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Cyclic strut-and-tie modelling of reinforced concrete structures

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ABSTRACT: Nonlinear cyclic force-displacement responses of three concrete cantilever beams and three large-scale concrete bridge knee-joint systems were analysed using nonlinear cyclic strut-and-tie models. Existing element models in computer program RUAUMOKO were employed for performing the nonlinear analyses. The analytical results were found to compare satisfactorily with the experimental data.

1 INTRODUCTION

A strut-and-tie model (STM) is a discrete representation of the stress field developed within a concrete structure when subjected to external actions. Representation of structures with STMs allow analysis and design of reinforced concrete structural types to be performed in a rational manner.

Within a STM, uniaxially stressed struts and ties having finite dimensions are used to represent the actual compressive and tensile stress fields respectively. The pin connections joining the struts and ties together correspond to the biaxially or triaxially stressed nodal zones.

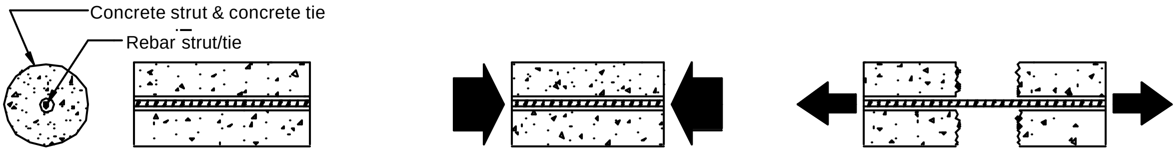
The strut-and-tie modelling technique has traditionally been employed in design practice to predict strength and to examine equilibrium of the applied loads, reactions, and internal forces for disturbed (D-) regions of structures with irregular geometry where the internal flow of force is not well known. Typically, such an investigation enables determination of suitable reinforcement detailing for the D-regions.

Previous investigations by To *et al.* (2001, 2002a & 2002b) have demonstrated that a well-formulated STM is capable of capturing the monotonic response of reinforced concrete structures beyond the elastic range. In addition, the internal force demands developed in structural components can be effectively assessed using this modelling technique. The study reported in this paper is the extension of the application of strut-and-tie modelling technique to capture the nonlinear cyclic force-displacement response of reinforced concrete structures.

The nonlinear computer program RUAUMOKO [Carr (1998)] was employed for performing the analyses, with adequate stress-strain models for concrete and steel reinforcing bars were adopted. It is important to note that the objective of this investigation was not to obtain an exact replication of the stress transfer mechanism that develops within a structure when subjected to cyclic action. Instead, the cyclic STMs described in this paper represent only a simple diagnostic tool for the seismic-resistant analysis and design of complicated structural types.

2 IDEALISED UNIAXIAL FIBRE MODEL

The development of the uniaxial fibre model used in this work was based upon the results of Tjokrodimuljo (1985). He tested seventeen reinforced concrete prisms with cyclic axial loads to investigate the hysteretic response of concrete and reinforcement in the flexural zones of reinforced concrete beams and columns. The uniaxial fibre model that was developed is shown in Figure 1. This model consists of three elements arranged in parallel, namely a concrete tie, a concrete strut and a rebar strut-tie.



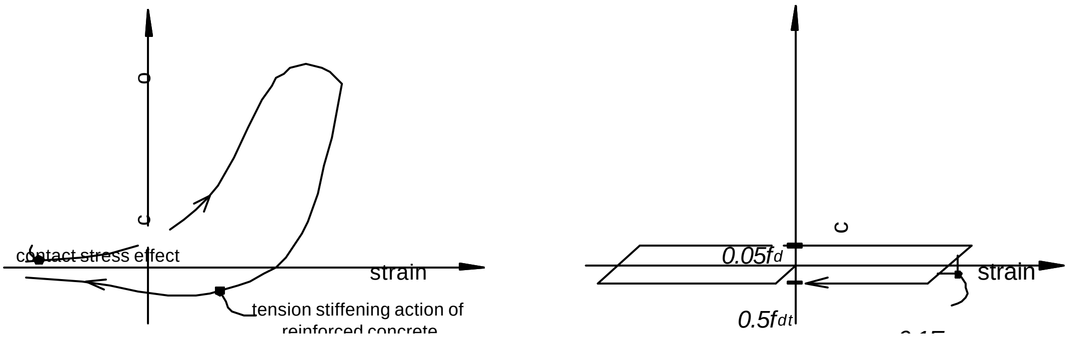
a) The original shape of an idealised uniaxial fibre.

b) Model subjected to axial compression.

c) Model subjected to axial tension.

Figure 1. Idealised uniaxial fibre model.

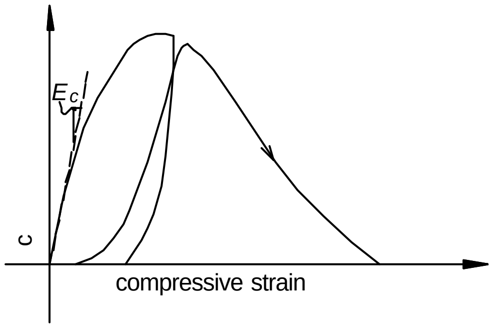
A typical form of the stress-strain characteristics of concrete as found by Tjokrodimuljo are shown in Figure 2a. In this figure the “contact stress effect” is illustrated by the non zero compressive stress at zero strain on the loading path. This is due to the wedging type action by dislocated aggregate particles in the cracks of the concrete and is partially responsible for the elongation of flexural members when subjected to cyclic action [Fenwick *et al.* (1996)]. In addition, “tension stiffening” can be observed. Experimental results show that this phenomenon continues during cyclic loading and the “stiffness” is in the order of 0.1 E_c. To replicate this behaviour, the elastic-perfectly plastic concrete “tie” element was placed in parallel with a concrete “strut” element which used the masonry strut hysteretic model developed by (Crisafulli 1997). The stress-strain characteristics of these are shown in Figure 2b and c. The stiffness and strength of the concrete “tie” were chosen to help providing the effects of “contact stress” and “tension stiffening”. The compressive strength for the contact stress effect was chosen as 0.05 f_d [Douglas (1996)] and the tensile strength of the concrete ties was taken as 0.5 f_{dt}, where f_d is the effective compressive strength of a concrete strut and f_{dt} is the effective tensile strength of a concrete tie. The hysteretic behaviour of this combination is shown in Figure 2d which compares well with that shown in Figure 2a. The modified Takeda hystetic model [Otani 1974], as shown in Figure 2e, was used to model the behaviour of the reinforcement. In Figure 2f the axial load-displacement traces developed by the fibre model are shown to compare satisfactorily with an experimental result obtained by Tjokrodimuljo.



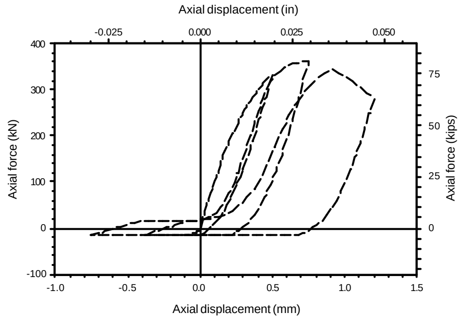
a) Response of concrete component in reinforced concrete member

b) Concrete tie

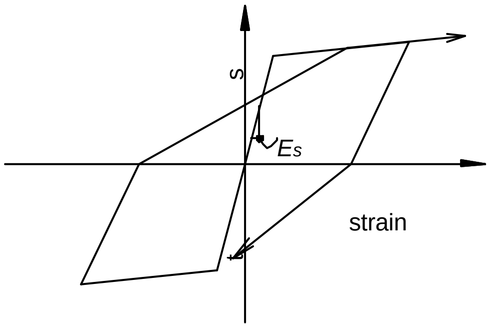
Figure 2. Model member stress-strain properties.



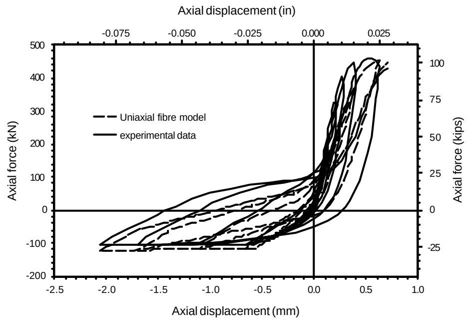
c) Concrete strut



d) Combined analytical response of concrete tie and concrete strut



e) Rebar strut-tie



f) Uniaxial fibre model

Figure 2 cont. Model member stress-strain properties.

3 STM FORMULATION PROCEDURE

The full procedure for computing the strut and tie model member properties is described in To *et al.* (2003). Highlights of the procedure to provide an insight into the modelling techniques are given in this section.

3.1 Formulation strategy

The strategy employed in the formulation of the procedure for cyclic STMs requires defining parts of a structure into separate regions. There are two types of regions, “B” and “D”. B(or Bernoulli) -regions are those parts of a structure where conventional flexural theory is assumed to hold and D(or Disturbed)-regions are where the internal stress-strain distribution is significantly disturbed by discontinuities in the physical geometry or externally applied actions.

3.2 B-region structures

For B-region structures such as beams and columns, a comprehensive section force analysis based upon the Bernoulli compatibility condition that plane sections remain plane is performed. Section force analytical results are then used to compute the effective area and strength of the model members, concrete struts, concrete ties, and rebar strut-ties. The model members are located at the force centroid of tension reinforcement measured at the first yield state for each flexural action direction. The first yield state is defined by the commencement of rebar yielding or the extreme concrete fibre reaching a compression strain value of 0.002, whichever occurs first.

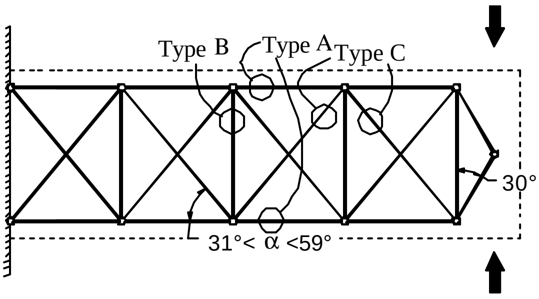


Figure 3. STM of a cantilever beam.

Depicted in Figure 3 is the STM of a cantilever beam which is a typical B-region structure. Note that the angle of inclination chosen for the diagonal struts $31^\circ \leq \alpha \leq 59^\circ$ are based on the recommendations by CEB-FIP (1978). Also illustrated in this figure are type A model members are the uniaxial fibre elements to represent the member flexural zone. In addition, type B model members are rebar strut-tie elements to model the transverse reinforcement and type C model members are constructed with a concrete strut and a concrete tie arranged in tandem to represent the diagonal concrete zone. The inclusion of concrete ties in type C model members is to account for structural stiffness at low load levels.

3.3 D-region structures

The STMs for D-region structures, such as the knee-joint region in the examples considered later, are modified from monotonic models reported in the literature [Ingham *et al.* (1997)]. Experimentally measured reinforcement strength was used as the effective strength of rebar struts-ties, while the effective strength of concrete struts was determined using Table 1 according to the anticipated strut condition when subjected to cyclic action [Sritharan and Ingham (2002)]. Furthermore, $0.5\sqrt{f'_c}$ [MPa] [Priestley *et al.* (1996)] was used to compute the effective strength of concrete ties. All members were located at the force centroid of the corresponding force-transfer mechanism.

Table 1. Effective strength of concrete struts applied to D-region of joint systems

Effective strut strength	Strut condition
$0.68\,f'_c$	This value can be adopted for struts locating in regions where minor cracking is expected. An example of this application would be the struts residing in a prestressed joint.
$0.51\,f'_c$	This value is appropriate for concrete struts when the neighbouring rebar in not subjected to significant strain hardening ($e_s \leq 0.01$).
$0.34\,f'_c$ *	This is the maximum permissible stress for concrete struts when there is potential development of significant inelastic strain ($e_s > 0.02$) in the neighbouring reinforcement. This value can also be applied to struts modelling bottle-shaped stress zones when no effective confinement is provided.

* For $0.01 < e_s < 0.02$, consider linear interpolation to obtain appropriate permissible stress.

4 CYCLIC STRUT-AND-TIE ANALYSES

In this section, details of three cantilever beams and three large-scale knee-joint test units subjected to repeated cyclic actions are presented. For the STMs, attention was given to discretisation of the appropriate joint force-transfer model, and the derived structural response. Results obtained from the strut-and-tie analyses were compared with experimental records to determine the effectiveness and advantage of the STM procedure.

4.1 **Cantilever beams**

The hysteretic response of three cantilever beams was calculated using cyclic STM procedure. Relevant structural details of the beams are listed in Table 2. All three units were doubly reinforced, having top and bottom longitudinal reinforcement with minimal concrete cover. In addition, the shear strength of the tested units was designed to exceed that required by the codes and a large shear span to effective depth ratio was adopted.

Table 2. Structural details of the cantilever beams.

	Beam 1*	Beam 2**	Beam 3***
Sectional width, b (mm)	200	200	228.6
Sectional height, h (mm)	500	500	406.4
Top reinforcing steel area, A'_s (mm^2)	1570.8	1570.8	1140.1
Bottom reinforcing steel area, A_s (mm^2)	1570.8	1005.3	593.8
Transverse reinforcing steel area, A_v (mm^2)	185.4	191.6	126.7
Shear span, l (mm)	1500	1329	1587.5
Stirrup spacing, s (mm)	100	100	88.9
Concrete compressive strength, f'_c (MPa)	33.2	27.7	31.58
Yield strength of longitudinal steel reinforcement, f_y (MPa)	311	280 (top) 298 (bot.)	451.6 (top) 458.5 (bot.)
Yield strength of transverse steel reinforcement, f_{vy} (MPa)	300	300	413.7

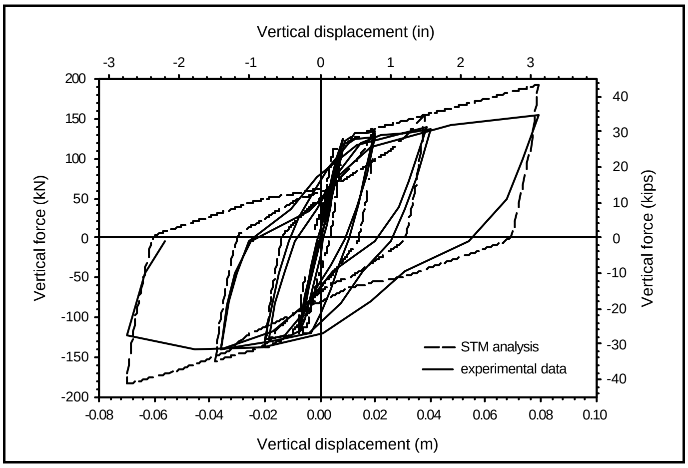
* Data extracted from Fenwick, *et al*. (1982) test unit beam 1a.

** Data extracted from Fenwick and Fong (1979) test unit beam 3a.

*** Data extracted from Ma *et. al.* (1976) test unit beam 3.

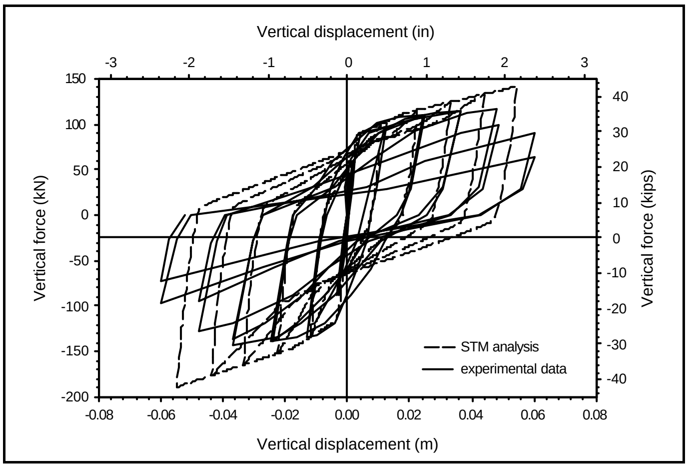
The STMs for all the three cantilever beams were of the same form and shown in Figure 3.

The cyclic force-displacement response envelopes that were generated using the STMs were compared to the corresponding experimental records in Figure 4. Results show satisfactory agreement between the two sets of data. The elastic and plastic stiffness as well as flexural yield strength of the cantilever beams were accurately predicted by the cyclic STM analyses. Slight discrepancies between the model data and experimental observations for the reloading and unloading branches were due to the Bauschinger effect not being considered in the modified Takeda hysteresis model.

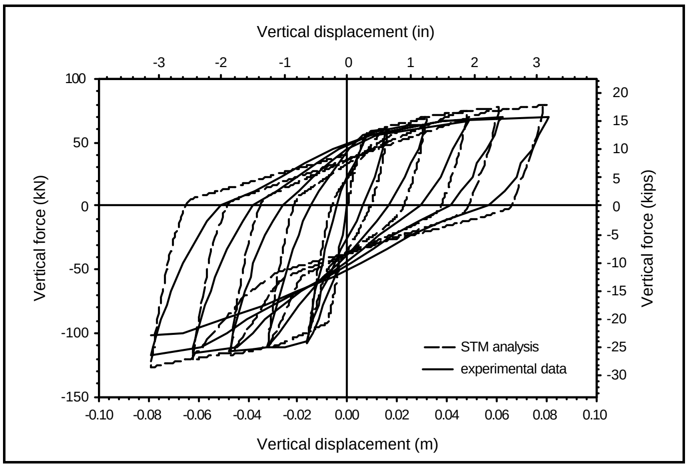


a) Beam 1 [Fenwick, *et al*. (1982)]

Figure 4. Comparison between cantilever beam test unit data and model response.



b) Beam 2 [Fenwick and Fong (1979)]

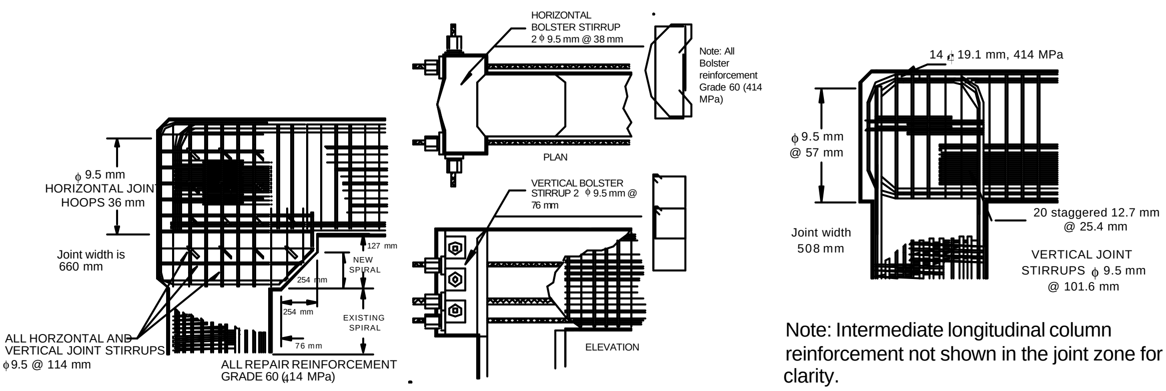


c) Beam 3 [Ma et. al. (1976)]

Figure 4 cont. Comparison between cantilever beam test unit data and model response.

4.2 **Knee-joint test units**

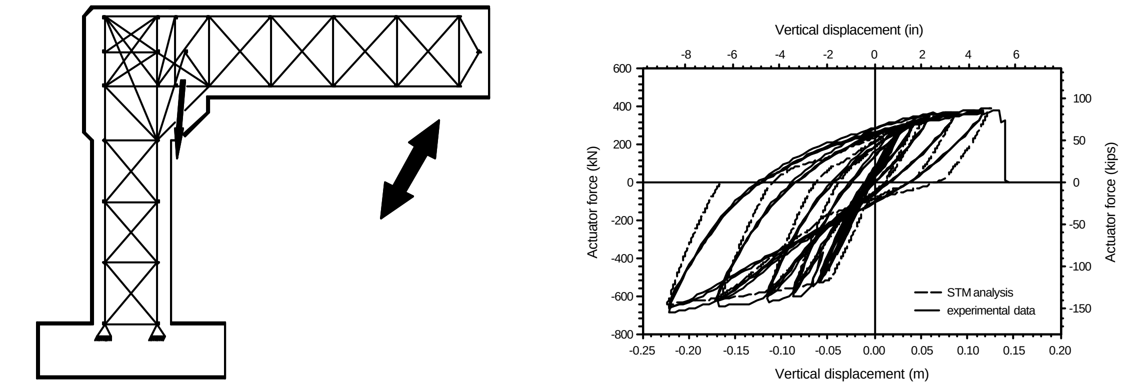
To demonstrate the performance of the strut and tie formulation for a structure with a D region, three large-scale bridge knee-joint units tested by Ingham *et al*. (1994) were modelled. The units had the same dimensions, a common column reinforcement detail, but different reinforcement details in the "joint" region. The reinforcement details of the three units; the "repaired joint", the "retrofitted joint" and the "redesigned joint" are shown in Figure 5.



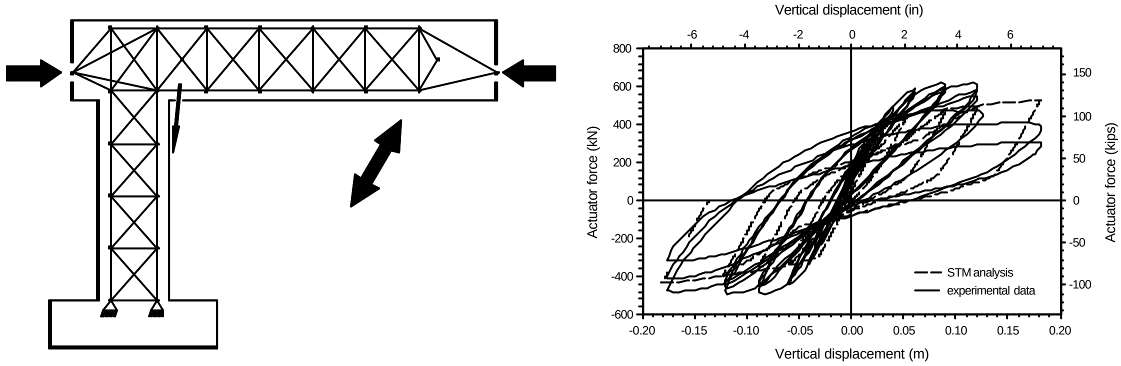
a) Repaired joint b) Retrofitted joint c) Redesigned joint

Figure 5. Knee-joint test units reinforcement details.

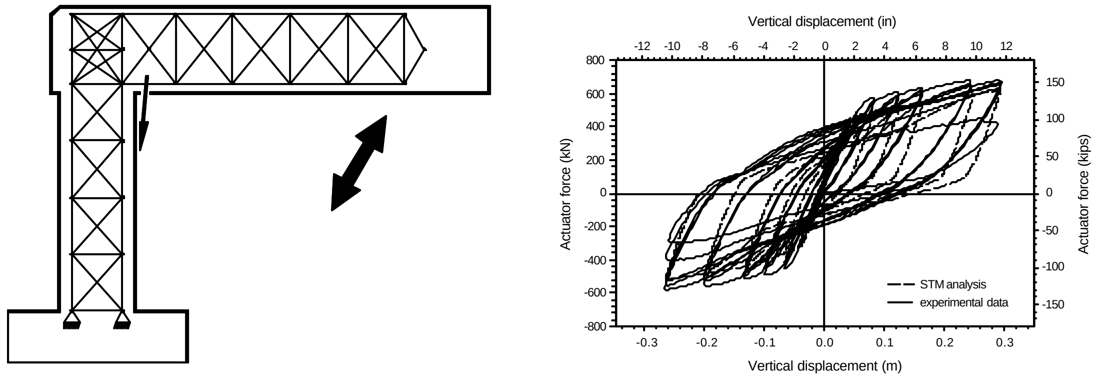
The strut and tie models for the analyses of these units are shown in Figure 6, based on the formulations proposed by Ingham *et al.* (1997). The effective strength of concrete struts located in the knee-joint region of the repaired, retrofitted and redesigned units were assumed to be 0.68 f_c', 0.51 f_c' and 0.68 f_c' respectively [To *et al.* (2001)].



a) Repaired joint



b) Retrofitted joint



c) Redesigned joint

Figure 6. Cyclic STMs and force-displacement response diagrams for knee-joint test units.

Also shown in Figure 6 are the experimental and STM force-displacement responses. In general, the analytical results are consistent with experimental observations, with the elastic and plastic stiffness of the test units satisfactorily captured. However, the models could not replicate the strength degradation exhibited by the retrofitted knee-joint as a result of cumulative concrete damage. This limitation could be overcome with the inclusion in the STM of a reduction in the internal lever arm of the longitudinal members with an increase of member ductility and loading cycles. Typically, such capability does not exist in computer softwares including RUAUMOKO. Furthermore, the unloading and reloading branches of the model force-displacement responses could be improved with the inclusion of the Bauschinger effect.

Despite some discrepancies between analytical and experimental results, the strut and tie models are able to capture the general cyclic performance of complicated structural systems, as it requires a consistent level of modelling detail to all structural forms.

5 CONCLUSIONS

- The current research paper reported an investigation regarding the application of strut and tie models to model the cyclic force-displacement responses of reinforced concrete structures.
- All the cyclic force-displacement response envelopes generated using strut and tie models were in satisfactory agreement with experimentally recorded data.
- The formulation procedure developed in the current investigation for cyclic strut and tie models was proved to be adequate.
- Analytical results could be improved by including the Bauschinger effect in the rebar struts-ties of STMs.

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