Modelling of post-tensioned precast reinforced concrete frame structures with rocking beam-column connections

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ABSTRACT: Recent earthquakes have identified that buildings designed to a performance based criteria need not only resist the ground excitation but remain, in many situations, undamaged once ground motions have abated. One promising approach is the use of post-tensioned frame structures assembled from precast reinforced concrete members.

A modelling approach has been developed which provides for the accurate simulation of the developed compression zone and accounts for the shift of the neutral axis in the contact area over the duration of earthquake motion. Furthermore, the beam elongation effects are captured as well as the effects on the structure due to the lengthening of the post-tension tendons.

A new multi-spring contact element has been developed and included in the finite-element program RUAUMOKO. The distribution of the stiffness along the contact area of the rocking elements is calculated based on a number of different integration schemes. The model is compared with experimental displacement controlled tests undertaken with beam-column subassemblies and experimental dynamic tests of a multi-storey frame structure.

1 INTRODUCTION

Recent earthquakes have identified that buildings designed to a performance based criteria need not only resist the ground excitation but remain, in many situations, undamaged once ground motions have abated. One promising approach is the use of post-tensioned frame structures assembled from precast reinforced concrete members. In case of an earthquake excitation these structures start to rock. A gap opens between the jointed elements and the unbonded tendon gets stretched. The post-tensioned tendons act as a self centring mechanism. The ductility demand of the structure is provided by the opening of the gaps. In the ideal case the structure survives the earthquake undamaged.

Additionally energy dissipation devices can be used at the joints of the connected elements. This provides additional hysteretic energy dissipation in case of rocking of the structure and gap opening.

For the design of these structures an accurate modelling approach is needed which provides for the accurate simulation of the developed compression zone and accounts for the shift of the neutral axis in the contact area over the duration of earthquake motion. Furthermore, the beam elongation effects have to be captured as well as the effects on the structure due to the lengthening of the post-tension tendons.

Several different experimental studies on this topic are published. Based on this work different modelling approaches were developed. But neither of the different published approaches models accurately all the above mentioned criteria with a reasonable amount of effort. Therefore a new multi-spring contact element has been developed and included in the finite-element program RUAUMOKO (Carr (2004)).
2 MULTISPRING CONTACT MODELLING APPROACH

2.1 Objective

The objective of the modelling approach was to develop an element, which simulates the contact area of two precast concrete members which were jointed together by, for example, unbonded post-tensioned cables or bars. In case of rocking of the connection leading to gap opening the model should simulate accurately the beam elongation as well as the local stresses and strains in the contact area.

2.2 Contact Element Setup

When the joint is closed the whole section is in compression. At joint opening representing rocking of the connection the neutral axis of the section moves from infinity, which is valid for a section in uniform compression, into the section. A compression zone forms, which decreases in size with increasing gap opening. If the contact zone would be infinitely stiff, the sections would rotate during joint opening around the pivot point. This would conform to the rocking according to *Housner (1963)* (see Figure 1 (a)). Recent works from *Pampanin et al. (2001)* show that in rocking precast reinforced concrete frame members this assumption is not accurate enough and the formation of a compression zone has to be taken into account in the simulation of these structures (see Figure 1 (b)).

(a) rotation around pivot point (*Housner (1963)*)  
(b) compression zone formation (*Pampanin et al. (2001)*)

Figure 1: Contact zone of two precast concrete members with open gap

To simulate the contact zone a multispring contact element was developed. The principle setting of the element in case of using up to 10 contact springs is shown in Figure 2. The contact springs are compression only springs and have no tensile stiffness. They are distributed along the section height of the contact area.

![Figure 2: Principle setting of multispring contact element with 10 contact springs](image)

The multispring contact element is placed between two frame elements to simulate the local contact deformations. The deformations due to bending and compression/tension of the frame element are based on the assumptions that the sections remain plane (Bernoulli Hypothesis).
2.3 Spring Distribution and Weighting

To calculate the position of the springs and their weighting two different integration schemes were used which can be used alternatively. First a distribution on the basis if the Gauss integration scheme was used. This integration scheme is widely used in finite element simulations. However it has the disadvantage that according to this approach no contact spring is positioned at the edge of the contact area. This can be of disadvantage by modelling the rocking of a section around the corner of a member, because the stiffness on the edge at the end of the contact area is zero. Therefore a second integration scheme, the Lobatto Integration, was used alternatively, which provides all the time one contact point at the edge of the section. The multispring contact element was set up for 2 to 10 contact points for each integration method. The values derived from the integration formulas are shown exemplarily for 2 and 10 contact points in Table 1 (Abramowitz/Stegun (1965)). The abscissas for the different integration formulas are tabled from the centre line to +1 to –1. The tabled values have be multiplied by half of the height of the section to get the abscissas of the contact springs, measured from the centre. The tabled values of the weights have to be multiplied by the half of the global stiffness of the contact zone of the section to get the individual spring properties.

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3 PRINCIPLE BEHAVIOUR OF MODELLING APPROACH

The principle behaviour of the new developed multispring contact element is presented using the model of a subassembly test which was performed at the University of Canterbury (Arnold/Davies/Spieth/Mander (2004), Davies (2004)). The test represents an inner node of a precast post-tensioned concrete frame. Each beam was jointed by post-tensioned high strength steel bars with the column element. This leads to two rocking connections at the beam/column interfaces. The test setup is shown in Figure 3 (a).

A principle drawing of the RUAUMOKO model is shown in Figure 3 (b). The beams and the column were modelled with linear elastic Giberson frame members. To model the post-tensioned bars linear elastic-plastic prestressed spring elements were used. The multispring contact elements were placed between the beams and the column at the position of the joints. No loads were applied on the model structure. The model was tested with displacement control at the top node.

![Figure 3: Test setup of subassembly test (Arnold/Davies/Spieth/Mander (2004), Davies (2004)) and RUAUMOKO Subassembly Model](image-url)
3.1 Influence of Used Integration Scheme and Number of Contact Springs

Figure 4 (a) shows a comparison of the simulated force-drift behaviour of the above described subassembly using the Lobatto integration scheme and 2 and 10 contact springs. For comparison the result of the displacement controlled experiment of the subassembly is included as well in the graph. Figure 4 (b) shows the same graph using a Gauss integration scheme. As a contact stiffness depth for the multispring contact element the longitudinal stiffness of one quarter of the beam height was taken.

Using the Lobatto Integration scheme and just 2 contact springs, the rocking of the connection is forced to go around the pivot point. No compression zone can form. This leads to a larger inner lever arm at the rocking section as in reality which results in a approximately 15 % higher force at the opening of the joint in the simulation than at the equivalent experiment. The stiffness of the simulation after opening of the gap is comparable to the experiment. With increasing number of contact springs the ability of the model increases to form a realistic compression zone. The stiffness is not any more concentrated at the corners of the rocking section. Using 10 contact springs the model captures very well the force - drift behaviour of the subassembly.

Using the Gauss Integration scheme and just 2 contact springs, the inner lever arm of the section is reduced by 40 %. This leads to a underestimation of the capacity of the rocking connection at the joint opening of approximately 10 %. For further gap opening the underestimation of the simulation compared to the experiment increases. With increasing numbers of contact springs stiffness closer to the edge of the rocking section is increased which leads to a increasing force at the joint opening and increasing stiffness of the simulated subassembly at higher drifts which results into a better accuracy.

The comparison of Figure 4 (a) and (b) shows that the Lobatto integration scheme converges from an upper bound with increasing number of contact springs and the Gauss integration scheme converges from a lower bound with increasing number of contact springs. It can be seen that by using 10 contact springs, the behaviour of the rocking subassembly can be simulated with very good agreement using either Lobatto of Gauss integration.

3.2 Influence of Contact Stiffness

Figure 5 shows a comparison of the simulated force-drift behaviour of the above described subassembly using the Lobatto integration scheme and a different contact stiffness. For comparison the result of the displacement controlled experiment of the subassembly is included as well in the graph.

The multispring contact element simulates the local deformations due to eccentric compression in the contact zone at gap opening resulting from rocking of the sections. The axial stiffness of the simulated joint represents the cross sectional area times an elastic modulus divided by the length of the joint.
However the joint length may be very small and the localized strains in the adjoining beams are effected by the opening of the joint. This can be done by taking an effective length to represent the elastic support to the joint by the adjoining beams. This has been achieved by taking the effective length as a fraction of the joint height. In order not to change the global stiffness of the model including the contact elements, the stiffness of the members adjacent to the contact element has to be increased to compensate the additional flexibility of the contact element. The cross sectional area used is that of the joint and the elastic modulus is that of the contact material. To show the influence of the contact stiffness the effective length was taken as h/2, h/3, h/4, where h is the height of the contact joint, and finally the length of the steel armouring alone (20mm).

The results of the simulations show, that the model is not very sensitive to the contact stiffness when the stiffness is chosen in a reasonable range. Assuming just the steel plates as compression influence area represents gives values which are slightly above the experimental results and assuming half of the height of the section as influencing length gives values which are slightly below the experimental results. The closest fit with the experiments show the simulations with using one quarter and one third of the section height as influencing length.

![Figure 5: Column force vs. drift of subassembly experiment and simulation using multispring contact element with Lobatto – Integration and different contact stiffness](image)

3.3 **Comparison of Neutral Axis Level in Contact Zone**

Depending on the imposed drift on the structures the position of the neutral axis in the contact zone changes its position. At no imposed drift the stresses in the contact zone of the post-tensioned sections are approximately uniformly distributed compression. The theoretical neutral axis in this case is at infinity. By imposing a drift to the section the compression stresses on one side increase and decrease on the other side. The theoretical neutral axis moves towards the section and reaches the edge of the section at the imposed drift when a gap is about to form. With increasing opening of the joint the neutral axis moves further inside the section. This is shown in Figure 6 where the neutral axis is positioned approximately in the middle of the section at the end of the shown compression zone. The displacements in the compression zone of the rocking sections and the gap opening were measured in the experiment with 3 LVDT’s (Figure 6).

![Figure 6: Measurement of deformations in the contact zone of the rocking sections](image)
The neutral axis in the rocking section was calculated from the measured displacements. Using the same approach the neutral axis in the simulated rocking sections was calculated. A comparison of experiment and simulation is shown in Figure 7. Here the new developed multispring contact element with 10 contact springs and the Lobatto Integration approach was used. As contact stiffness the longitudinal stiffness of a quarter of the section height was taken. For better understanding the height of the section is shown as well at the right side of the graph. The comparison shows a very good agreement of experiment and simulation. At 0% drift the position of the neutral axis is at infinity. With increasing drift the neutral axis moves towards the section and reaches the section edge at approximately at a drift of 0.1 % when the gap starts to open. With further gap opening the compression zone becomes smaller and the neutral axis moves to the top or bottom of the section respectively. At large drifts the compression zone has a depth of approximately 10% of the section height. This behaviour is captured by the simulation realistically as the comparison with the experiment shows.

![Figure 7: Neutral axis height in rocking section; comparison experiment with simulation (using Lobatto Integration with 10 contact springs); height of section h=560mm](image)

3.4 Influence of Elastic Limit of the Contact Area

Another decisive influencing parameter on the behaviour is the local compressive strength of the contact zone. This sensitivity study focuses on the level of the limit of elastic behaviour of the contact area. This represents the stress level when the armouring steel starts to yield or the concrete in the contact zone starts to deform plastically.

Simulations with different levels of elastic limits were performed. The load displacement behaviour of the contact zone was modelled as bilinear with the plastic stiffness as 0.1 times the elastic stiffness. The elastic stiffness of the contact zone was calculated as the axial stiffness of the section depth of one quarter of the section height as discussed above. Simulations were run with a elastic limit of the contact zone of 45 and 300 N/mm². Cycles of 1 %, 2 % and 3 % in sequence were performed. The results of the simulations are shown in Figure 8. The curves of each cycle are shown in separate graphs. For comparison the result of the displacement controlled experiment of the subassembly is included as well in the graph.

The simulation shows that for a plastic limit of 300 N/mm² no plastic deformation in the contact zone can be seen for drifts up to 3 %. At the simulation with a elastic limit of 45 N/mm² the plastic deformation in the contact zone starts at a drift of less than 1 %. This leads to a reduced loads at unloading of the first cycle and the systems shows a hysteretic energy dissipation. At the second cycle the loads at the opening of the gap are reduced. The loading curve follows the unloading curve of the previous cycle. Due to the further plastic deformation at the second cycle the loading curve on the third cycle is reduced further as it follows again the unloading curve of the previous cycle.
4 APPLICATION OF MODELLING APPROACH ON SYSTEMS WITH DRAPED CABLES

4.1 Description of Model

A principle drawing of the RUAUMOKO model and the experimental setup is shown in Figure 9. The beams and the column were modelled with linear elastic Giberson frame members. The Multispring Contact Elements were placed between the beams and the column at the position of the joints. To model the post-tensioned bars linear elastic prestressed spring elements were used.

Figure 9: Test Setup and RUAUMOKO Model of Subassembly with Draped Cables

(a) Slaving of the nodes  
(b) Friction elements

Figure 10: Modelling alternatives to ensure the position of the draped post-tensioned bars in relation to the beam centre line
To take into account the friction between the unbonded post-tensioned bars and the beam a new friction element was developed. Instead of slaving the nodes to ensure the position of the draped post-tensioned bars in relation to the beam centre line, the friction distance elements were installed. This is shown in Figure 10. As in the experiment point loads of 130 kN were applied on both beams. The model was loaded displacement controlled at the top node.

4.2 Comparison Between Experiment and Simulation

The friction force $F_F$ was assumed as $F_F = 0.5 F_N$ where $F_N$ is the force between the post-tensioned bar and the duct. The result of the numerical simulation in comparison with the experimental result is shown in Figure 11 (a). A different approach is to model the friction between the post-tensioned bar and duct as independent from the normal force as a constant value at each connection point. In this case the friction force $F_F$ was assumed to be 25 kN at each connection point. The results are shown in Figure 11 (b). The friction forces for the simulations shown in Figure 11 were taken to adjust the depth of the hysteretic energy dissipation loop of the simulation to the experimental results. A comparison of the two modelling approaches shows that by assuming a constant friction force along the bar length a better agreement between simulation and experiment can be reached in the shape of the hysteretic loop.

![Figure 11](image1.jpg)

(a) $F_F = 0.5 F_N$  
(b) $F_F = 25$ kN = constant

Figure 11: Column force vs. drift of subassembly experiment and simulation using multispring contact element with 10 contact springs; modelling friction force between post-tensioned bar and beam as $F_F = 25$ kN

![Figure 12](image2.jpg)

(a) Test setup  
(b) RUAUMOKO model

Figure 12: Test Setup and RUAUMOKO Model of Shake Table Frame Test
5 APPLICATION OF MODELLING APPROACH ON DYNAMIC EXPERIMENT OF FRAME STRUCTURE

A structure with these opening joints near the ends of the beams and the basis of the columns has been built and tested on the University of Canterbury shake table (Murahidy (2004 a, 2004b)). The experimental results are described in detail in the above references. In Figure 12 (a) the elevation of the frame is shown together with the prestressing tendon locations. Figure 12 (b) shows the computational model with the location of the multi spring elements and the prestressing tendons. Further computational work on this model is proceeding and will be presented shortly.

6 CONCLUSION

Modelling approach for post-tensioned precast reinforced concrete frame structures with straight and draped post-tensioned bars was developed. The multispring element was programmed as one element into the time history program RUAUMOKO which allows its use with low computational and data preparation costs. To achieve this without the use of multiple elements a multi spring element was developed. This allows accurate modelling of the global behaviour of rocking frame structures including beam elongation effects and takes account of local effects such as shift of neutral axis and compression zone depth. Further this makes modelling of local damage in the joint contact zone possible. A guide was developed for the effective stiffness of the contact zone. A modelling approach was developed to allow for the friction between draped post-tensioned bars and their ducts and was implemented in RUAUMOKO.

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