

Strategy for anti-buckling design of transverse reinforcement

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2005 NZSEE
Conference

ABSTRACT: This paper presents a strategy to enhance the current provision for anti-buckling design of lateral ties. The resulting design method will restrict the buckling-induced reduction of average compressive stress of main bars to an allowable limit until the desired level of ductility is attained. First, a method to determine the maximum compressive strain likely to be experienced by the main bars is described. Based on an average compressive stress-strain relationship of reinforcing bars, evaluation of *bar buckling parameter* (a function of slenderness ratio and yield strength of the bar) required to restrict the loss of compressive stress at the maximum compressive strain to a tolerable limit is then explained. For a bar of known diameter and yield strength, the maximum allowable tie spacing can then be determined. Next, lateral stiffness required to restrain the buckling tendency of the main bars at the tie locations is expressed as a function of the geometrical and mechanical properties of the main bars. Similarly, the anti-buckling stiffness of the lateral ties is also derived as a function of the mechanical and geometrical properties of the lateral ties. Finally, a design framework to decide the spacing, amount and arrangement of lateral ties is established.

1 INTRODUCTION

Lateral ties enhance the performance of RC structures in three different ways; by providing additional shear resistance, by confining the core concrete, and by restraining the buckling tendency of the main bars. New Zealand Standard [NZS3101 1995] provides separate design criteria to address these three roles of lateral ties. To ensure that the shear demand of a section is adequately met, the total area A_{sh} and spacing s of lateral ties are designed to satisfy Eq. (1), where V_n is the nominal shear demand, V_c the shear contribution of concrete, f_{yt} the yield strength of the ties and z the distance between the resultant compressive and tensile forces in the cross-section; i.e. the arm-length.

$$\frac{A_{sh}}{s} \geq \frac{(V_n - V_c)}{f_{yt} \times z} \quad (1)$$

$$\frac{A_{sh}}{sh_c} = \frac{(\mu_\phi - 33p_t m + 22) A_g f_c'}{110 A_c f_{yt} \phi_c' A_g} \frac{P_e}{\phi_c' A_g} - 0.006 \quad (2)$$

$$\rho_s = 1.4 \left[\frac{(\mu_\phi - 33p_t m + 22) A_g f_c'}{110 A_c f_{yt} \phi_c' A_g} \frac{P_e}{\phi_c' A_g} - 0.006 \right] \quad (3)$$

NZS3101 recommends Eq. (2) to calculate the total area A_{sh} of rectangular hoops and Eq. (3) to calculate the volumetric ratio ρ_s of circular hoops (or spiral) needed to confine the core concrete. In these equations, p_t is the reinforcement ratio expressed as A_{st}/A_g , where A_{st} is the total area of the main bars and A_g is the gross area of the column cross-section, and m is expressed as $f_y/0.85f_c'$, where f_y is the yield strength of the main bars and f_c' is the concrete compressive strength. Similarly, h_c is the concrete core dimension perpendicular to the hoop direction measured to outside of the hoops, A_c is the concrete core area, P_e is the axial compressive force, μ_ϕ is the curvature ductility, and ϕ is the strength reduction factor.

In order to prevent premature buckling of the compressed main bars, the diameter of the lateral ties is not allowed to be less than 5 mm and the vertical spacing of the ties in potential plastic-hinge regions is restricted to $6d_b$, where d_b is the diameter of main bars. NZS 3101 also postulates that, in order to fulfill the anti-buckling role satisfactorily, the designed rectangular hoops and circular hoops or spirals must satisfy Eqs. (4) and (5) respectively, where A_t is the area of each tie leg and d_c is the core diameter measured to the outside of the hoops/spiral.

$$\frac{A_t}{s} = \frac{1}{96} \frac{\sum A_s f_y}{f_{yt} \times d_b} \quad (4)$$

$$\rho_s = \frac{A_{st}}{110d_c} \frac{f_y}{f_{yt}} \frac{1}{d_b} \quad (5)$$

The shear force carried by lateral ties can be quantified using the truss theory, which has been unanimously implemented in all seismic design codes. As far as the confinement criteria are concerned, the code provisions are strongly supported by the results of many experimental and theoretical studies [Sheikh and Uzumeri 1982; Mander et al. 1988_{a,b}; Watson et al. 1994; Pujol et al. 2000]. NZS3101 rightly follows two major steps in its anti-buckling design recommendations: the first is to limit the maximum spacing of lateral ties and the second is to ensure that the amount of lateral ties is enough to overcome the buckling tendency of the main bars at the tie location. Nevertheless, these anti-buckling recommendations were developed based on experience and intuition without any strong quantitative backup. Given that the anti-buckling criteria supersedes the confinement criteria for axial load levels lower than 25% of the compressive strength [NZS3101: Part 2 1995] and the majority of bridge piers and beams and columns of building frames are subjected to smaller axial loads, the need to establish reliable and convincing anti-buckling design criteria cannot be overemphasised. This paper tries to supplement the codes in this aspect.

In ductile RC members that are designed to fail in flexure, the main bars almost invariably buckle in the plastic-hinge regions before failure. Hence, it will be futile to aim for a complete avoidance of buckling from the failure mechanism. The intent of the design of lateral ties should be to delay the buckling-induced instability until the desired level of ductility is acquired. As the lateral deformation of main bars under compression is a gradual process, the definition of the buckling initiation point is rather obscure and is difficult to predict. It sounds more convincing to deal with buckling implicitly in terms of its consequences (i.e., the ill-effects) instead. As is well known, buckling results in a gradual loss of average compressive stress [Dhakar and Maekawa 2002_a]. This is of major concern in capacity design as it leads to the reduction of the section moment capacity, which reflects itself in the load-displacement relationship with a softening of the restoring force, thereby impairing the ductility. In order to ensure that the main bars carry sufficient compressive stress to avoid a significant reduction of section moment, lateral tie design recommendations should be established by following three major steps: (i) estimation of maximum compressive strain expected in the main bars; (ii) determination of maximum allowable tie spacing so that the loss of compressive stress at the maximum expected strain is within the permissible limit; and (iii) deciding the amount and arrangement of lateral ties to ensure that the buckling length of main bars does not exceed the tie spacing.

2 STRATEGY FOR THE DETERMINATION OF ALLOWABLE SPACING

2.1 Maximum compressive strain of main bars

The maximum compressive strain to be experienced by the main bars in an RC section may correspond either to the failure of the section or to the design curvature ductility. Flexural failure of a section refers to the stage when the confined core concrete crushes; i.e., when the compressive strain in the extreme fiber in the core concrete equals the confined ultimate strain ϵ_{cu} . Note that the cover concrete would not carry any stress at the ultimate stage due to spalling. Hence, the extreme compression fibre of the core concrete would be adjacent to the main bars in compression, thereby

rendering the maximum compressive strain of the main bars ϵ'_{su} equal to ϵ_{cu} . The value of ultimate strain for confined concrete ϵ_{cu} corresponds to the fracture of the confining hoop and can be estimated by Eq. (6) [Mander et al 1988_{a,b}; Paulay and Priestley 1992], where ϵ_{sm} is the steel strain at maximum tensile stress, f_{cc} is the compressive strength of the confined concrete, and ρ_s is the volumetric ratio of the confining lateral ties.

$$\epsilon'_{su} = \epsilon_{cu} = 0.004 + 1.4 \rho_s f_{yt} \epsilon_{sm} / f_{cc}' \quad (6)$$

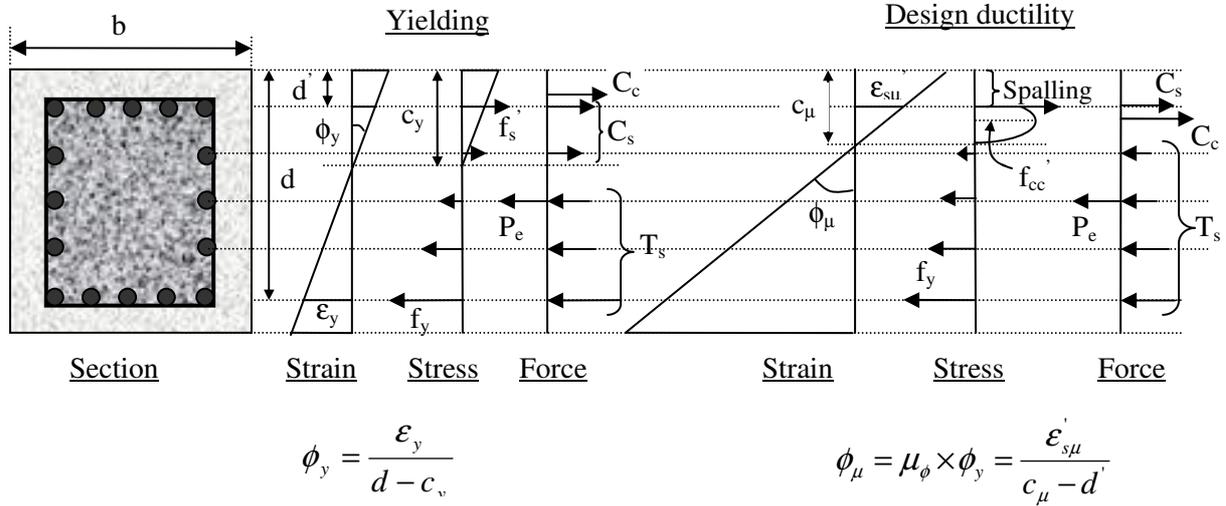


Fig. 1 Maximum compressive strain of main bars at design ductility

On the other hand, the section, if designed properly, may be able to achieve the design curvature ductility without reaching the ultimate stage. Fig. 1 shows the strain, stress, and force distributions across the section at yielding and at the design ductility level. It is assumed in the illustration that the cover concrete is spalled off when the section response is at the design ductility level. The maximum compressive strain $\epsilon'_{s\mu}$ likely to be experienced by the main bars during the section response at design ductility can be expressed as in Eq. (7), where the variables are defined in Fig. 1.

$$\frac{\epsilon'_{s\mu}}{\epsilon_y} = \frac{c_\mu - d'}{d - c_y} \mu_\phi \quad (7)$$

It needs a parametric study based on section analysis to investigate the variation of c_μ and c_y with respect to parameters such as axial load, reinforcement ratio and material strengths and to come up with a reasonable approximation of the maximum compressive strain $\epsilon'_{s\mu}$ at the design ductility. If a more precise estimation of the useful compressive strain range is needed, designers may opt to calculate the values of ϵ'_{su} and $\epsilon'_{s\mu}$ from Eqs. (6) and (7) respectively and take the lower of these two values.

2.2 Maximum allowable buckling length of main bars

As mentioned earlier, the maximum tie-spacing should be determined to ensure that the loss of compressive stress at the maximum compressive strain is within a tolerable limit. In order to assess the loss of compressive stress due to buckling, an average compressive stress-strain relationship is needed. The author adopts the bare bar constitutive model proposed by Dhakal and Maekawa [2002_a]. A general layout of the average compressive stress-strain relationship is sketched in Fig. 2. The main feature of this model is that it acknowledges that the average compressive behaviour of a bar is governed not only by the slenderness ratio L/d_b but is also influenced by the yield strength of the bar f_y . Hence by using this model, the influence of the yield strength of main bars in the buckling resistance of lateral ties can also be captured, which is an added advantage. As shown in the figure, the compression response of a bar is completely described by a single compound variable called the *bar*

buckling parameter λ_b which is defined in Eq. (8), where L , d_b and f_y are buckling length, bar diameter and the yield strength of the bar in MPa, respectively.

$$\lambda_b = \frac{L}{d_b} \sqrt{\frac{f_y}{100}} \quad (8)$$

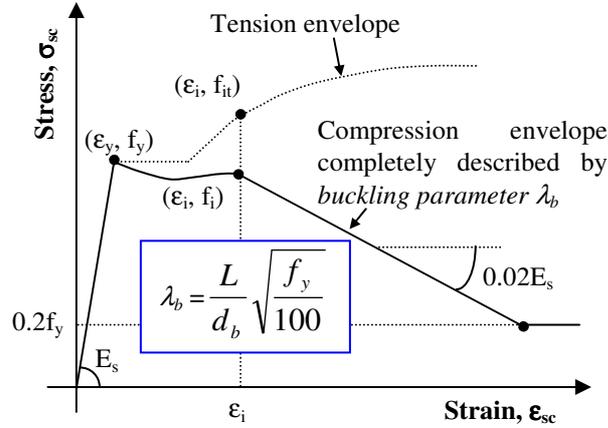


Fig. 2 Compressive stress-strain envelope for reinforcing bars [Dhakai and Maekawa 2002_a]

The loss of compressive stress due to buckling is the difference between the stresses in the tensile curve and the compression envelope at a given strain. As this model normalizes the average compressive stress-strain relationship with respect to the tension envelope, it can readily be used to calculate the buckling-induced loss of average compressive stress. For a smaller value of the *bar buckling parameter* λ_b , the average stress-strain curve deviates less from the tension envelope; i.e., the loss of average compressive stress becomes smaller. However, an acceptable amount of loss within the applicable compressive strain range needs to be decided. If the maximum compressive strain likely to be experienced by the main bars is known, the buckling parameter corresponding to a given loss of compressive stress can be calculated.

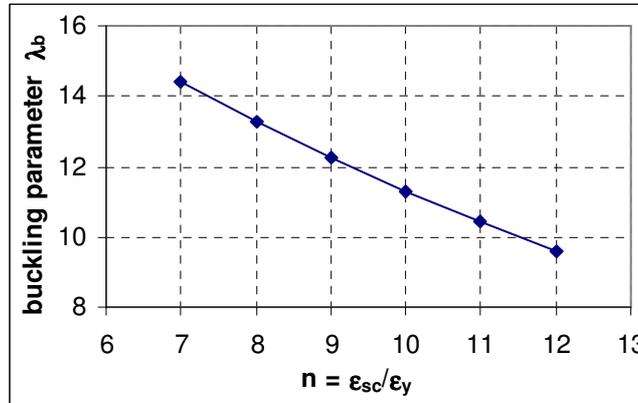


Fig. 3 Variation of buckling parameter with the normalized maximum compressive strain

For example, Fig. 3 shows the variation of λ_b with respect to the ratio of the maximum possible compressive strain in the main bars to the yielding strain for a 10% loss of compressive strength. Similar curves can be generated for any prescribed percentile loss of compressive strength. The maximum allowable value of buckling parameter $\lambda_{b,max}$ required to restrict the loss of compressive stress within a prescribed range can then be readily obtained, which serves as a target for the anti-buckling design of lateral ties. Thus, the maximum allowable buckling length becomes:

$$\lambda_b \leq \lambda_{b,max} \quad \rightarrow \quad L \leq \lambda_{b,max} \times \frac{d_b}{\sqrt{f_y/100}} \quad (9)$$

2.3 Maximum allowable tie spacing

The buckling length of main bars in RC members depends on the geometrical and mechanical properties of the lateral ties and the main bars [Kato et al. 1995; Dhakal and Maekawa 2002_b]. If the lateral ties are properly designed, the main bars may buckle between two successive ties; i.e., the buckling length, in this case, will be equal to the tie spacing. Therefore, Eq. (9) can also be written as Eq. (10), which serves as a design recommendation to restrict the maximum spacing s_{max} of the lateral ties.

$$s_{max} \leq \lambda_{b,max} \times \frac{d_b}{\sqrt{f_y/100}} \quad (10)$$

A designer-friendly table can be generated to specify the maximum allowable tie-spacing to main bar diameter ratio for bars of different yield strength. Note that the buckling length may include more than one tie-spacing if the lateral ties are not stiff enough to restrain the buckling tendency of the main bars. Hence, Eq. (10) alone does not ensure that λ_b does not exceed $\lambda_{b,max}$. If the actual spacing is less than half/one-third/one-fourth of that specified in Eq. (10), a lesser number of lateral ties may be provided because the buckling length requirement given in Eq. (9) will not be violated even if the ties at one/two/three consecutive levels break.

3 STRATEGY FOR THE NUMBER AND ARRANGEMENT OF LATERAL TIES

3.1 Lateral-stiffness required to restrain buckling of main bars

In order to supplement the spacing requirement given by Eq. (10), the number and arrangement of lateral ties should be such that the buckling of longitudinal bars must be restricted within two consecutive ties; i.e., the buckling length L should not be more than one tie-spacing. The lateral stiffness k_t required to confine the buckling of a main bar between two consecutive ties has been theoretically derived by some researchers [Bresler and Gilbert 1961; Dhakal and Maekawa 2002_b], and all agree that the required lateral stiffness k_t can be computed using Eq. (11), where EI is the average flexural rigidity of the main bar at the onset of buckling.

$$k_t \geq 0.75 \frac{\pi^4 EI}{s^3} \quad (11)$$

As buckling happens after compression yielding, the Young's modulus E_s cannot be used to evaluate the flexural rigidity. In the past, different expressions have been used to evaluate the reduced stiffness of reinforcing bars during buckling [Bresler and Gilbert 1961; Papia et al 1988, Dhakal and Maekawa 2002_b]. Here, the expression proposed by Dhakal and Maekawa [2002_b] is adopted, which has been extensively verified for more than 40 different kinds of tests. This expression is given in Eq. (12), where I is the second moment of area of the bar and f_y is the bar's yield strength expressed in MPa.

$$EI = \frac{E_s I}{4} \sqrt{\frac{f_y}{100}} \quad (12)$$

If the value of the average flexural rigidity EI is substituted from Eq. (12) to Eq. (11), we get Eq. (13) which gives a simple expression to evaluate the lateral stiffness required to restrain the buckling tendency of a compressed bar.

$$k_t \geq 0.18 \frac{E_s I}{s^3} \sqrt{\frac{f_y}{100}} \quad (13)$$

Now, Eqs. (10) and (13) together form a complete anti-buckling design recommendation so that the designed system of lateral ties is able to avoid the premature buckling of main bars until the intended ductility is achieved. However, the actual anti-buckling stiffness of different systems of lateral ties must be quantified accurately to check if it satisfies the requirement given in Eq. (13).

3.2 Evaluation of anti-buckling stiffness of lateral ties

When lateral ties are provided in the form of rectangular hoops, diagonal hoops and intermediate cross-ties, buckling of a main bar at the hoop level is possible only if (i) a hoop leg supporting the bar bends to accommodate the outward deflection of the bar; or (ii) hoop legs along the buckling direction elongate to accommodate the outward deflection of the bar. A main bar located not at the corner of a rectangular/diagonal hoop and not connected to a cross-tie will try to induce bending deformation in the supporting leg spanning perpendicular to its buckling direction. As shown in Fig. 4b, the bending stiffness k_b of a transverse tie leg supporting a main bar that divides the tie length l_t into two segments of length a and b is given by Eq. (14), where E_t and I_t refer to the modulus of elasticity and the moment of inertia of the lateral ties, respectively.

$$k_b = \frac{3E_t I_t}{l_t^3} \left(\frac{l_t}{a} \times \frac{l_t}{b} \right)^3 \quad (14)$$

This equation assumes that the tie leg is fixed at both ends. Note that the value of k_b is infinite when either a or b is zero. Hence, buckling of main bars at the corner of a rectangular hoop does not induce any bending deformation on the transverse leg. The bending stiffness k_b decreases as the position of the main bar moves towards the centre and, at the other extreme, k_b becomes a minimum when the main bar is located at the mid-span, i.e., $a = b = 0.5l_t$. Therefore, checking the bending stiffness of a tie leg against the buckling of a main bar located closest to the mid-span and not supported by any cross-tie or diagonal hoop is a sufficient condition. Also note that the smaller the tie length l_t , the larger the bending stiffness k_b . Hence, if a main bar is supported by two hoops (e.g., a peripheral hoop and an inner hoop) the inner hoop will provide a higher bending stiffness, and the peripheral hoop need not be required to restrain this main bar.

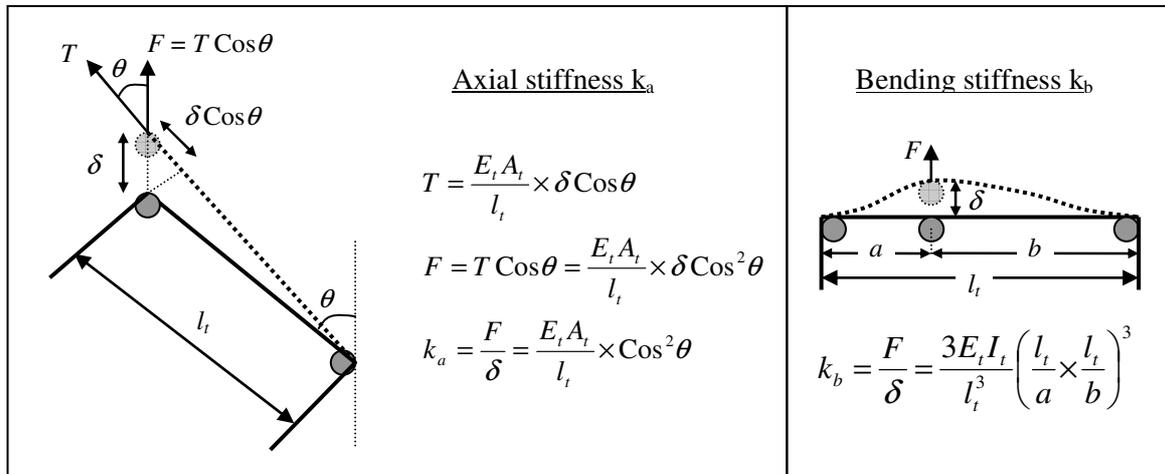


Fig. 4 Anti-buckling stiffness of rectangular hoops (a) Axial stiffness; (b) Bending stiffness

If the flexural stiffness of a tie leg is larger than the required stiffness given by Eq. (13), this transverse tie leg will act as a rigid link. The buckling tendency of all main bars resting on this tie-leg, including those at the corners, will then induce axial tension in the two side legs of the rectangular hoop. Hence, the axial stiffness of the side legs must overcome the buckling tendency of all main bars resting on the transverse leg of the hoop. The axial stiffness of a tie-leg of length l_t and cross-sectional area A_t , inclined at an angle θ to the buckling direction is derived in Fig. 4a. If n_l number of hoop-legs are involved in counteracting the buckling tendency of n_b number of main bars, then the effective axial stiffness k_a of the hoop is given by Eq. (15).

$$k_a = \frac{E_t A_t}{l_t} \cos^2 \theta \times \frac{n_l}{n_b} \quad (15)$$

This equation also applies for diagonal hoops and intermediate cross-ties. The value of n_l is 1 for a cross-tie and 2 for a rectangular hoop or a diagonal hoop. Similarly, the value of n_b is usually 1 for a cross-tie or a diagonal hoop (more for an octagonal hoop) and 2 or more for a rectangular hoop, and θ is zero except for diagonal hoops and inclined cross-ties. When the lateral ties include a combination of more than one rectangular hoop or diagonal hoop or intermediate cross-tie, the buckling tendency of the supported main bars will be restrained by different components of the lateral ties depending on their arrangements. For example, a bar tied to an intermediate cross-tie or to a diagonal hoop may be restrained by its axial stiffness, and need not contribute to the bending and axial stiffness requirements of the peripheral rectangular hoop. The values of n_b , n_l and θ for some common arrangements of main bars and lateral ties are illustrated in Fig. 5.

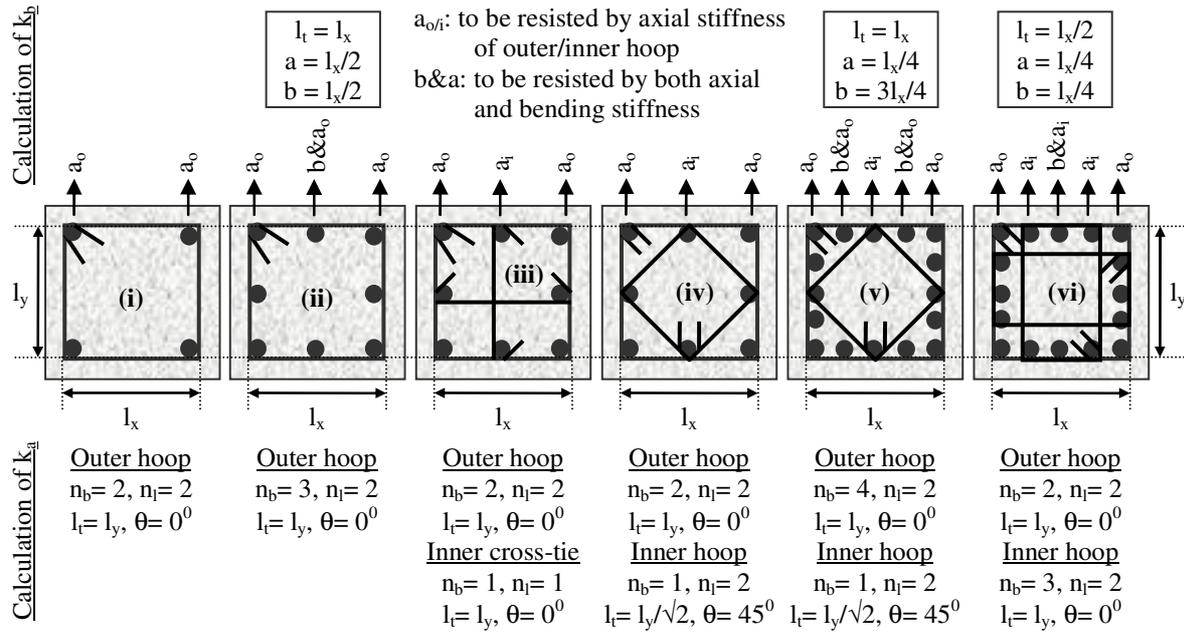


Fig. 5 Calculation of anti-buckling stiffness for different arrangements of lateral ties

4 CONCLUSION AND RECOMMENDATION

The outcomes of the foregoing discussions can now be integrated to propose an enhanced design method of lateral ties that satisfies the anti-buckling requirement. First, the spacing of the designed tie system must not be greater than that specified in Eq. (10). Next, the anti-buckling stiffness computed by Eqs. (14) and (15) must be greater than the required lateral stiffness given in Eq. (13). If the loading is biaxial and the reinforcement arrangement is not symmetrical, the main bars may buckle in any of the two directions, and the anti-buckling stiffness along both directions should be checked. In square cross-sections where the main bars and lateral ties are arranged symmetrically with respect to both axes, verification in only one direction is sufficient.

Note that the proposed enhancement is based on strong theoretical background, and the important components of this design enhancement proposal are experimentally verified [Dhakar and Maekawa 2002_{a,b}]. It offers a significant improvement over the existing intuition-based anti-buckling design provisions. Nevertheless, the proposed enhanced design method is not in its final stage, and needs to be supplemented by the following: (i) derivation of maximum compressive strain in main bars as a function of ductility demand; and (ii) evaluation of anti-buckling stiffness of circular hoops and spirals.

REFERENCES:

- Bresler, B., and Gilbert, P. H. (1961). Tie requirements for reinforced concrete columns. *Journal of ACI*, 58(5), 555-570.
- Dhakar, R. P., and Maekawa, K. (2002_a). Modeling for post-yield buckling of reinforcement. *J. Struct. Engrg.*, ASCE, 128(9), 1139-1147.
- Dhakar, R. P., and Maekawa, K. (2002_b). Reinforcement stability and fracture of cover concrete in reinforced concrete members. *J. Struct. Engrg.*, ASCE, 128(10), 1253-1265.
- Kato, D., Kanaya, J., and Wakatsuki, K. (1995). Buckling strains of main bars in reinforced concrete members. *Proc. of the EASEC-5*, Australia, 699-704.
- Mander, J. B., Priestley, M. J. N., and Park, R. (1988_a). Theoretical stress-strain model for confined concrete. *J. Struct. Engrg.*, ASCE, 114(8), 1804-1826.
- Mander, J. B., Priestley, M. J. N., and Park, R. (1988_b). Observed stress-strain behavior of confined concrete. *J. Struct. Engrg.*, ASCE, 114(8), 1827-1849.
- NZS 3101: Part 1: 1995 (1995), *Concrete Structures Standard - The Design of Concrete Structures*. Standards New Zealand, Wellington.
- NZS 3101: Part 2: 1995 (1995), *Concrete Structures Standard – Commentary on the Design of Concrete Structures*. Standards New Zealand, Wellington.
- Papia, M., Russo, G., and Zingone, G. (1988). Instability of longitudinal bars in RC columns. *J. Struct. Engrg.*, ASCE, 114(2), 445-461.
- Paulay, T., and Priestley, M. J. N. (1992). *Seismic Design of Reinforced Concrete and Masonry Buildings*. Wiley Interscience, New York.
- Pujol, S., Sozen, M., and Ramirez, J. (2000). Transverse reinforcement for columns of RC frames to resist earthquakes. *J. Struct. Engrg.*, ASCE, 126(4), 461-466.
- Sheikh, S. A., and Uzumeri, S. M. (1982). Analytical model for concrete confinement in tied columns. *J. Struct. Div.*, ASCE, 108(12), 2703-2722.
- Watson, S., Zahn, F. A., and Park, R. (1994). Confining reinforcement for concrete columns. *J. Struct. Engrg.*, ASCE, 120(6), 1798-1823.