

## Mass-eccentric building structures: effects of asymmetric distribution of axial forces in vertical resisting elements

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**ABSTRACT:** Torsional behavior of asymmetric buildings is one of most frequent sources of structural damage and failure during strong ground motions. Plan irregularity of buildings is frequently due to asymmetric distribution of mass that results in rotational motions of the floor slab in addition to translations, even for stiffness and strength symmetric systems. Moreover, asymmetric distributions of mass lead to an asymmetric distribution of axial forces in resisting elements, so that they present different lateral strength capability because of the influence of interaction phenomena. However, simple single storey asymmetric models used so far are not capable to evidence the influence of uneven distribution of axial forces; in fact, they have been developed under the assumption that resisting elements are able to sustain uni-directional horizontal forces only and no allowance for vertical forces is usually made. Therefore, a refined advanced numerical model of one-storey asymmetric building structure has been developed which is able to overcome limitations of the above-mentioned simplified models: this new idealization can take into account the presence of vertical forces due both to gravity loads and to vertical input ground motion as well as the effects of inelastic interaction between axial force and bi-directional horizontal forces. Results obtained from this new model are compared to those from previous models, thus evidencing a significant effect of plan-asymmetry of axial forces due to gravity loads on the overall system lateral-torsional inelastic behaviour.

### 1 INTRODUCTION

Plan asymmetric building structures subjected to lateral input ground motions are affected by torsional coupling, i.e. floor rotations in addition to translations, which generally results in larger lateral forces and deformations experienced by resisting elements (frames, shear wall etc.). Furthermore, for structures designed to undergo inelastic behaviour under strong earthquakes, torsional motions is one of the most frequent cause of severe damage and failure, since they lead to additional displacements and ductility demand in resisting elements.

Plan irregularity may be a consequence of plan asymmetric distributions of stiffness and strength among vertical resisting elements, but it is also frequently due to asymmetric distribution of mass over the floor slabs that results in torsional coupling, even for stiffness and strength symmetric systems. Buildings with this type of asymmetry are referred as mass eccentric systems.

In earlier investigations, the effects of torsional coupling on seismic response, for both stiffness and mass eccentric systems, have been widely investigated by means of simple one-storey models (e.g. Goel and Chopra 1991, Rutenberg 1998). They have been considered suitable to clarify the influence of key structural parameters and to carry out design criteria applicable also to some classes of multi-storey asymmetric buildings. However, simplified models neglect important effects that may influence inelastic behaviour of resisting elements and, in turn, of the entire structure. Namely, resisting elements are assumed to sustain uni-directional horizontal forces only; therefore, no allowance for vertical forces due both gravity loads and vertical input ground motions is usually made (Gherzi and

Rossi 1998, Stathopoulos and Anagnostopoulos 1998, Tso and Wong 1995, Tso and Smith 1999). Moreover, interaction among bi-directional horizontal and vertical forces in resisting elements cannot be taken into account in those analyses.

In recent years, some investigations have demonstrated that the contemporaneous action of axial force, due to both gravity loads and seismic loads, and of bi-axial bending moments, primarily due to horizontal earthquake forces, may influence to a large degree inelastic behaviour of resisting elements, with possible effects on torsional response (Kang-Ning 1996, Como et al. 2000).

Therefore, with the aim at fulfilling limitations of the above-mentioned simplified models, a refined numerical model of one-storey plan asymmetric building structure has been developed: it can take into account presence of vertical forces due both gravity loads and vertical input ground motions as well as effects of inelastic interaction between axial force and bi-directional horizontal forces (De Stefano and Pintucchi 2002a, De Stefano and Pintucchi 2002b).

The developed model seems to be very effective for analysing mass eccentric systems. In fact, asymmetric distribution of mass leads to an asymmetric distribution of axial forces in resisting elements, so that they present different lateral strength capability because of the influence of interaction phenomena. The new model, contrary to the simple models used so far, allows for analysing the effects of plan-asymmetry of axial forces due to gravity loads on the overall lateral-torsional inelastic behaviour.

Results obtained from the parametric analysis reported in this paper evidence the inadequacy of previous simplified models in predicting peak ductility demand and, principally, plan distribution of ductility demands among resisting elements. Effects on torsional response, as represented by floor rotations, are also evaluated.

## 2 MODEL SPECIFICATION

In this section the main characteristics of the numerical model used in this work are presented. It represents a single floor slab sustained by six vertical elements located as shown in Figure 1. Each resisting element is assumed to be massless and is considered to provide stiffness and strength along any direction. As regards behavior of resisting elements, an elastic perfectly plastic constitutive relationship according to the normality rule has been considered and interaction phenomena arising in the inelastic range are accounted for by means of an ellipsoidal yield domain.

It is assumed that the floor slab is infinitely rigid in its own plane, while it is flexible in the orthogonal direction. As a consequence, the system moves in the horizontal plane as a rigid body, having three degrees of freedom, namely  $x$ - and  $y$ - translations of the mass centre and floor rotation about vertical axis; furthermore, since floor slab is flexible in the vertical direction, displacements in the  $z$ -direction at locations of resisting elements,  $uz_j$ , are independent. Therefore, on the whole, the structure motion is described by  $3+n$  displacement coordinates, being  $n$  the number of the resisting elements. Moreover, horizontal inertia forces and moments involve system total mass  $M$  and rotational mass, whereas inertia forces resulting from the  $z$ -direction accelerations at locations of resisting elements involve masses evaluated by means of their tributary areas.

It should be noted that, if the structure is in the elastic range, the first three dynamic equilibrium equations, describing motion of floor slab in the horizontal plan, are coupled because of the asymmetry of the structure, but they are uncoupled from the  $n$  dynamic equilibrium equations in the vertical direction; in the inelastic range, even horizontal and vertical equilibrium equations couple because of the ellipsoidal domain.

In order to understand results presented below, it should be finally recalled that, contrary to the new model presented, the previous ones always neglect vertical forces and do not consider dynamic equilibrium equations in vertical direction.

### 3 PARAMETRIC ANALYSIS

#### 3.1 Analysed system

Analysed system is a one-way mass eccentric building model, being the mass centre  $C_M$  shifted at a distance  $E_M=0.10L$  along  $x$ -axis from the geometric and stiffness centre  $C_S$ , as shown in Figure 1. Total mass  $M$  has been distributed nonuniformly over the floor slab, being mass density on the right side of floor slab  $\gamma_1$  smaller than that on the left side  $\gamma_2$ ; as a consequence, the system mass centre  $C_M$  is shifted along the  $x$ -axis from the geometric centre of the floor slab. Mass radius  $\rho$  has been taken equal to  $\rho = 0.33$ .

As regards masses acting in vertical direction, which are obtained from tributary areas, masses of Element 1 and 2 are  $m_1 = m_2 = 0.175M$ , masses of Element 3 and 4 result  $m_3 = m_4 = 0.25M$  and  $m_5 = m_6 = 0.075M$ . Therefore, axial forces are larger in Elements 1 and 2 than in Elements 5 and 6, so that their plan distribution in resisting elements is asymmetric (i.e.  $N_1 = N_2 = 0.175Mg$ ,  $N_5 = N_6 = 0.075Mg$ ).

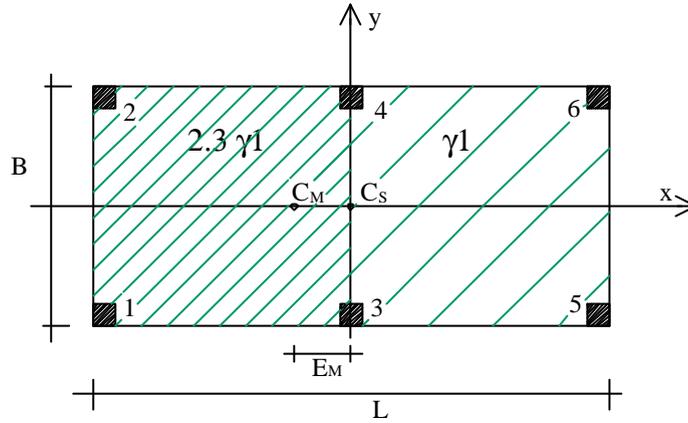


Figure 1: Analysed mass-eccentric system.

As regards total lateral stiffnesses, for the sake of simplicity, they are assumed to be equal in the two horizontal directions. Lateral uncoupled periods in  $x$ - and  $y$ - direction  $T_x = T_y$  have been varied, ranging between 0.1s and 1.5s. Subsequently, vertical period  $T_z$  has been evaluated from the lateral ones by means of a relation given in (Como *et al.* 2000), which has been seen to correlate well vertical to lateral period of real building structures.

Stiffness and strength characteristic of resisting elements are symmetric with respect to the floor geometric centre, i.e. it has been assumed that structure is designed for total mass uniformly distributed over the floor slab, being the shift of mass occurred during the building life. Therefore, cross-section areas of Elements 1 and 2, equal to those of Elements 5 and 6, are supposed to be half of those of Elements 3 and 4: as a consequence, lateral stiffnesses in  $x$  and  $y$ -direction as well as vertical stiffness of Elements 3 and 4 are greater than those of Elements 1, 2, 5 and 6. It has been assumed that  $k_3 = k_4 = 4k_1 = 4k_2 = 4k_5 = 4k_6$  and  $k_{z3} = k_{z4} = 2k_{z1} = 2k_{z2} = 2k_{z5} = 2k_{z6}$ . It should be noted that, for the obtained stiffness distribution, the ratio between torsional stiffness radius  $d$  to mass radius  $\rho$  is equal to 1.22, being the system torsionally-stiff.

As regards strengths, total lateral capacities  $F_{ox}$  and  $F_{oy}$  have been defined, for each input ground motion, from the relevant inelastic acceleration spectrum of the horizontal component having the higher peak ground acceleration, for three different values of imposed displacement ductility demand  $\mu$  ( $\mu=2,4,6$ ), corresponding to building structures that are expected to undergo different levels of inelasticity. Subsequently, lateral strengths of each element have been obtained by distributing  $F_{ox}$  and  $F_{oy}$  proportionally to its stiffness. Finally, vertical strengths of each resisting element has been assigned by applying a safety coefficient  $s=3$  against gravity loads that are expected to act on each vertical element, assuming mass uniformly distributed over floor slab.

### 3.2 Input ground motions

An ensemble of five pairs of horizontal components of real earthquakes, which can represent, on the average, the seismic action adopted by the Eurocode 8 for medium stiff soil conditions, has been used as input ground motion. The main characteristics of the selected records are reported in Table 1.

For each pair of records, the component having the higher peak ground acceleration (PGA) has been arbitrarily assumed to act along the y-direction. The other component has been taken as x-direction input ground motion.

**Table 1: Earthquake records used as input ground motion**

Earthquake	Date	Station	Duration (sec)	Primary component PGA (g)	Secondary component PGA(g)
Imperial Valley	18.05.40	El Centro	53.40	0.348	0.214
Kern County	21.07.52	Taft	54.40	0.179	0.156
Montenegro	15.04.79	Petrovac	19.60	0.438	0.305
Valparaiso	03.03.85	El Almendral	72.02	0.284	0.159
Northridge	17.01.94	Newhall	59.98	0.590	0.583

## 4 RESULTS

The maximum displacement, due to floor rotations, at the flexible edge - normalized with respect to the maximum y-displacement of the geometric centre of the floor slab - has been plotted as a function of the lateral uncoupled period  $T$ , for different levels of expected inelasticity  $\mu$ . This kinematic parameter assumed as torsional response indicator has been denoted as RCCG. Results obtained considering axial forces in resisting elements and interaction phenomena, as given by the new model, have been compared to those obtained in previous analyses, i.e. neglecting both the above-mentioned aspects: mean values of the parameter RCCG, averaged over all input ground motion considered, are given in Figure 2.

Trends of results obtained for the analysed mass eccentric systems confirm those obtained elsewhere (De Stefano and Pintucchi 2002c) by means of a wide parametric analysis conducted for stiffness eccentric systems: interaction phenomena induce a reduction in torsional response, as represented by the above kinematic parameter, for almost all lateral periods.

Regarding damage experienced by vertical resisting elements, interesting results have been obtained. A comparison between peak and plan distribution of ductility demands among resisting elements as evaluated by the two models (i.e. with and without interaction phenomena) is carried out.

The evaluation of ductility demand for resisting elements subjected to multi-component actions has been simply conducted by extending the well-known concept of displacement ductility demand under uni-directional excitations. Namely, a parameter defined as radial ductility  $Dd$ , given by the maximum value of the ratio between the moduli of displacement vector and of yielding displacement vector in the same direction, has been computed for all resisting elements:

$$Dd = \max \left( \frac{u_x(t)^2}{u_{ox}^2} + \frac{u_y(t)^2}{u_{oy}^2} + \frac{u_z(t)^2}{u_{oz}^2} \right)^{\frac{1}{2}} \quad (1)$$

Figure 3 represents curves of radial ductility for all resisting elements (numbered as shown in Fig.1 and denotes here as Elem.1 – Elem.6), as a function of lateral uncoupled period  $T$ , considering El Centro input ground motion and different levels of design ductility  $\mu$ .

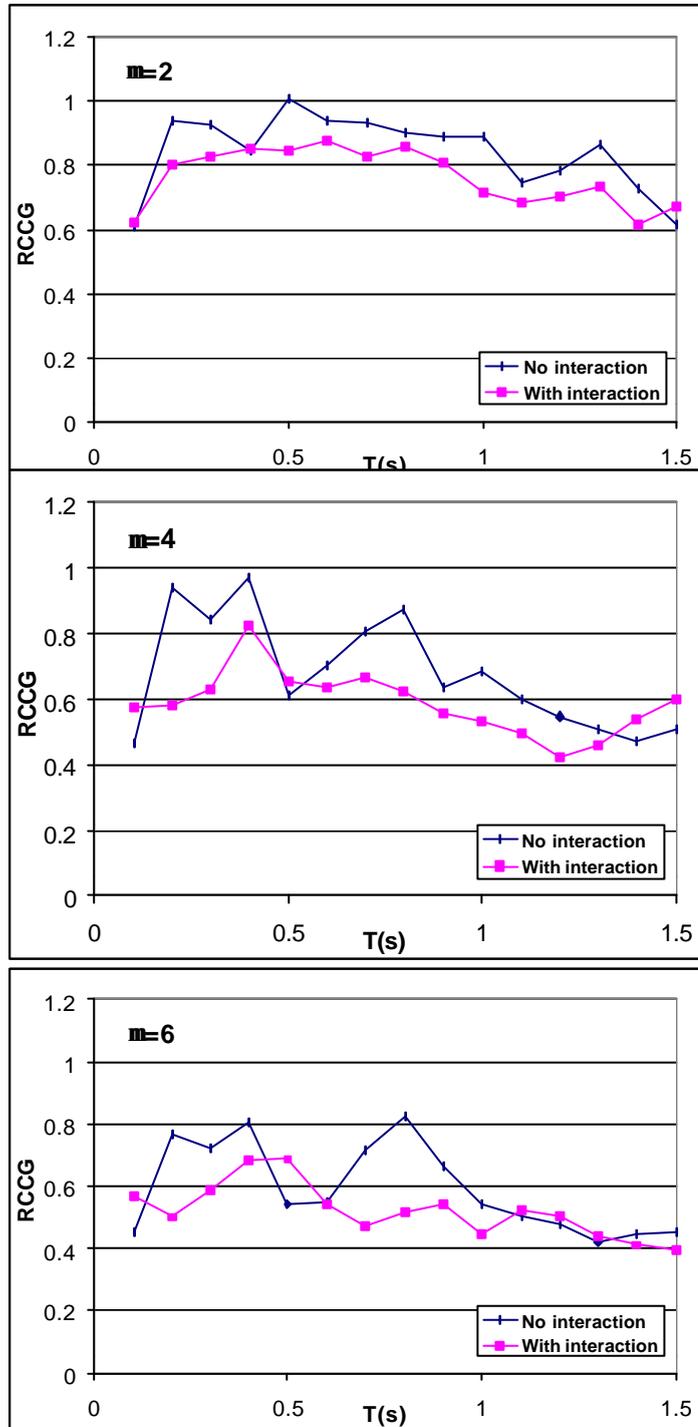


Figure 2: Mean values of the parameter RCCG as a function of lateral uncoupled period  $T$ , for  $\mu = 2, 4$  and  $6$ .

It can be seen that effects of interaction phenomena are significant, leading generally to amplifications in ductility demands of resisting elements. Moreover, trend of results from the refined model shows a significant non-uniformity in distribution of radial ductility demand among resisting elements, contrary to what emerges from the simplified models. Although radial ductility of Elem. 5 and 6 evaluated for the two models are comparable both qualitatively and quantitatively, radial ductility of Elem. 1 and 2, which are affected by larger axial forces, is consistently high, if interaction phenomena are considered.

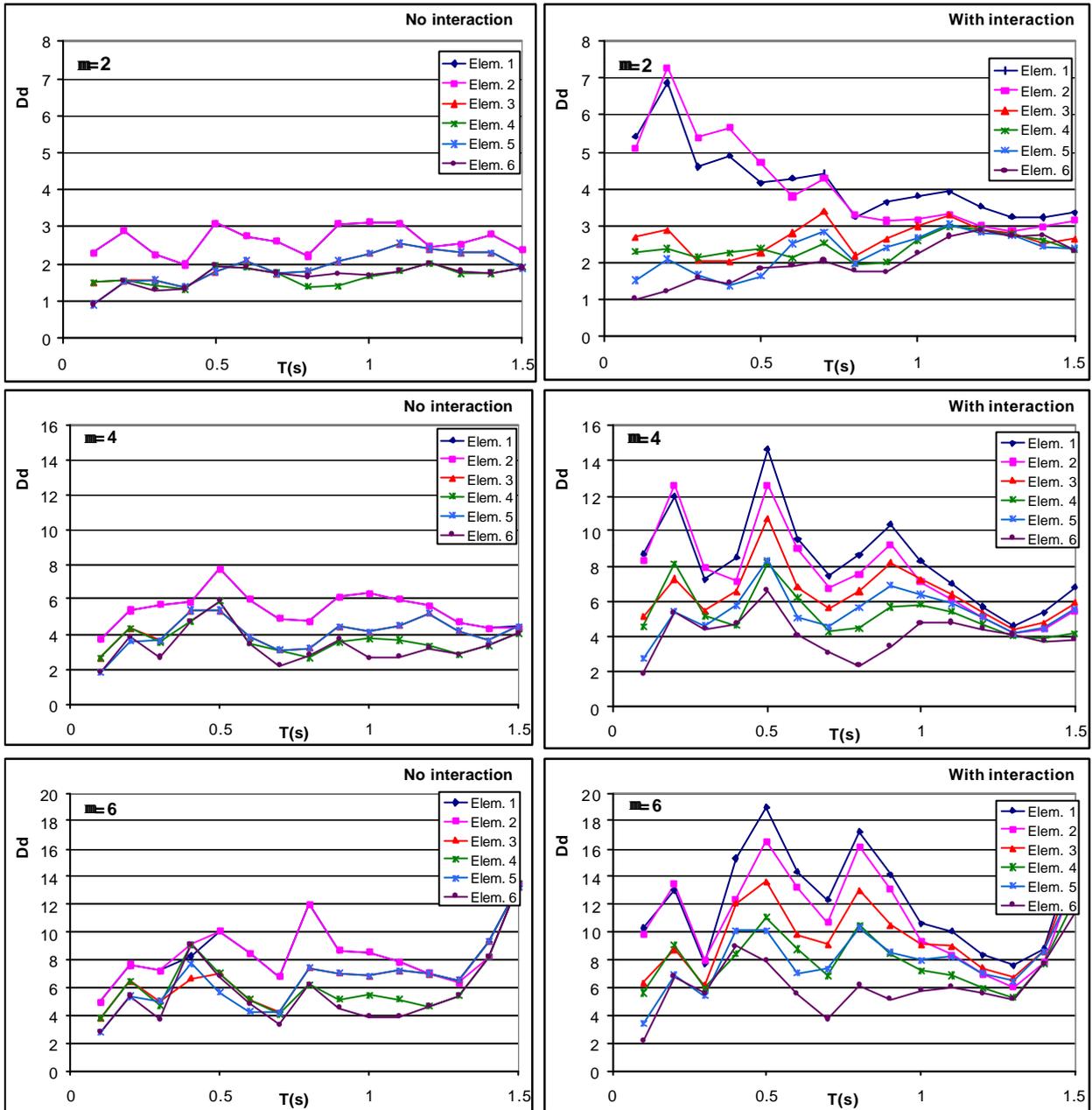


Figure 3: Comparison of ductility demand distribution among resisting elements, as obtained by the previous model (no interaction) and the new one (with interaction), under El Centro record.

Moreover, the new model evidences variations in ductility demand between resisting elements aligned in the y-direction which cannot be detected with previous models. In conclusion, the case shown for El Centro record demonstrates that a plan asymmetric distribution of axial forces in resisting elements, which goes together with nonuniform plan distribution of mass in buildings, may induce a consistent disuniformity in distribution of ductility demand among all resisting elements. This circumstance, which may results in severe local damage and failure, cannot be captured by simplified models used so far, which are inadequate from this point of view.

Figure 4 shows element ductilities, averaged over the five earthquakes used as input ground motions. As for the case of El Centro earthquake, trends of mean values evidence high differences in ductility demands of resisting elements, even if differences in elements aligned along the y-axis are less important.

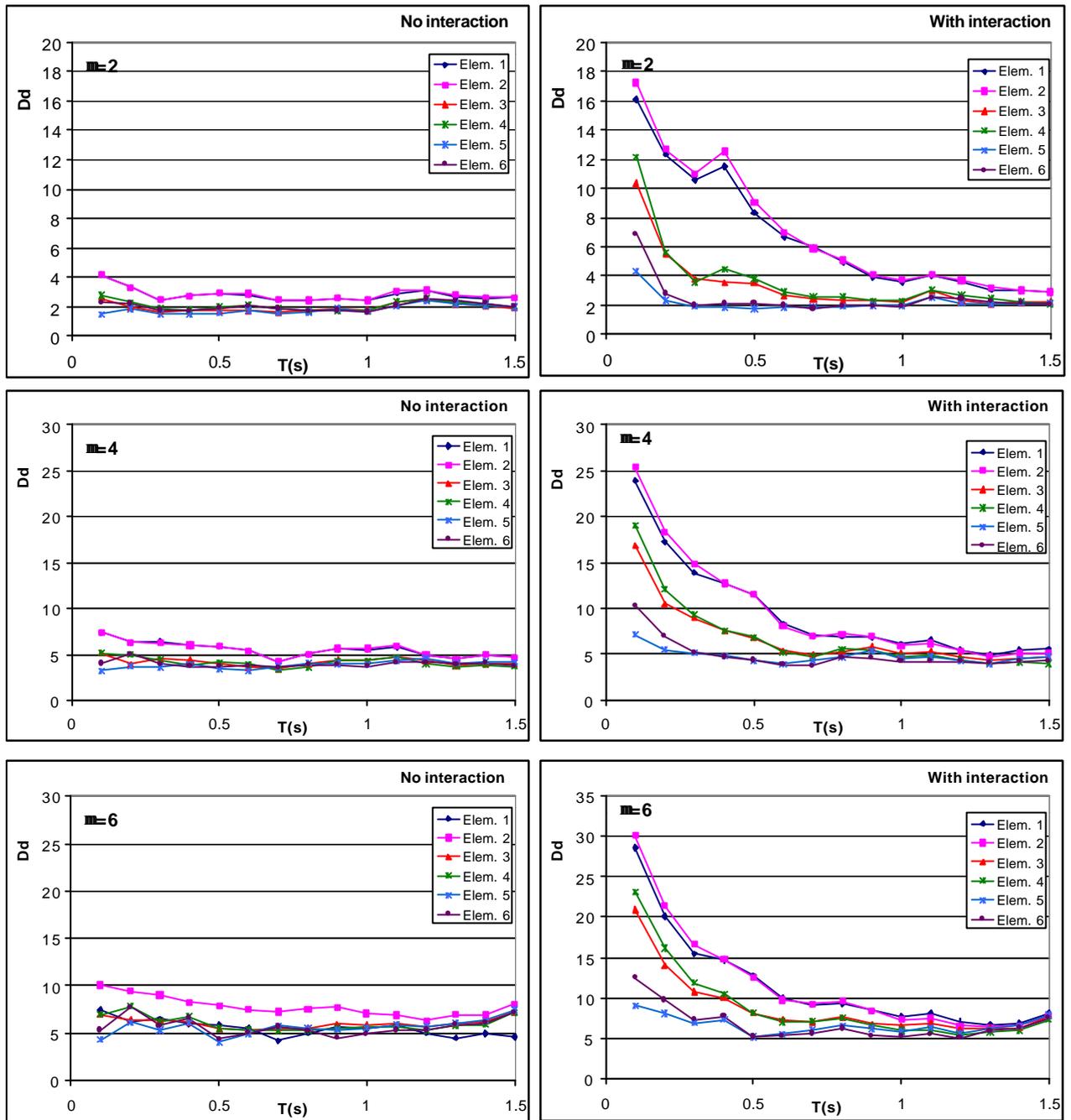


Figure 4: Mean values of ductility demand in resisting elements, as obtained by the previous model (no interaction) and the new one (with interaction).

## 5 CONCLUSIONS

Asymmetric distributions of mass leads to an asymmetric distribution of axial forces in resisting elements, which, as a consequence, present different lateral strength capability because of the influence of interaction phenomena. However, simple single storey asymmetric models used so far are not capable to evidence the influence of this aspect, as they have been developed under the assumption that resisting elements are able to sustain uni-directional horizontal forces only and no allowance for vertical forces is usually made.

Therefore, a refined advanced numerical model of one-storey asymmetric building structure, which is able to overcome limitations of the above-mentioned simplified models, has been used in this paper to analyse torsional behaviour of mass-eccentric systems.

The new idealization can take into account the presence of vertical forces due both to gravity loads and to vertical input ground motion as well as the effects of inelastic interaction between axial forces and bi-directional horizontal forces. In this manner, the model allows to evidence the effects of plan-asymmetry of axial forces due to gravity loads on the overall lateral-torsional inelastic behaviour.

Results obtained from the parametric analysis presented in this paper point out the inadequacy of previous simplified models in evaluating peak values but, principally, in predicting plan distribution of ductility demands among resisting elements. Analyses conducted by means of the new model shows a significant nonuniformity of ductility demands among resisting elements, due to their different levels of axial forces, which can result in very high ductility demands on some resisting elements.

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