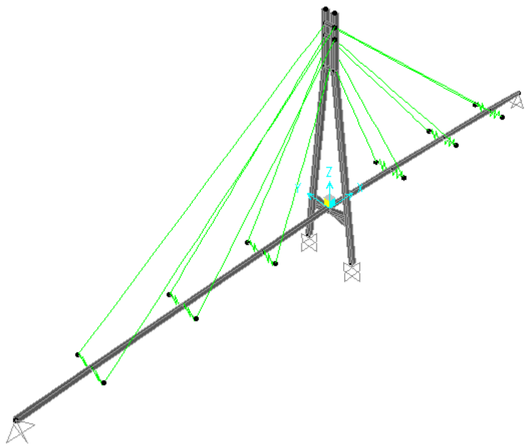

Figure 3. FE model of the bridge using rigid elements connecting cables to the deck

Main elements to be modelled in case of cable bridges are bridge deck, pylon, cables and connections of cables and deck. There are many ways to model cable bridges to obtain a realistic representation of their dynamic behaviour. A reasonable representation of bridge deck can be achieved by using beam elements with rigid links joining the cable elements with deck elements (Chang et al. 2001). The bridge was modelled in SAP2000 (SAP 2000) and the FE model is shown in Figure 3. The girder and pylon were modelled using the Euler-Bernoulli type finite elements that ignore shear deformations. This simplification does not have significant effect on modal properties of the FE model of the bridge under study. Cables were modelled as catenary elements provided in SAP2000. An initial non-linear static analysis was performed to account for the geometric non-linearity caused by the cable sag and was followed by a linear dynamic analysis to obtain natural frequencies and mode shapes. A linear analysis that uses stiffness from the end of non linear static analysis for cable stayed structures has been demonstrated to provide accurate results (Abdel-Ghaffar and Khalifa 1991).

3 EXPERIMENTAL PROGRAM AND SYSTEM IDENTIFICATION

Experimental work has been carried out using 12 uni-axial Honeywell QA 750 accelerometers to measure structural response, 3 uni-axial Crossbow MEMS accelerometers to measure shaker input force and a desktop fitted with NI DAQ 9203 data acquisition card. Data was collected at a sampling rate of 200 Hz. Three APS ElectroSeis Model 400 shakers (APS Dynamics 2010) were used in synchronised mode to impart a combined excitation force of up to 1.2 kN to the structure.

Full scale tests can be conducted by output only (no measured force) or input-output (measured force) methods. The cable stayed bridge under study has been tested using both of these methods. The first test was conducted using jumping to establish the initial estimation of the natural frequencies of the bridge. Two people jumped several times on the bridge in unison to excite the structure and thereafter the bridge was allowed to freely vibrate for two minutes. This was done to establish the range of excitation frequencies for subsequent forced vibration tests. The forced vibration test was then conducted using three dynamically synchronised shakers and their input force was measured using accelerometers mounted on the armature. A sweep sine excitation frequency ranging from 1 to 15 Hz with total sweep duration of 700 s was adopted to adequately excite the structure. To obtain the natural frequencies in vertical as well as horizontal directions, the shakers were tilted at 90 degrees so that the structure can be excited in both the directions. Figure 4 shows the location of the shakers and accelerometers on the bridge and Figure 5 shows the QA 750 accelerometers placed on the bridge deck to measure structural response.

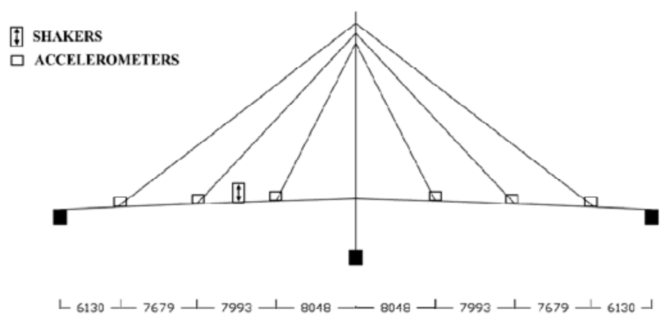

Figure 4. Location of shakers and accelerometers on bridge (dimensions in mm)

Figure 5. Accelerometers placed on the bridge to measure structural response

For system identification in the frequency domain, frequency response function (FRF) is the most commonly used method. A FRF is a measure of systems response at the output to the input signal (Friswell and Mottershed 1995). A FRF of the data obtained during vertical shaker test is shown in

Figure 6. It can be noted from the figure that there are peaks at 1.64 Hz, 1.90 Hz, 3.66 Hz, 6.32 Hz and 7.42 Hz. Similarly, two peaks were observed for horizontal shaker excitation at 4.85 Hz and 5.36 Hz.

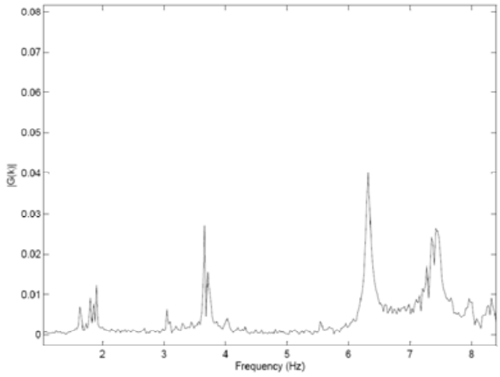

Figure 6. FRF during vertical shaker test

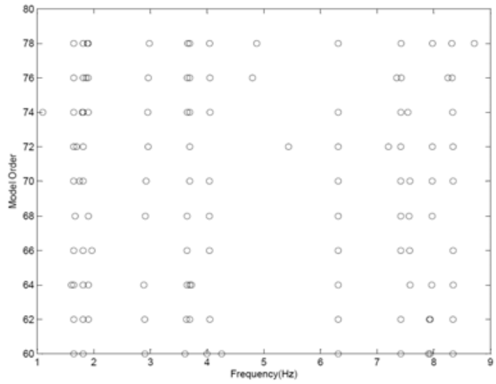

Figure 7. Stability diagram for vertical test

The core of most identification algorithms is a least square solution which gives a relationship between the input and output of an unknown system. The subspace state space identification (SSI), a powerful technique for modal identification in the time domain, is used for system identification in this study. The general subspace algorithm follows the development by van Overschee (1996) and can be applied to both input only and input-output methods. State-space system matrices are obtained based on system input and outputs by the SSI method. The system natural frequencies, damping ratios and mode shapes can then be derived from these system matrices.

The determination of the order of the state space model is a challenging problem. Theoretically, the system order should be twice the number of the degrees of freedom (DOF) of interest. But due to measurement noise, a higher model order is normally selected to extract the modes of interest with higher confidence. The model order selected for this study ranges from 60 to 80 for vertical and horizontal shaker configurations. As a result, the system identifies some spurious modes which do not represent the actual structural modes. To overcome this problem, stability diagrams are employed. As the system order increases, the structural modes identified by the algorithm should remain stable (Bodeux and Golinval 2001). Stability tolerances were selected based on the experience and data quality. A tolerance of 1% for frequency change was selected for this study. The stability diagram for vertical shaker configuration is shown in Figure 7. It can be seen from the stability diagrams that the first five modes are stable and can be identified from the vertical tests. Likewise, the first two modes can be identified from the horizontal tests. Five vertical mode shapes and two horizontal mode shapes identified from modal tests are shown in Figure 8.

Table 1 summarises the natural frequencies identified from FRF, SSI method and those identified by the initial FE model. It can be seen from the results that the identified frequencies from the FRF and SSI methods match very well. But the frequencies obtained from the initial FE model differ from the experimental frequencies by up to 9.2% and average error is found to be -3.93.

Table 1. Natural frequencies identified by FRF and SSI and in initial FE model

Mode No.	Mode Type	Frequency (Hz)			Error between SSI and FE (%)
		FRF	SSI	FE model	
1	1st Vertical	1.64	1.63	1.60	-1.7
2	2nd Vertical	1.90	1.89	1.77	-6.2
3	3rd Vertical	3.66	3.70	3.68	-0.5
4	1st Horizontal	4.85	4.87	4.40	-9.7
5	2nd Horizontal	5.36	5.33	5.22	-2.0
6	4th Vertical	6.32	6.31	6.09	-3.5
7	5th Vertical	7.42	7.42	7.14	-3.7
Average error =					**-3.93**

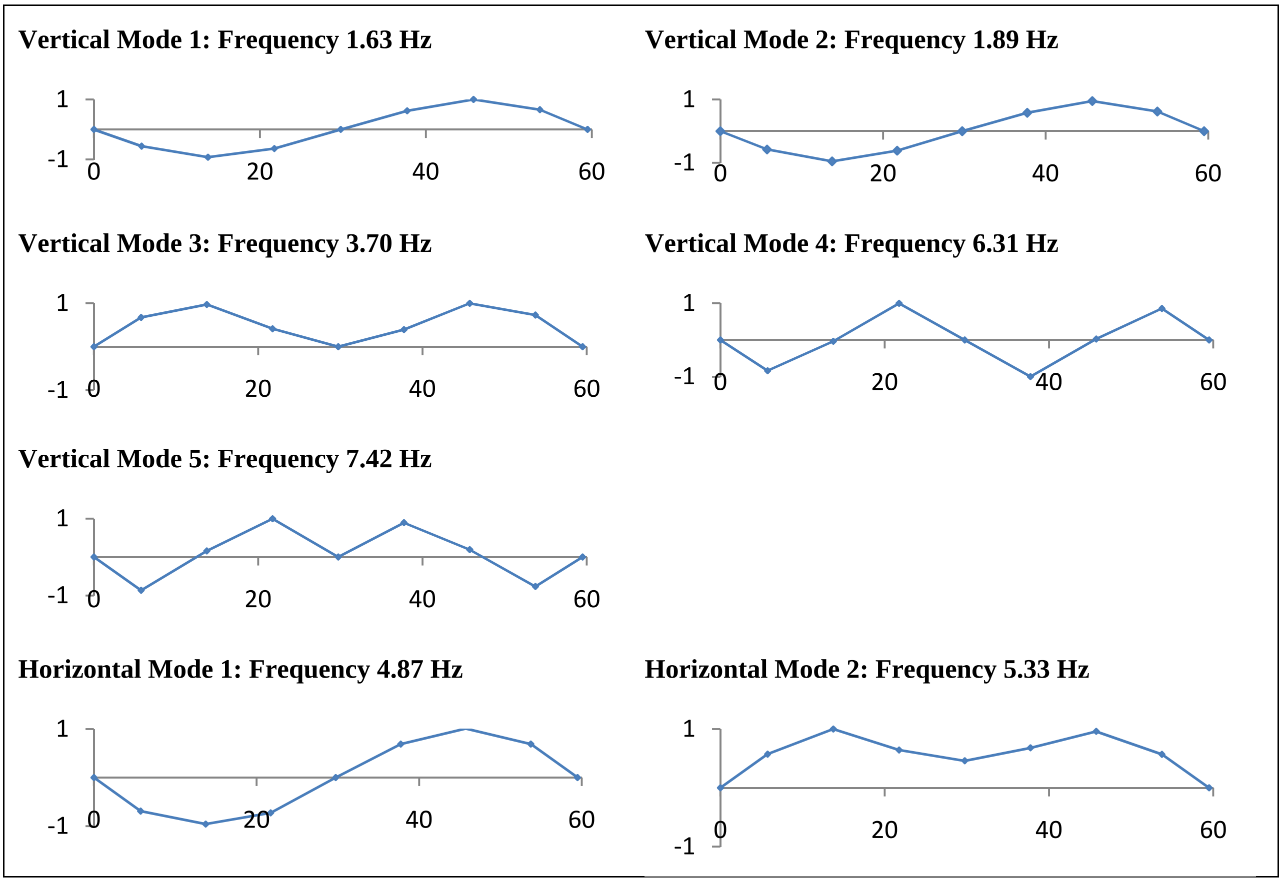

Figure 8. Five vertical modes and two horizontal modes identified by modal tests (frequencies according to SSI method)

In model updating, dynamic measurements such as natural frequencies and mode shapes are normally correlated with their FE model counterparts to calibrate the FE model. There is a degree of uncertainty in the assessment of the actual properties of the materials used in the full scale structure as well as the most realistic representation of the element stiffness in the initial FE model. The challenge of finding a set of suitable parameters having physical justification necessitates the need for use of suitable optimization tools.

4 GLOBAL OPTIMIZATION ALGORITHMS

In this paper, three GOAs namely particle swarm optimization (PSO), genetic algorithms (GAs) and simulated annealing (SA), were investigated. These are population based algorithms using the concept of survival of the fittest to find the optimal solution. A brief introduction to each of these methods is given here.

PSO (Kennedy and Eberhart 1995) is a population based stochastic optimization method. The basic idea is that if one of the population members detects a desirable path for the most fertile feeding locations, the rest of the swarm should follow. Each particle in the swarm should be influenced by the rest of the swarm but also be able to independently explore to a certain extent to increase the diversity. This search process is modelled by particles that have a position and velocity vector in multidimensional space and each position coordinate represents a parameter value.

GAs (Holland 1975) mimic the process of natural evolution. A population of candidate solutions are randomly generated and fitness of each solution in the population is evaluated. Based on the fitness values, multiple solutions are stochastically selected from current generation. No new solutions are formed during this process of selection. Afterwards, two operators, i.e. cross over and mutation, are used for creating new solutions. In the process of cross over, new solutions are created from the mating pool by exchanging information between the strings obtained from the selection process. The mutation operator is used to create entirely new solutions, which prevent the algorithm from being trapped in a local minimum.

SA (Kirkpatrick 1984) resembles the cooling process of molten metals in which the atoms can move freely at high temperatures but with the decrease in temperature the movements of the atoms are reduced. The atoms finally form crystals with a minimum possible energy but if cooling is too fast, this crystalline state may not be achieved. In a minimization problem, this technique simulates the process of slow cooling of molten metals to find the minimum function value, which is normally attained by using a Boltzmann probability distribution.

5 BRIDGE MODEL UPDATING

The selection of parameters for model updating is a crucial step. Only those parameters should be selected which the responses are sensitive to and whose values are uncertain in the initial model. If too many parameters are selected for model updating, the problem may become ill conditioned as the number of knowns may be less than number of unknowns. The parameters selected in this study are stiffness of the pylon in the longitudinal and horizontal direction, stiffness of the deck in the horizontal and vertical direction, and mass of the deck. A sensitivity analysis has been carried out for the parameters. The sensitivities of modal frequencies to the updating parameters are shown in Figure 9, which indicates that the frequencies are sensitive to the parameters.

It is hard to estimate the variation bounds of the parameters during model updating and this is normally selected using engineering judgement. The lower and upper bounds for stiffness values of the pylon have been selected as ±10%, whereas stiffness bounds for the deck stiffness have been selected as -10% and +40% as it is expected that the stiffness of the bridge deck is underestimated, mostly due to uncertain concrete stiffness. The variation bound for the mass of the deck has been selected as ±20%.

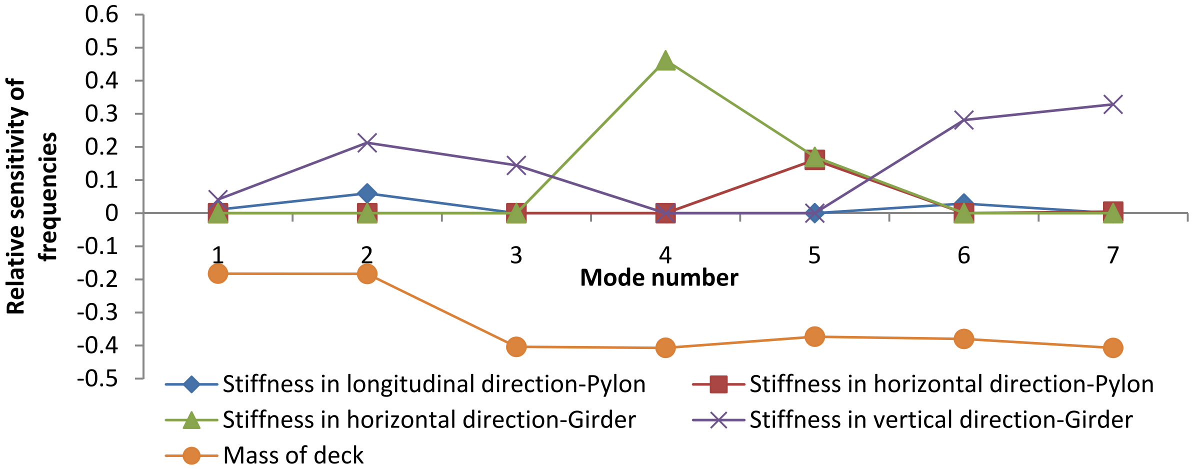

Figure 9. Sensitivity analyses of modal frequencies to selected updating parameters

An important observation made in this study is that the first two vertical modes (Figure 8) have similar mode shapes and an initial SAP model with no discretization of the cable elements did not show these two modes. It was decided to discretize the cables into two elements after which all experimentally observed modes were replicated in the FE model.

Mass and stiffness matrices have been extracted from SAP2000 to MATLAB. Due to the discretization of the cables, many cable modes of vibration exist in the FE model making it difficult to pair identified experimental and analytical frequencies. Model assurance criteria (MAC) is used to pair mode shapes. MAC is defined as:

$$\mathrm{MAC_i} = \left|\emptyset'_{\mathrm{ai}}.\emptyset_{\mathrm{ei}}\right|^2 / \left(\emptyset'_{\mathrm{ai}}.\emptyset_{\mathrm{ai}}\right).\left(\emptyset'_{\mathrm{ei}}.\emptyset_{\mathrm{ei}}\right) \quad (1)$$

where $\emptyset_{\mathrm{ei}}$ and $\emptyset_{\mathrm{ai}}$ represent experimental and analytical mode shapes, respectively.

The next step is to define residual vectors for the experimental and analytical values as

$$e_i = (\omega_{a,i} - \omega_{e,i})/\omega_{e,i} \qquad (2)$$

where ω represents the frequency and subscripts a and e refer to analytical and experimental, respectively. The first five vertical modes and first two horizontal modes have been considered while updating.

The three different global optimization methods described earlier have been applied to the problem. The results are compared in terms of the accuracy of the updated frequencies. The optimization algorithm was run till the point where the improvement in objective function value in consecutive five iterations becomes less than 10^{-4}. The updated frequencies by all the three optimization algorithms are given in Table 2 and updated frequency errors when compared to experimental results are given in Table 3.

It can be noted that all algorithms have given improved results compared to the initial frequency estimates. The average error for PSO, GA and SA has improved to -0.13 ,-0.35 and 0.43, respectively, from the initial average error of -3.93 between the initial model and experimental frequencies. PSO has proved to be best in decreasing the error and matching the analytical frequencies to the experimental ones. PSO has proved to be computationally more efficient as, on average, GA took 35 iterations to converge, whereas PSO took only 20 iterations to converge. The MAC criterion was used for correlation of mode shapes and was greater than 0.97 for the initial FE model modes, which shows that the initial model has a good distribution of stiffness and mass. The MAC values after updating have been given in Table 3 and show that MAC value is not preserved during process of updating.

Table 2. Updated frequencies by PSO, GA and SA

Mode. No.	Frequencies (Hz) Initial model	Experiment	PSO	GA	SA
1	1.60	1.63	1.63	1.63	1.64
2	1.77	1.89	1.82	1.81	1.83
3	3.68	3.70	3.73	3.74	3.79
4	4.40	4.87	4.87	4.85	4.85
5	5.22	5.33	5.33	5.33	5.38
6	6.09	6.31	6.36	6.33	6.40
7	7.14	7.42	7.49	7.46	7.54

Table 3. Updated frequency differences and MAC values for PSO, GA and SA

	PSO		GA		SA	
Mode. No.	**Frequency Difference (%)**	**MAC**	**Frequency Difference (%)**	**MAC**	**Frequency Difference (%)**	**MAC**
1	0.2	0.99	-0.0	0.99	0.3	0.99
2	-3.7	0.99	-4.1	0.99	-3.4	0.99
3	0.9	0.99	1.2	0.99	2.4	0.99
4	0.0	0.99	-0.4	0.99	-0.3	0.99
5	-0.1	0.98	0.0	0.97	1.0	0.97
6	0.7	1.00	0.3	1.00	1.4	1.00
7	1.0	0.99	0.6	0.99	1.6	0.99
Average error=	**-0.13**		**-0.35**		**0.43**	

The updated parameters should be physically meaningful; otherwise it is difficult to justify the results with respect to actual structure. Only the results updated by PSO are discussed here. The vertical stiffness of the bridge deck has increased by 21.2%, and horizontal stiffness by 31.9%, respectively. This is due to the fact that the initial model does not take into account the effect of the parapets and service ducts running through the length of the bridge deck. The other three parameters related to the pylon stiffness in the longitudinal and horizontal direction, and mass of deck do not have appreciable changes and their values have been found to change by -8.0%, +9.9% and -2.6%, respectively.

6 CONCLUSIONS

Full scale testing and model updating of a full scale cable stayed bridge has been performed in this study. Five vertical and two horizontal modes of vibration have been successfully identified. It has been noted that discretization of cables is an important step for correlating experimental and analytical mode shapes. Three different optimization algorithms, namely PSO, GA and SA have been used to check their effectiveness for model updating. It has been found that PSO have given better results than GA and SA. PSO proved to be better in terms of computational efficiency as it took less iterations to converge. The results show that the methodology proposed herein has potential in model updating of full scale structures.

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