

# Modelling rocking structures using standard finite elements

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**ABSTRACT:** A technique is proposed for the purpose of analysing rocking structures using finite element methods. The proposed technique involves a minimum number of approximations and makes similar assumptions to those used in Housner's rocking rigid block model. The example presented in this paper shows that the proposed method is an improvement on a number of previous analyses of rocking structures. Contact forces are consistent with contact conditions and no fictitious damping or other restoring forces are relied upon in the uplift stage. The effectiveness of the proposed method is evaluated by comparing displacement time histories obtained 1) by a finite element approach, 2) by solving Housner-type governing differential equations, 3) from published results of a previous study. Good matching of all three was achieved; however there are still opportunities for further refinement and improvement to the cumbersome parameter selection process.

## 1 INTRODUCTION

The challenge of modelling and exploiting rocking behaviour for structural purposes has intrigued earthquake engineers for about half a century. It is thought that by allowing rocking motion to occur in a structure, the motion itself will act as a base isolation system reducing base shear, acceleration and deformation of the structure. There have been many documented cases where tall structures allowed to rock, have survived large earthquakes while other seemingly more stable structures suffered great damage (Housner, 1963). Whilst this strengthens the case for isolation by rocking; the lack of understanding of the underlying concepts has meant that engineering adoption of a rocking mechanism remains in its infancy.

The first real attempt at developing a mathematical model of the rocking problem was made by Housner in 1963, as he tried to explain how fortuitous rocking response aided the survival of several elevated water tanks during the 1960 Chilean earthquake. Subsequent to this, there have been several improvements made to the basic rocking model including additional actions such as sliding, bouncing and free flight (Hogan, 1994; Ishiyama, 1982; Shenton & Jones, 1991a, 1991b). Although the later models are closer to reality, there are still many grey areas in the transition between rocking phases. Confident prediction of response as a consequence of a specific action remains unattainable and untested.

In addition, these models are limited to a rigid rocking block and have become very mathematically obscure and abstract, with limited relevance in the engineering context. It is because of this a different approach to the problem has been proposed in this paper. The objective is to make use of finite element modelling (FEM), a versatile tool that is well understood by engineers, using a carefully constructed model to enable application to the rocking problem. This paper aims to evaluate the rocking behaviour of a simple A-frame in its simplest form with the minimum number of approximations, on the way highlighting some of the faults with the current common practice. Displacement time histories obtained via the proposed method will be compared to time histories

obtained by solving the classical Housner problem (a widely respected benchmark solution corresponding to the case of a rigid block rocking with fully plastic impacts), and with the problem as solved by Beck et al. (Beck & Skinner, 1974).

## 2 DIFFICULTIES ASSOCIATED WITH SOLVING THE GENERAL ROCKING PROBLEM USING FEM

In the first instance, the pure rocking problem is a stiff, geometrically nonlinear problem, requiring an iterative solution process, taking into account the structure's displaced geometry at all times. The boundary conditions for a rocking problem are nonlinear. Hence, the response of the structure is very much history dependent and errors are therefore cumulative. Thus it is essential to determine the intermittent contact events extremely precisely in the time domain for an accurate analysis. To further complicate matters, the main energy dissipation mechanism in a pure rocking system is in the nature of radiation damping which occurs virtually instantaneously. This is very much ideologically incompatible with the energy dissipating devices commonly incorporated in the time integration procedures employed in conventional finite element packages.

Lastly, perhaps the greatest practical obstacle for the implementation of rocking problem with FEM, is that impact events associated with rocking of tall slender objects have been experimentally observed to be near "fully plastic" events. The implication is that, even when an arrangement is found to be able to dissipate the equivalent amount of energy as an impact in an acceptable time interval, it must also respond fully plastically (i.e. with negligible bounce back and any vertical vibration suppressed). In the absence of this, it would be questionable whether the model in fact modelled rocking behaviour at all closely, but simply represented a structure constrained in a fashion that resulted in the exhibition of rocking-like behaviour.

## 3 THE HOUSNER ROCKING PROBLEM

For the purpose of familiarising ourselves with the rocking problem, Housner's original rigid block (Housner, 1963), subjected to horizontal ground acceleration, is shown in a displaced position in figure 1 below.

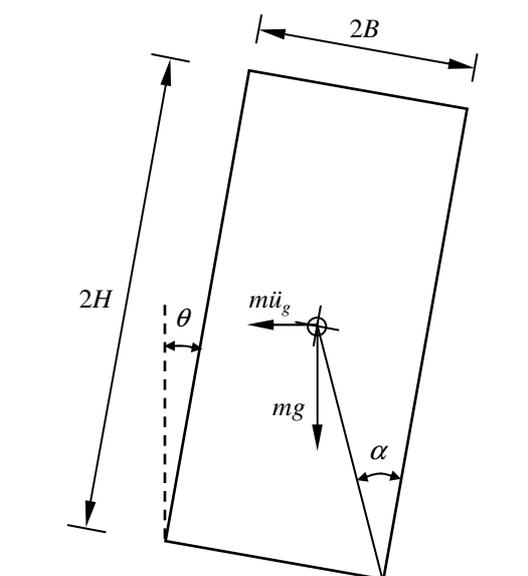


Figure 1- Schematic of the rigid rocking block

The major assumptions in his model were:

- The block is to be considered as a rigid body.
- There is no sliding between the block and the base.
- Impacts occur at discrete points, namely at the corners of the block.
- Upon impact, the collision is to be perfectly inelastic, so that there will be no “bounce-back” and the block rocks over in a smooth, continuous fashion.
- As a consequence of the smooth continuous rotation assumption above, angular momentum about the point of impending contact will be conserved over a very short time interval containing the impact.

With these assumptions, a piece-wise governing equation of motion (Equation 1) was set up with respect to the angle of rotation and can be solved via numerical integration techniques or by small rotations approximation.

$$\ddot{\theta} + p^2 \left( \frac{\ddot{u}_g(t)}{g} \cos(\alpha - \text{sgn}(\theta) \cdot \theta) + \text{sgn}(\theta) \cdot \sin(\alpha - \text{sgn}(\theta) \cdot \theta) \right) = 0 \quad (1)$$

where  $p^2 = \frac{mgR}{I_0}$

$m$  = overall mass of the block (kg);

$I_0$  = rotational inertia of the block about its rotating corner ( $\text{kgm}^2$ );

$\ddot{u}_g(t)$  = horizontal ground acceleration as a function of time ( $\text{ms}^{-2}$ );

$g$  = gravitational constant ( $\text{ms}^{-2}$ );

$\text{sgn}$  = signum function;

and  $\alpha$ ,  $\theta$  and  $R$  are as defined in figure 1.

An expression for the apparent coefficient of restitution for tall slender blocks was deduced (Equation 2). A small number of researchers have since experimentally demonstrated that for certain cases, this expression does indeed resemble reality, and references to their experiments have been provided for information. (Chik-Sing Yim et al., 1980; McManus, 1980)

$$r = \left[ 1 - \frac{mR^2}{I_0} (1 - \cos 2\alpha) \right]^2 = \left( \frac{\dot{\theta}_{\text{after impact}}}{\dot{\theta}_{\text{before impact}}} \right)^2 \quad (2)$$

where  $\sqrt{r}$  = coefficient of restitution ( $e$ )

#### 4 THE MODEL STRUCTURE

The model structure chosen for this simulation is a 61m high, 12.2m wide, A-frame with a mass of 680.2 tonnes lumped at the apex. Incidentally, this is identical in dimensions to the model structure presented in a study leading up to the construction of the South Rangitikei Railway Bridge in 1974 (Beck & Skinner, 1974). The only difference is that in this study the weight of the leg of the piers has been neglected and the concentration has been on the case where there is no additional viscous damping device. This model is intended to represent a tall, slender, bridge-like structure, where the majority of the mass is located at a point somewhere above the pier foundations.

For the purpose of evaluating the proposed method’s performance, the model was subjected to the 1940 El Centro NS ground excitation, and the rocking response was subsequently obtained by solving the nonlinear differential equations developed in the same manner as Housner’s rocking block, and by FEM with the proposed supplementary technique.

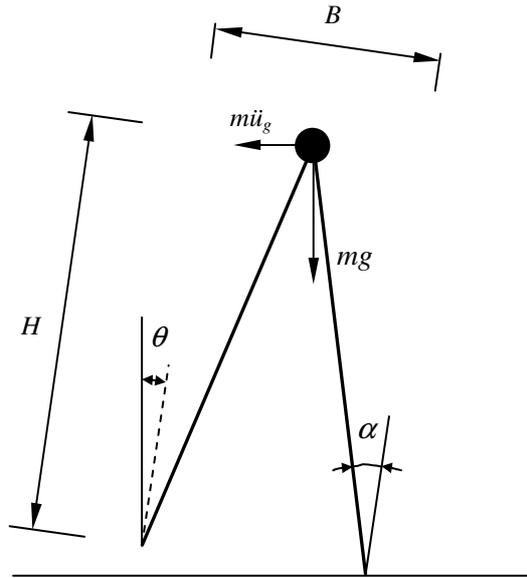


Figure 2 – Schematics of the rigid rocking A-frame

#### 4.1 Mechanics of the A-frame and developing the benchmark solution

Applying the same assumptions as in Housner's rigid block problem, a piecewise governing equation of motion with respect to the angle of rotation for the A-frame was developed. (Equation 3)

$$\ddot{\theta} + p^2 \cdot \left( \frac{\ddot{u}_g(t)}{g} \cos(\alpha - \text{sgn}(\theta) \cdot \theta) + \text{sgn}(\theta) \cdot \sin(\alpha - \text{sgn}(\theta) \cdot \theta) \right) = 0 \quad (3)$$

and the corresponding expression for the theoretical apparent coefficient of restitution is:

$$r = (1 - 2 \sin^2 \alpha)^2 \quad (4)$$

where all symbols have their usual meanings.

Given the ground acceleration record, equation 3 was then solved using MATLAB's ODE45 procedure and equation 4 was applied when a contact event was detected. Extreme care was taken to identify any contact events and linear variation of ground acceleration between available data points was assumed. Tolerance limits were set, adhering to guidelines suggested by Shampine et al in their book (Shampine et al., 2003, p27). The solution forms the basis of the benchmark solution, which will be compared with the proposed FEM method.

## 5 THE PROPOSED FINITE ELEMENT MODEL

A ground spring arrangement as in figure 3 has been proposed to simulate the contact surface between the rocking A-frame and the ground. The goal of the ground springs arrangement design was to simulate Housner's rocking assumptions as closely as possible. The basic concept of the arrangement, is that it is to act like an inverted Tuned Mass Damper; tuned to suppress vertical vibrations of the frame after a contact has been made, encourage a no-bounce-back impact, provide a contact force only when there is physical contact, and dissipate enough energy in the system to mimic the approximate radiation damping consistent with fully plastic impacts.

Consequent to the above, the A-frame should then be able to rock over smoothly in a FEM analysis making the minimum number of approximations and employing only simple springs and viscous dampers. OPENSEES (Mazzoni et al., 2000) was chosen as an appropriate finite element program for this study due to its scriptable, open and transparent nature, with SAP2000 nonlinear v8.3.1 subsequently used for further verification.

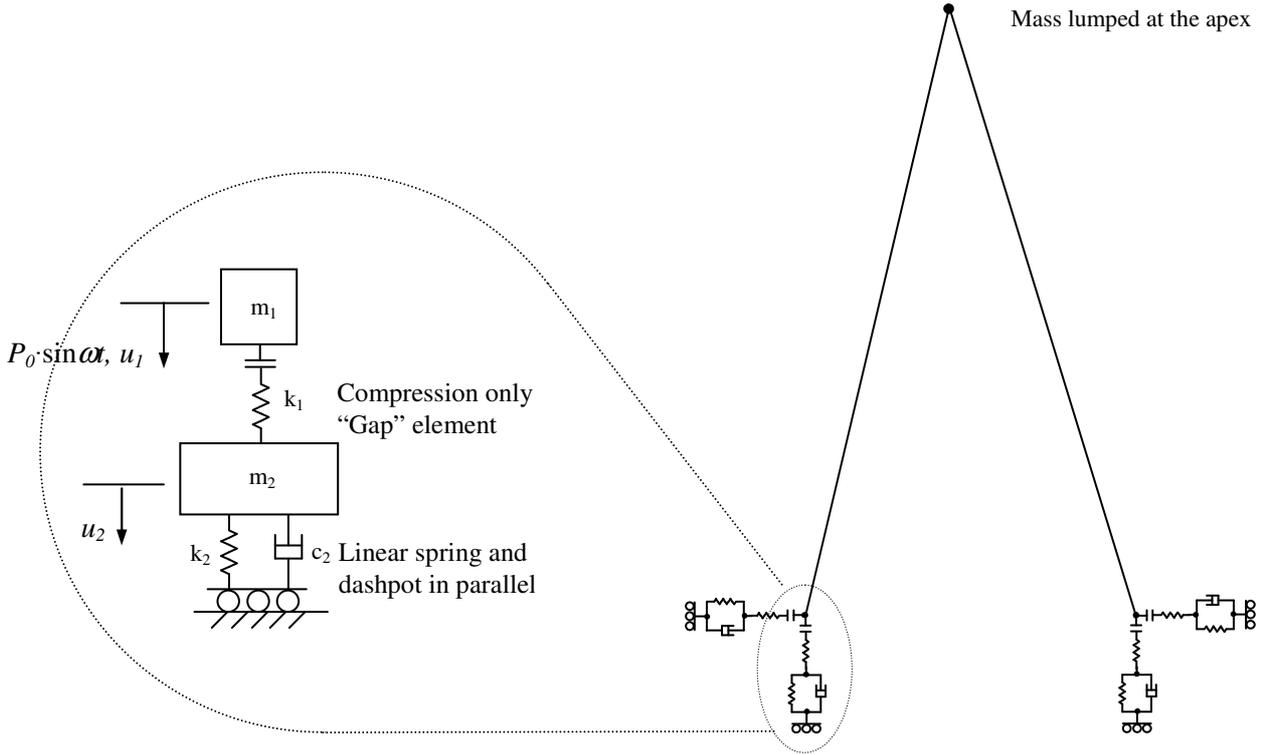


Figure 3 – Definitions of the ground springs variables and the finite element model of the A-frame

### 5.1 Choosing the parameters for the ground springs

For the purpose of choosing parameters for the ground springs, the peak steady state response of a similar mechanical model under sinusoidal excitation was studied. The model studied has the nonlinear gap element in figure 3 replaced with a linear spring in order to simplify the problem. This was shown to be insignificant as only an approximate response of the system was required. A sinusoidal excitation was chosen as we are interested in the system's characteristics in the frequency domain. The two ranges of input frequencies of particular interest are the very-high frequencies, corresponding to an impact event, and the low frequencies corresponding to the overall rocking of the structure. The ideal frequency response relationship for the ground springs will have the very-high frequencies damped out to suppress the vibrations associated with an impact, a near uniform response near the rocking period and a low amplification across all other frequencies.

Consider a sinusoidal force of amplitude  $P_0$  acting on the top mass ( $M_I$ ). Equations of motion for the system can be written in complex notation as:

$$\begin{aligned} (k_1 - m_1 \omega^2) u_1 e^{i\omega t} - k_1 u_2 e^{i\omega t} &= P_0 e^{i\omega t} \\ -k_1 u_1 + (k_1 + k_2 - i\omega c_2 - m_2 \omega^2) u_2 &= 0 \end{aligned} \quad (5)$$

Rearranging the second equation and substituting into the first gives,

$$u_1 = P_0 \cdot \frac{(k_1 + k_2 - m_2 \omega^2) - i(\omega c_2)}{[\omega^2 m_1 (-k_2 + \omega^2 m_2) + k_1 (k_2 - \omega^2 (m_1 + m_2))] + i[\omega c_2 (\omega^2 m_1 - k_1)]} \quad (6)$$

Now, through introduction of non-dimensional factors as outlined, and some algebraic manipulation, the peak steady state amplitude response ratio (ARR) of  $M_I$  (imitating the A-frame) is arrived at as,

$$ARR = \sqrt{\frac{(2\mu\xi g)^2 + (1 + \mu(f^2 - g^2))^2}{(2\mu\xi g)^2(1 - g^2)^2 + [\mu g^2(g^2 - f^2) + (\mu f^2 - g^2(1 + \mu))]^2}} \quad (7)$$

where $\mu$	$= m_2/m_1$	= mass ratio = absorber mass/A-frame mass
$\omega_a$	$= k_2/m_2$	= natural frequency of the absorber
$\Omega_n$	$= k_1/m_1$	= natural frequency of the gap system
$f$	$= \omega_a / \Omega_n$	= frequency ratio (natural frequency)
$g$	$= \omega / \Omega_n$	= forced frequency ratio
$x_{st}$	$= P_0 / k_1$	= frequency ratio (natural frequency)
$c_c$	$= 2m_2\Omega_n$	= “Critical” damping of the absorber system
$\xi$	$= c_2 / c_c$	= damping ratio of the absorber system
ARR	$= u_1 / x_{st}$	= amplitude response ratio

By optimising this expression for a given  $\mu$  (mass ratio) to the ideal frequency response curve shape outlined in this section, families of sensible stiffness and damping parameters can be determined. Microsoft Excel’s solver proved to be a convenient tool for this task. A trial and error approach based on reasonable static displacement of the system under gravity was then used to determine the best value of  $\Omega_n$ . The exact value of the remaining parameters could then be found using the relationships above.

It is advisable that the horizontal mass ratio of the ground spring absorber be kept small, to minimise unwanted horizontal ground acceleration due to the mass of the ground absorber, but not so small as to induce excessive vibration in the ground springs in the horizontal direction, when the A-frame is detached.

## 5.2 Advantages and Disadvantage of the proposed technique

The main advantage of this proposed technique is the provision of reasonable certainty that rocking behaviour is modelled correctly. Contact forces only exist when the structure is in contact, unlike other analysis methods in the past, where always-on viscous dampers, unjustified classical damping matrices or empirical stiffness relationships based on small angle push-over theory were relied upon (Mander & Cheng, 1997; Palermo et al., 2004). Since rocking is such an amplitude and history-dependent problem, any deviation from the true time-history path due to incorrect constraints at the supports is likely to lead to downstream errors and possibly grossly inaccurate results (which may, however, appear quite plausible). In addition, with the analysis conducted with the help of finite element methods, there is potential that this can be expanded to other more complicated structures, and the rigid body assumptions may be allowed to relax to collect useful information such as forces, moments and stresses in structural members.

It should be noted that the proposed method is not without faults and limitations. Conceptually, there are still inherent incompatibility issues of rocking with time integration solutions (Abaqus Inc., 2003,s2.4.2). The proposed method has demonstrated its ability to emulate Housner’s fully plastic impact model, and by suitable tuning of the support parameters could be made to model impacts with other characteristics. Before embarking on a detailed analysis of this type it is desirable to verify that the response of the structure in question conforms to the “smooth rocking” paradigm of the Housner type (and does not enter the bouncing, sliding or free-flight regimes). The process of finding the correct ground spring parameters was cumbersome, and relied on trial and error in parts of the calculations. Finally, more testing is required to confirm its validity with different sized structures and ability to predict overturning.

It should be noted that this method is not suitable for foundation tipping type problems where the rotation of the structure is due to deformation of surrounding soil. Different assumptions should be made and different information is available for problems of this sort.

## 6 RESULTS FROM THE EXAMPLE ANALYSIS

The parameters used in the analysis can be found in the table below:

**Table 1. Summary of parameters used in the finite element model**

$m_1$ (Lumped mass of the A-frame) = 680156.1 kg		
	<b>Vertical ground springs</b>	<b>Horizontal ground springs</b>
$\mu =$	10	0.01
$f =$	0.9598	6.4923
$\xi =$	0.229	59.9
$\Omega_n =$	6.2 Hz	10 Hz
$k_1 =$	1.032E+09 N/m	2.685E+09 N/m
$k_2 =$	9.509E+09 N/m	1.132E+09 N/m
$m_2 =$	6801561 kg	6801.561 kg
$c_2 =$	1.165E+08 Ns/m	3.324E+08 Ns/m

Displacement time histories of the example structure as outlined in section 4 under 1940 El Centro NS ground excitation are as follows:

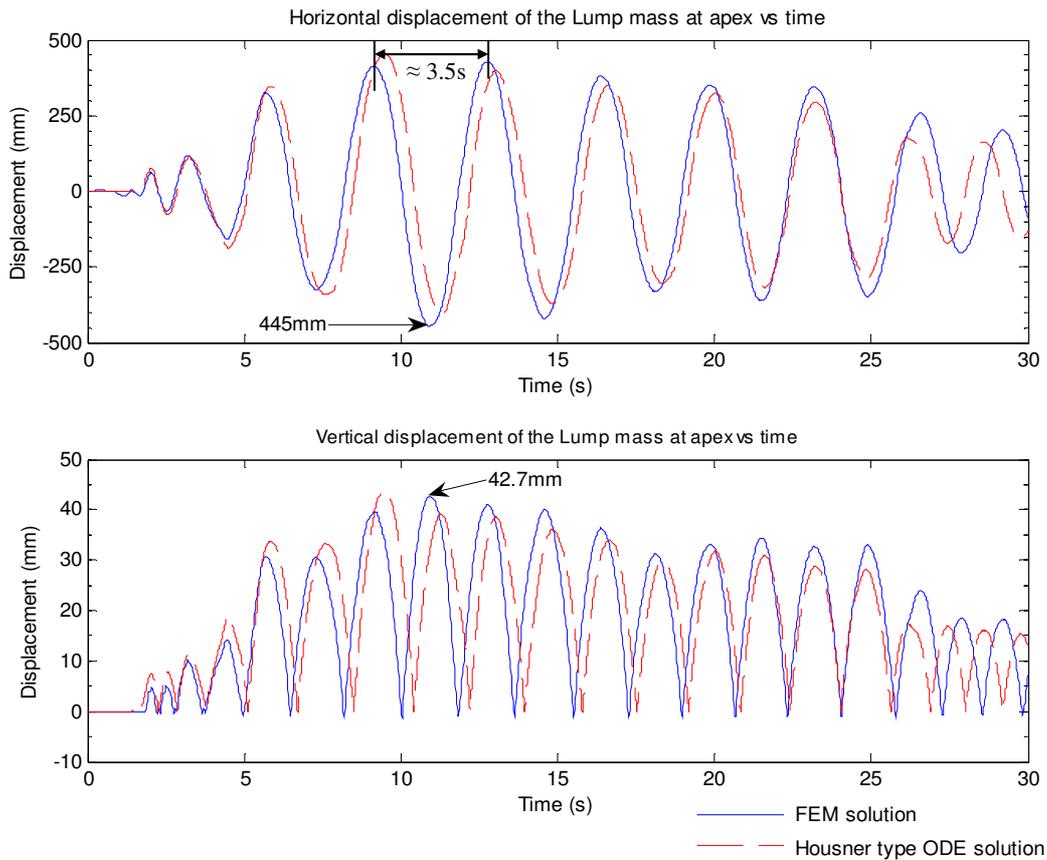


Figure 4 – Displacement time histories of the A-frame under 1940 El Centro NS Earthquake

As can be observed, the displacement time histories as obtained by FEM provided good matching with the benchmark Housner ODE solution, not only in magnitude but also in the time domain. Vertical and horizontal displacements are also consistent with rocking motion. Comparison can also be drawn with

the almost identical problem presented by Beck and Skinner in 1974. In their study, the peak horizontal displacement of the structure was 444.5mm (17.5 inch) with a perceived “period” of approximately 3.3 seconds.

## 7 CONCLUSION

An example of the proposed technique for the use of the finite element method for modelling rocking problems has been presented in this paper. The presented example showed good agreement with the Housner type rocking solution, and also compared well with the previous analysis of a similar structure by Beck et al. The objective of modelling rocking behaviour in its simplest form with simple elements in a finite element environment has been achieved. The solution obtained via the proposed ground spring arrangement is consistent with the contact conditions, and unlike other studies did not rely upon any fictitious damping force in the uplift stage. There is potential for this method to be developed further to include flexibility in the structure and assumptions other than those of Housner. More research is required for adaptation of the method to structures of different form and structures of significantly different dimension.

In closing, rocking behaviour is a very nonlinear process with complex interactions. Care needs to be taken to ensure the isolation aspect of rocking mechanisms is correctly included in the analysis. Results should be interpreted with caution and the analysis program verified on simple cases before tackling more complex problems. This paper has shown an example where this was the primary concern, and it is recommended that this is also the starting point for any analysis on other rocking structures.

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