Earthquake scaling for inelastic dynamic analysis of reinforced concrete ductile framed structures

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ABSTRACT: An earthquake (or a suite of earthquakes) is needed when carrying out inelastic dynamic analysis of a reinforced concrete ductile framed structure. The earthquake should be scaled so as to make its spectral accelerations to match the elastic design acceleration spectrum for the structure to match the elastic code design base shear implied in New Zealand loading code, NZS4203. Techniques and relevant scaling methods are available. However, the maximum base shear obtained from the scaled earthquake for all current scaling methods may not match the code design base shear.

To solve this problem, six different scaling methods are applied to scale four different earthquakes for a ductile framed reinforced concrete structure designed according to the current New Zealand codes using capacity ductile design. By comparing the maximum responses in terms of the maximum base shears, inter-storey drifts and spectral accelerations to the scaled earthquakes with the corresponding design values, the advantages and disadvantages for those scaling methods are identified, leading to a scaling method for which both the design base shear and design inter-storey drift can be matched simultaneously. Finally, a procedure for earthquake scaling is recommended.

1 INTRODUCTION

For convenience some terms used in this paper are defined first of all. The design base shear refers to the design base shear of the elastic structures ($\mu = 1.0$). The design inter-storey drift is the maximum value of the inter-storey drifts for each level of a structure obtained using the SRSS combination method with the design displacement spectrum derived from the design acceleration spectrum of the current New Zealand loading code (NZS4203 1992) as shown in Eq. (12). The maximum base shear is the largest absolute value obtained of the base shear available from an elastic dynamic time-history analysis. The maximum inter-storey drift is the largest absolute value observed over all stories in the structure during the elastic time-history analysis.

According to the Equal Displacement Principle (Newmark & Veletsos 1960) for long period structures, the inelastic structure has a similar magnitude of maximum displacement to that of an identical structure that is constrained to remain linearly elastic. This implies that the design level earthquakes for inelastic time-history analyses should be those which are able to produce the maximum response values which are equal to that at the design level, i.e. the design base shear and the design displacement (inter-storey drift) for this elastic structure. The maximum base shear and maximum displacement (inter-storey drift) of the elastic structure are used to assess the intensity of an excitation for the inelastic structure and should match the corresponding design responses for the scaled earthquakes.

There are two ways for the maximum base shear to satisfy the design requirement in base shear for the scaled earthquakes. One is to have the maximum base shear match the design value directly. The other is to match the design base shear by matching the design spectral accelerations for the first few modes of free-vibration, which dominate the maximum base shear of the structure. However, the problem is that the maximum base shear obtained from the scaled earthquakes for all current scaling methods may not match the code design base shear.
In order to determine an appropriate earthquake scaling method, six different scaling methods that are outlined in section 6 are applied to scale four different earthquakes. By comparing the maximum responses (the maximum base shears, inter-storey drifts and spectral accelerations) to the scaled earthquakes with the corresponding design requirements, the advantages and disadvantages for those scaling methods are identified, leading to a new scaling method for which both the maximum base shear and maximum inter-storey drift to the scaled earthquakes are close to the corresponding design values.

2 THE DUCTILE FRAMED STRUCTURAL MODEL

The ductile reinforced concrete 3 bay moment resisting frame is 12 storeys high with a 9.2metre span length and inter-storey height of 3.65 metres designed according to the current New Zealand codes (NZS4203 1992; NZS3101 1995) using the capacity design philosophy (Park & Paulay 1975; Paulay 1977, 1979, 1980, 1988; Paulay & Priestley 1992) with a structural displacement ductility of 5.0. The unreduced (basic) live load $Q_b$ was 2.5 kPa for each floor except for the roof levels where the assumed live load was zero. Fig. 1 illustrates the dimensions of the structure, which was assumed to be situated on intermediate subsoil near the centre of Christchurch, New Zealand.

It was assumed that the frames would be required to resist the component of the earthquake motion in the plane of the frame only. The component in the perpendicular direction was assumed to be taken by some other resisting system, for example by structural walls or by transverse frames. 92% of the base shear is distributed to be the equivalent static lateral forces at each level, proportioned to the lumped weights and the heights at levels under consideration, and an additional horizontal force of the 8% left is added to the roof level force considering the effects of higher modes.

3 FOUR UNSCALED EARTHQUAKE RECORDS

A peak acceleration-to-velocity ($A/V$) ratio of an earthquake excitation is a very important parameter. The $A/V$ ratio is closely correlated with the energy content of the record in the frequency domain. High $A/V$ ratio records, in general, have larger input energy in the short-period range and vice versa for low $A/V$ records (Tso et al 1993). Four different earthquake excitations with different durations of excitation were used in this research as shown in Table 1. The $A/V$ ratios for the Bucharest (1977-NS), El Centro (1940-NS), Northridge (Sylmar-949NW) and Kobe (1995-NS) are 0.29g, 0.92g, 0.71g and 0.91g respectively with (1/Second) unit, trying to cover the different characteristics of earthquake records.

Fig. 2 shows the accelerograms for the four unscaled earthquake excitations, i.e. Bucharest (1977-NS), El Centro (1940-NS), Northridge (Sylmar-949NW) and Kobe (1995-NS).

<table>
<thead>
<tr>
<th>No</th>
<th>Country</th>
<th>Location</th>
<th>Date</th>
<th>Zone</th>
<th>$A/V$ Peak</th>
<th>$A/V$ 0.2</th>
<th>$A/V$ 0.5</th>
<th>$A/V$ 0.8</th>
<th>Duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Romania</td>
<td>Bucharest</td>
<td>4/03/1977</td>
<td>N-S</td>
<td>0.21g</td>
<td>0.73</td>
<td>0.29g</td>
<td></td>
<td>16</td>
</tr>
<tr>
<td>2</td>
<td>Imperial Valley California</td>
<td>El Centro</td>
<td>18/05/1940</td>
<td>N-S</td>
<td>0.35g</td>
<td>0.38</td>
<td>0.92g</td>
<td></td>
<td>20</td>
</tr>
<tr>
<td>3</td>
<td>Northridge</td>
<td>Sylmar</td>
<td>17/01/1994</td>
<td>949-NW</td>
<td>0.80g</td>
<td>1.12</td>
<td>0.71g</td>
<td></td>
<td>20</td>
</tr>
<tr>
<td>4</td>
<td>Kobe, Japan</td>
<td>MA, observatory</td>
<td>17/01/1995</td>
<td>N-S</td>
<td>0.84g</td>
<td>0.92</td>
<td>0.91g</td>
<td></td>
<td>20</td>
</tr>
</tbody>
</table>
3. Scale the record so as to match the design spectrum in a range of period of free vibration of interest.

4. Scale the record to match the design spectrum at all frequencies.

Pradono (1998) used the spectrum scaling method at the first mode period and Lin (1999) used the frequency scaling method in their research. For the frequency scaling method, an unscaled earthquake record is transformed into the frequency domain and the response at every frequency is scaled so that the record matches the design spectrum without changing the phase characteristics. The scaled record is then transformed back into the time domain.

Chang (1998) used three different scaling methods belonging to methods 2 and 3 above. By comparing the maximum base shears of structures under the scaled excitations with the corresponding design base shears, appropriate scaling factors were determined producing maximum base shears closer to the design ones. The three scaling methods are shown below:

1. Scaled at the fundamental period of free vibration of the structures.

2. An average of scale factors for the fundamental, second and third modal periods of free vibration of the structures.

3. Weighted average of scale factors for the fundamental (weight 2), second (weight 1) and third (weight 1) modal periods of free vibration of the structures.

Satyarno (2000) scaled the maximum base shear of a structure under a natural earthquake record to match the design base shear of the structure directly.

Fig. 1: Twelve storey three bay frame member and node numbering and level definition

Fig. 2: The accelerograms for the four unscaled earthquake excitations
5 THE DESIGN-SPECTRUM SCALING AND DESIGN-RESPONSE SCALING METHODS

Earthquake scaling techniques can be summarised, in general, into two categories. One is to scale the earthquake so as to match the design spectrum, which is design-spectra based and is called design-spectrum scaling. The other is to scale the earthquake so as to match the design-response (base shear or inter-storey drift) directly, which is design-response based and is called design-response scaling.

A design-spectra scaling method scaling factor can be obtained from the weighted average of the acceleration ratios for the first few significant modes in order to make the spectral accelerations for the scaled excitation as close as possible to the design spectral accelerations. The weighting factors represent the influence for the mode under consideration. The scaling factor can be calculated from the following equation;

\[ SF = \sum_{n=1}^{N} \lambda_n R_n, \quad n = 1, 2, \ldots, N \]  

where \( SF \) is the scaling factor considering the first \( N \) modes, \( \lambda_n \) is the weighting factor for mode \( n \), \( R_n = \frac{a_n^d}{a_n} \) ratio of design spectral acceleration to the spectral acceleration of the unscaled excitation for mode \( n \).

The design-response scaling factor is the ratio of the design response values to the maximum elastic response values for a structure under an unscaled excitation. The responses may be the maximum inter-storey drift or the maximum base shear.

6 THE SIX SCALING METHODS USED IN THIS STUDY

Six different scaling methods were used. Four are design-spectral-acceleration based scaling methods that endeavour to match the design spectral acceleration, and two try to match the design base shear and design inter-storey drift directly.

The four scaling methods of weighted average acceleration ratios used in the study are shown below. Method 1 considers only the first mode. Method 2 assigns arbitrary weighting factors 92%, 5% and 3% for the first three modes respectively. Methods 3 and 4 are natural period and effective mass weighted average of acceleration ratios respectively.

Method 1: \( SF = R_1 \)  
Method 2: \( SF = 0.92R_1 + 0.05R_2 + 0.03R_3 \)  
Method 3: \( SF = \frac{T_1R_1 + T_2R_2 + T_3R_3}{T_1 + T_2 + T_3} \)  
Method 4: \( SF = \frac{M_1^*R_1 + M_2^*R_2 + M_3^*R_3}{M_1^* + M_2^* + M_3^*} \)

where the effective mass for mode \( n \) is determined by Eq. (6).

\[ M_n^* = \frac{(L_n^*)^T}{M_n^*} \]  

where \( M_n^* = [\phi_n]^T[M][\phi_n] \) and \( L_n^* = [\phi_n]^T[M][r] \). \( \{r\} \) is the displacement vector of the structure due to a unit ground displacement in the direction of the earthquake excitation.

The design base-shear and design inter-storey-drift scaling methods used in this study are expressed by Eqs. (7) and (8) respectively.

Method 5: \( SF = \frac{V_{\text{design}}}{V} \)
Method 6: 
\[ SF = \frac{ID_{\text{design}}}{ID} \]  
(8)

where \( V_{\text{design}} \) and \( ID_{\text{design}} \) is the design base shear and design maximum inter-storey drift, and \( V \) and \( ID \) are the maximum base shear and the maximum inter-storey drift from an unscaled earthquake excitation.

The design base shear forces were determined by Eq. (9), which is equal to the design spectral accelerations multiplied by the structure seismic weights.

\[ V_{\text{design}}^{\nu_1=1.0} = C_{\alpha(T)}^{\text{design}} \cdot W = C_{(T,\mu=1.0)} \cdot S_p \cdot R \cdot Z \cdot L_u \cdot W \]  
(9)

where \( C_{\alpha(T)}^{\text{design}} \) is the design spectral acceleration coefficient. \( C_{(T,\mu=1.0)} \) is the basic seismic hazard acceleration coefficient for an intermediate soil site for \( \mu=1.0 \) (elastic); the structure performance factor, \( S_p = 1.0 \) for an elastic time history analysis; the risk factor, \( R = 1.0 \) for building category IV, the zone factor \( Z = 0.8 \) for the Christchurch, the limit state factor \( L_u = 1.0 \) for the ultimate limit state.

The design inter-storey drift was obtained by combining the inter-storey drifts for the first few modes calculated from the design spectral displacements and the mode shapes as shown in Eqs.(10), (11). The square root of sum of squares (SRSS) is used for the response combination. In a two-dimensional structure no two lateral frequencies are close, and therefore no strong correlation between modal responses is likely (Carr 1994).

\[ ID_n = \sqrt{\sum_{i=1}^{N} (ID_{n,i})^2} \]  
(10)

where \( ID_{n,i} \) is the inter-storey drift at level \( n \) for mode \( i \) \( (i=1, 2, 3, \ldots N) \)

\( ID_n \) is inter-storey drift for level \( n \) \( (n=1, 2, 3, \ldots 12) \).

The design inter-storey drift for a structure \( ID_{\text{design}} \) is:

\[ ID_{\text{design}} = \text{Max}\{ID_1, ID_2, ID_3, \ldots, ID_n\} \]  
(11)

The design 5% damped elastic spectral displacements were derived from the Eq. (12).

\[ Sd_n^d = \frac{(T_s)^3}{4\pi^2} \cdot Sd_n^d \]  
(12)

where \( Sd_n^d \) and \( Sd_n^d \) is the design spectral displacement and acceleration for the single degree of freedom of natural period \( T_s \).

7 ASSUMPTIONS USED FOR THE ELASTIC TIME-HISTORY ANALYSIS

To obtain the maximum responses (base shear and inter-storey drift) of the structure under the four earthquakes scaled by using the six scaling methods, elastic time-history analyses were carried out using RUAUMOKO (Carr 1998). The assumptions and mathematical modelling used were:

1. The floors were assumed to be rigid in their own planes. All the nodes at each level were coupled so that each had the same horizontal displacement.
2. The foundations were assumed to be fixed, i.e. no soil-structure interaction effects were considered.
3. A lumped mass model was used.
4. Effects of large displacements allowing for the P-Delta effects on member flexibility and the nodal coordinates were updated in every time step.
5. Initial stiffness Rayleigh damping model was used to represent the damping in the structure. 5% damping ratio was assigned to the first and twelfth modes.
8  MAXIMUM ELASTIC RESPONSES OF THE THREE STRUCTURES UNDER THE SCALED EARTHQUAKE EXCITATIONS

8.1  Maximum Base Shears

Maximum base shear of the structures to the records obtained are shown in Fig. 3. It can be seen from Fig. 3 that for the base shear scaling method, the four dotted lines showing the maximum base shears for the four earthquakes converge to the solid line representing the design base shear. In other words, the maximum base shears for the base shear scaling method are very similar to the design base shears. This is independent of the characteristics of the excitations and the natural periods of free-vibration of structures. Hence the base shear scaling method can be used for earthquake scaling when design base shear is required to be matched.

Fig. 3: Maximum base shears for the four different earthquake excitations scaled by using the six different scaling methods

Note: R1 - first mode method. 0.92R1 - first three modes with arbitrary weighting factors 0.92, 0.05 and 0.03. PW - period weighted average of the first three modes. EMW - effective mass weighted average of the first three modes. BS – base shear method. ID – inter-storey drift method.

As shown in Fig. 3, the period weighted scaling method (PW) predicts very large base shears relative to the design level base shears for the Bucharest (1977-NS) excitation. However, the four design-spectra scaling methods, symbolised as R1, 0.92R1, PW and EMW, predict very much smaller base shears than the design values when subjected to the Northridge (Sylmar-949NW) excitation. This indicates that, for any period structure, these four scaling methods may predict significantly small base shears and this depends on the characteristics of earthquake excitations. Hence these four scaling methods may not be reliable for earthquake scaling when design base shear is required to be matched.

When applying the inter-storey drift (ID) scaling method as shown in Fig 3, the maximum base shears for the Kobe (1995-NS) earthquake are significantly smaller than the corresponding design base shears. This means that the maximum base shears for the inter-storey drift (ID) scaling method depend on the characteristics of earthquake excitations. This scaling method should not be used for earthquake scaling when design base shears are required to be matched for say medium and long period structures.

8.2  Maximum Inter-storey Drifts

The distributions of the inter-storey drift versus the six different scaling methods for the structure under the four different earthquakes are shown in Fig. 4, in which solid lines represent the design inter-storey drifts obtained by combining the design spectral displacements by the SRSS method.

It can be seen that in Fig. 4, when the structure is subjected to each of the four different earthquakes, the inter-storey drift scaling method presents the same maximum inter-storey drift as the corresponding design inter-storey drift represented using solid lines. This relationship is independent
of the characteristics of the earthquakes. Hence this scaling method can be applied for earthquake scaling when design inter-storey drift is required to be matched for a natural earthquake.

When subjected to the Northridge (Sylmar-949NW) excitation, the base shear scaling method produces significantly smaller maximum inter-storey drift than the design inter-storey drift. This indicates that for this structure, the base shear scaling method may produce significantly smaller inter-storey drifts than design values and this is earthquake type dependent. Hence, this base shear scaling method should not be used for earthquake scaling when the design inter-storey drifts are required to be matched for, say, medium and long period structures.

Compared with the design inter-storey drifts the four design-spectra (R1, 0.92R1, PW and EMW) scaling methods produce very much smaller maximum inter-storey drifts than the design inter-storey drifts when subjected to the Northridge (Sylmar-949NW) excitation. This indicates that the four design-spectra (R1, 0.92R1, PW and EMW) scaling methods are not able to match the design inter-storey drifts, depending on the characteristics of earthquakes, and therefore should not be used for earthquake scaling when design inter-storey drift is required to be matched.

Fig. 4: Distribution of maximum inter-storey drift versus the six different scaling methods for 12-storey structure under the four different earthquakes

Note: **R1** - first mode method. **0.92R1** - first three modes with arbitrary weighting factors 0.92, 0.05 and 0.03. **PW** - period weighted average of the first three modes. **EMW** - effective mass weighted average of the first three modes. **BS** – base shear method. **ID** – inter-storey drift method.

8.3 5% Damped Spectral Accelerations and Displacements of the First Six Modes of the Three Structures Versus the Six Different Scaling Methods for the Four Earthquakes

In Figs. 5 to 8, the 5% damped spectral accelerations and displacements for the first six modes of the structure for the six different scaling methods for the four different earthquakes are presented.

For this structure, larger spectral accelerations than the design values were observed for the first four modes when using the base shear and inter-storey drift scaling methods.

The maximum base shears and inter-storey drifts are dominated by the first few modes, say the first 4 modes for the 12-storey structure. The earthquake records for this cases discussed in the previous two
paragraphs, for which if the spectral accelerations for the first 4 modes were similar or smaller than the design values for some scaling methods, will produce smaller base shears and inter-storey drifts relative to those at the required design level. Further, in the case of the frequency scaling method, which produces very similar spectral accelerations to the design values, the maximum base shears and inter-storey drifts under the scaled earthquake records would be significantly smaller than those at design level. This indicates that the use of the frequency scaling method will lead to records of low intensity producing small values of the maximum base shears and inter-storey drifts compared with the design requirements.

Fig. 5: 5% damped spectral responses of the first six modes for Bucharest (1977-NS) earthquake scaled by using the six different scaling methods

Fig. 6: 5% damped spectral responses of the first six modes of the three structures for El Centro (1940-NS) earthquake scaled by using the six different scaling methods

Fig. 7: 5% damped spectral responses of the first six modes of the three structures for Northridge (Sylmar-949NW) earthquake scaled by using the six different scaling methods

Fig. 8: 5% damped spectral responses of the first six modes of the three structures for Kobe (1995-NS) earthquake scaled by using the six different scaling methods
9 RELATIONSHIPS BETWEEN THE MAXIMUM ELASTIC RESPONSES AND THE SCALING FACTORS

Fig. 9 shows the relationships between the maximum roof level displacement, base shear, inter-storey drift and the scaling factors for the structure under the four earthquakes scaled using the six scaling methods. Each one of the four earthquakes has six excitations for the six scaling factors obtained from the six scaling methods. There is a strong linear relationship between the maximum roof level displacement, base shear, inter-storey drift and scaling factors for the structure.

Fig. 9: Relationship between the maximum elastic responses and the scaling factors for the three structures under the four excitations

10 RECOMMENDED PROCEDURE FOR EARTHQUAKE SCALING

It is evident that the base shear and the inter-storey drift scaling methods are reliable for earthquake scaling when the design base shear and the design inter-storey drift is required to be matched.

In order to have an appropriate scaling method for which both the maximum base shear and maximum inter-storey drift to the scaled earthquakes are as close as to and are not less than the corresponding design values as expressed by either Eq. (13) or Eq. (14), either the base shear scaling method or the inter-storey drift scaling method whichever gives the larger scaling factor should be used for earthquake scaling, based on the linear relationships between the maximum elastic responses and scaling factors as shown in Section 9.

\[ V_{SF=1.0} > V_{\text{design}} \quad \text{and} \quad ID_{SF=1.0} = ID_{\text{design, spectra}} \]  
\[ V_{SF=1.0} = V_{\text{design}} \quad \text{and} \quad ID_{SF=1.0} > ID_{\text{design, spectra}} \]  

where

- \( V_{\text{design}} \): design base shear determined by using equivalent static method.
- \( ID_{\text{design, spectra}} \): the maximum inter-storey drift from SRSS of the spectral displacements.
- \( ID_{SF=1.0} \): the maximum inter-storey drift for natural earthquake excitation.
- \( V_{SF=1.0} \): the maximum base shear for natural earthquake excitation.

A seven-step procedure for earthquake scaling is recommended:

Step 1: Obtain the natural periods of free vibration and the mode shapes for the first few modes. The sum of the effective mass for the modes to be combined should exceed 90% of the total mass.

Step 2: If the natural periods of the structures are similar to or less than 1.35 seconds, the scaling factor \( SF^* \) may be determined using the base shear scaling method shown in Eq. 15. Otherwise proceed to step 3.
\[ SF = \frac{V_{\text{design}}}{V_{SF=1.0}} \]  

(15)

where \( V_{\text{design}} \): design base shear determined by using equivalent static method.

\( V_{SF=1.0} \): the maximum base shear for natural earthquake excitation.

Step 3: Calculate the design spectral displacements for the chosen first few modes using the design displacement spectrum.

Step 4: Compute the design spectra SRSS inter-storey drift using the design spectral displacements and the mode shapes for the first few modes.

Step 5: Calculate the design base shear of the elastic structure (\( \mu = 1.0 \)) using the equivalent static method.

Step 6: Carry out dynamic elastic time-history analysis for the structure subjected to the natural earthquake to obtain the maximum inter-storey drift and the maximum base shear.

Step 7: Determination of the scaling factor \( SF \) shown in Eq (16) below.

\[ SF = \max \left[ \frac{ID_{\text{design, spectra}}^{\text{design, spectra}}}{ID_{SF=1.0}}, \frac{V_{\text{design}}^{\text{design}}}{V_{SF=1.0}} \right] \]  

(16)

where \( ID_{\text{design, spectra}}^{\text{design, spectra}} \): design inter-storey drift from SRSS of the spectral displacements. \( V_{\text{design}}^{\text{design}} \): design base shear determined by using equivalent static method. \( ID_{SF=1.0} \) and \( V_{SF=1.0} \): the maximum inter-storey drift and base shear from elastic time-history analysis for natural earthquake excitation.

11 SUMMARY AND CONCLUSIONS

The first period, the arbitrary weighting factor, the period weighted and the effective mass weighted scaling methods trying to match the design acceleration spectra, the design base shear scaling and the design inter-storey drift scaling methods trying to match the design base shear and design inter-storey drift respectively were used for check to see whether these scaling methods are reliable for earthquake scaling by comparing the maximum base shears, the maximum inter-storey drifts and the 5% damped spectral responses (accelerations and displacements) with the corresponding design values implied in New Zealand loading code.

From the above study, the following conclusions may be drawn:

1. The application of any scaling method attempting to match the scaled record to the design acceleration spectra may lead to large variations between the maximum base shear and the design base shear. This design-acceleration-spectra scaling method and the frequency scaling method are not recommended when design base shear is required to match that implied in the current New Zealand loading code.

2. The base shear scaling method is reliable for earthquake scaling when design base shear is required to be matched.

3. The inter-storey drift scaling method is reliable for earthquake scaling when design inter-storey drift is required to be matched.

4. The design base shear scaling method is recommended to be used for earthquake scaling for short period structures.

5. A general earthquake scaling procedure has been proposed, for which both the maximum base shear and maximum inter-storey drift to the scaled earthquakes are close to the corresponding design values.
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