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The generation of in-elastic response spectra for earthquake acceleration records

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ABSTRACT: In 1960 Newmark showed that the displacements of inelastic structures subjected to earthquake excitation were similar to those of the same structure when it behaved elastically. Code writers have taken this to develop the equal displacement concept that has been the mainstay of seismic design codes for the past 40 years. Modifications have been made to the approach for structures with short natural periods of free vibration, to use the equal energy and equal acceleration concepts when deriving the inelastic design spectra. It will be shown in this paper that many of these assumptions are not particularly true even for the earthquake accelerograms used by Newmark. With all the advances in the analysis methods and design philosophies, such as capacity design and performance based design, made over the past 40 years that it is appropriate than the basic assumptions used in deriving the inelastic design spectra need to be re-appraised. This paper will outline a method of deriving the inelastic design spectra for any earthquake excitation allowing for almost any stiffness and strength degradation models to be used to represent the structural behaviour.

1 INTRODUCTION

The current method of producing the inelastic design spectra used in many building codes including the New Zealand Loadings Code [NZ4203:1992] is to use the equal displacement concept of [Newmark, 1960] which implies that the inelastic structure will have the same displacement as the elastic structure and this implies [Carr, 1994] that if the structure has a design ductility of 4 then the yield force will be a ¼ of the elastic design force. This means that the inelastic acceleration design spectrum will be the elastic acceleration spectrum divided by the ductility factor. For shorter natural period structures the equal energy concept is used. As the natural period tends to zero the inelastic acceleration spectrum will tend to the elastic acceleration spectrum, the equal acceleration concept. In the NZ Loadings Code a linear interpolation of the reduction factor is used between the equal displacement method at natural periods greater than 0.7 seconds and the equal acceleration concept as the natural period tends to zero. In many inelastic analyses carried out over the past 30 years there have been many instances where the equal displacement concept was observed to have been inapplicable. As a result, a combination of the inelastic capabilities of the analysis program Ruaumoko [Carr, 2001] and the response spectra methods to produce a program, INSPECT [Carr, 2002] to produce inelastic response spectra for any earthquake accelerogram using almost any of the 44 hysteretic stiffness degradation rules and for any of the 5 strength degradation rules used in Ruaumoko.

2 INELASTIC SPECTRA FOR A SPECIFIED DUCTILITY FACTOR

The method used for computing the inelastic response spectra follows the methods used for computing an elastic response spectrum with the addition of an iterative approach to achieve the target ductility factor and provision to allow for the hysteretic and strength degradation behaviour of the system. As the behaviour of the structure is non-linear the Constant Average Acceleration method [Carr, 2002] is used to integrate the equations of motion as the linear elastic methods used in most elastic response spectra programs are inapplicable. The first step is to compute the elastic acceleration and displacement response spectra for the earthquake accelerogram for the specified level of viscous damping. The initial yield force for the target ductility uses the reduction ratios derived for the current inelastic design acceleration response spectra, i.e. for long period structures where the natural period of free-vibration T is greater than 0.7 seconds, the acceleration spectrum is divided by the ductility ratio. For natural periods less than 0.1 seconds the yield force is taken as being equal to the acceleration spectrum. For intermediate natural periods a linear interpolation is made between the reduction factor at a free-vibration period of 0.7 seconds and the value of 1.0 at a free-vibration period of 0.1 seconds.

The next step is to compute the displacement for each natural period of free-vibration and to compare the computed ductility with the target ductility. The strength is then adjusted using a logarithmic relationship between the required strength and the ductility ratio between the target ductility and the computed ductility. The displacement is then re-computed and the strength is again adjusted until the computed displacement is within one percent of the target ductility, or if the iteration number exceeds 200. The relationship between the yield strength Y and the ductility Mu is given by

$$Y = Y_0 \, Mu^C$$

where Y_0 is the elastic spectral acceleration and C is the least squares slope of the relationship between $\log(Y_i/Y_0)$ and $\log(D_i)$ where Y_i and D_i are the Yield force and Displacement at each iteration i. Most computation requires of the order of 6 to 20 cycles and experience has shown that increasing the maximum number of cycles to greater than 200 does not improve the computed ductility which means that at some frequencies the relationship between strength and computed ductility is not strongly related.

Once convergence to the target ductility is achieved then the displacement of the oscillator gives the Spectral Displacement, the Yield Strength provides the equivalent of the current Strength (Acceleration) Design Spectra, the maximum total acceleration recorded provides the Spectral Acceleration and the program also computes the amount of plastic work done during the response and, provided a duration of free-vibration is allowed to occur following the duration of the earthquake excitation an estimate can be made of the residual displacement of the oscillator. The last piece of information is becoming of interest to designers as it gives an estimate of the permanent displacement that may result in the structure following the earthquake.

The elastic natural period of free-vibration is then incremented and the whole procedure is repeated. Once the whole range of natural periods of free-vibration has been covered the spectra for that target ductility is complete. The procedure can now be repeated for the next target ductility.

If the displacements computed for the target ductility are the same as the elastic displacements then the equal displacement concept espoused by Newmark would apply. However, as will be seen in the presented results this does not appear to apply even for the El Centro may 1940 earthquake which was supposed to be an earthquake that justified the Equal Displacement concept.

3 INELASTIC RESPONSE SPECTRA.

The El Centro May 1949 North-South component will be used to generate examples of the inelastic spectra to show both the capabilities of the INSPECT program and also to show the differences in the results from those that have been assumed by designers over the past 40 years. In this paper only two of the available hysteresis rules are used, the Elasto-plastic rule which is the same as the Bi-linear rule with the post elastic stiffness **r** = 0.0 and the Modified Takeda rule where the bi-linear factor **r** = 0.05 and Alpha and Beta are 0.4 and 0.5 respectively. These two hysteresis loops are illustrated in Figure 1. The examples in this paper have no strength degradation with either ductility or number of in-elastic cycles.

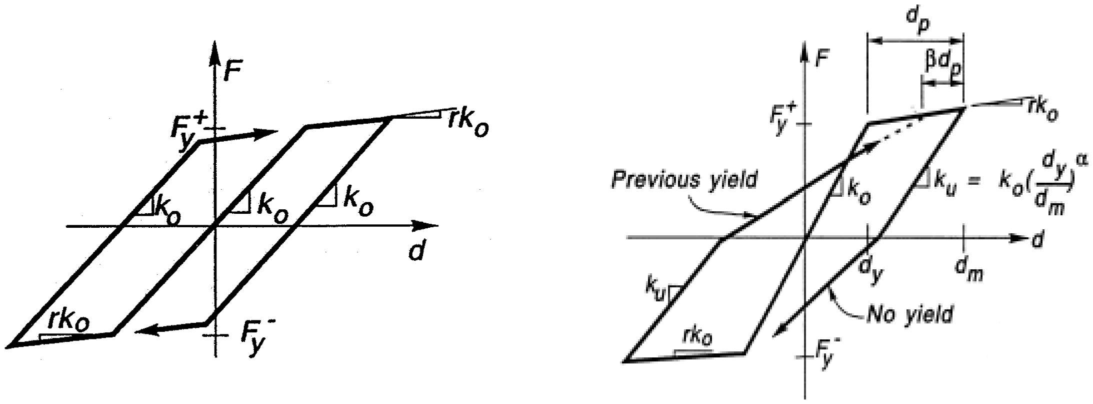

Figure 1. Bi-linear and Modified Takeda Hysteresis Loops.

Figures 2 and 3 show the Displacement Spectra for the El Centro accelerogram for both hysteresis loops and for the elastic system (ductility 1) and for ductility 2 and ductility 4. It is evident that the *Equal Displacement Concept* of Newmark does not hold for if it did then all three lines would follow a similar relationship. The current New Zealand code (NZ4203:1992) assumes that the concept holds for natural periods of free vibration greater than 0.7 seconds where it is evident from the plots that the spectra show a fairly large divergence at the longer natural periods. This implies that the current methods of reducing the inelastic design acceleration spectra from the elastic acceleration spectra by dividing by the structure ductility factor are not appropriate. It is also obvious that the shape of the hysteresis loop also has an effect on the spectral displacements for the inelastic structure. This effect of the loop shape has been observed by the author in dynamic analyses over many years and is contrary to the belief that the shape of the loop does not have a great effect on the inelastic displacements.

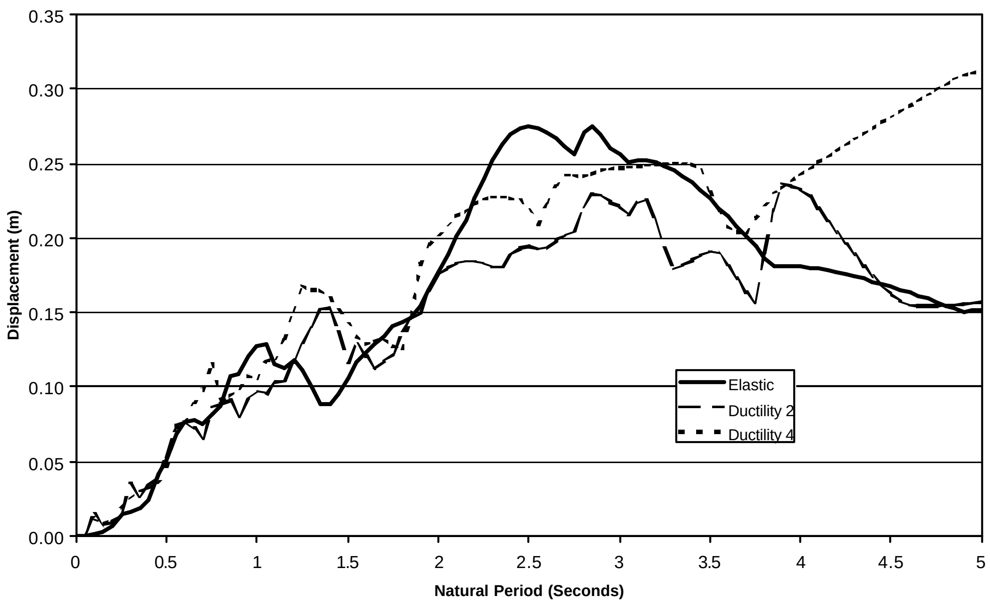

Figure 2. Spectral Displacement – Elasto-plastic

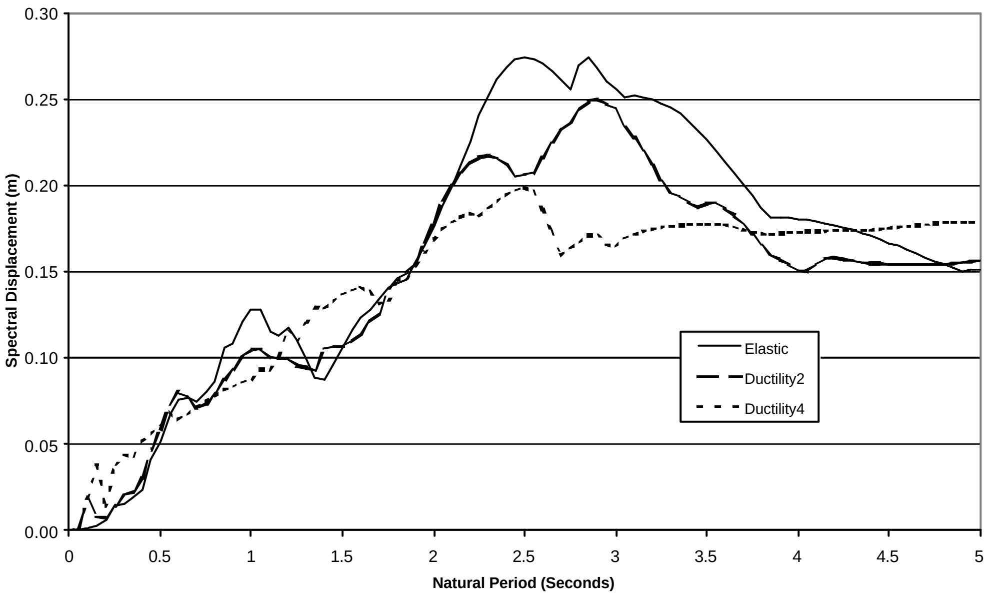

Figure 3. Spectral Displacement – Takeda

The next two plots, Figures 4 and 5, show the Spectral Accelerations and the Yield Forces for the El Centro earthquake for ductilities 1, 2 and 4 using the Elasto-plastic hysteresis. Again, if the equal displacement concept was true then for the longer natural periods the spectral values for ductility 2 would be half the elastic response and for ductility 4 a quarter of the elastic response. The Yield Force Spectra for the elastic case is the same as the Spectral Acceleration but the other lines show the reduced yield forces required to give the target structure ductility. For longer period structures, the shape of the Yield Spectra and the Acceleration Spectra are similar but for the shorter natural period structures they are very different. Figures 6 and 7 show similar results using the Modified Takeda hysteresis loop. For both hystersis loops the yield force distributions are similar to the acceration spectra but both the magnitudes and the shapes of the curves are not the same. The spike seen in the Spectral Acceleration plot for ductility 4 is the result of not achieving the target ductility (about a 2% error from the target ductility of 4.0) at the 0.1 second natural period. It must be noted that at large ductilities, i.e. greater than about 2.0 it appears to be quite common to have spectral accelerations than exceed the elastic accelerations at these short natural periods of free vibration. Such an effect is also observable in Figure 4 for the Elasto-plastic hysteresis.

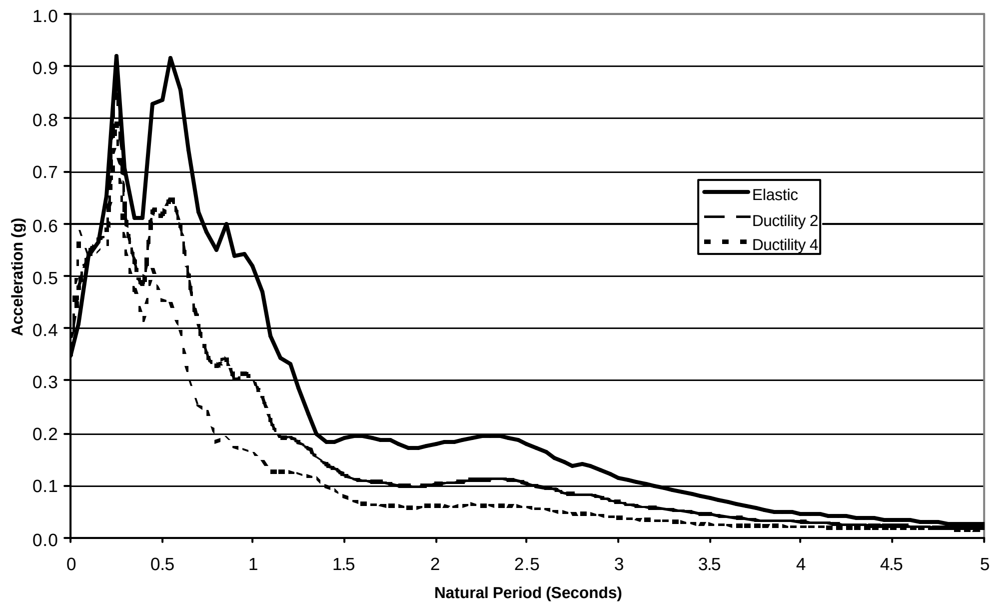

Figure 4. Spectral Acceleration – Elasto-plastic

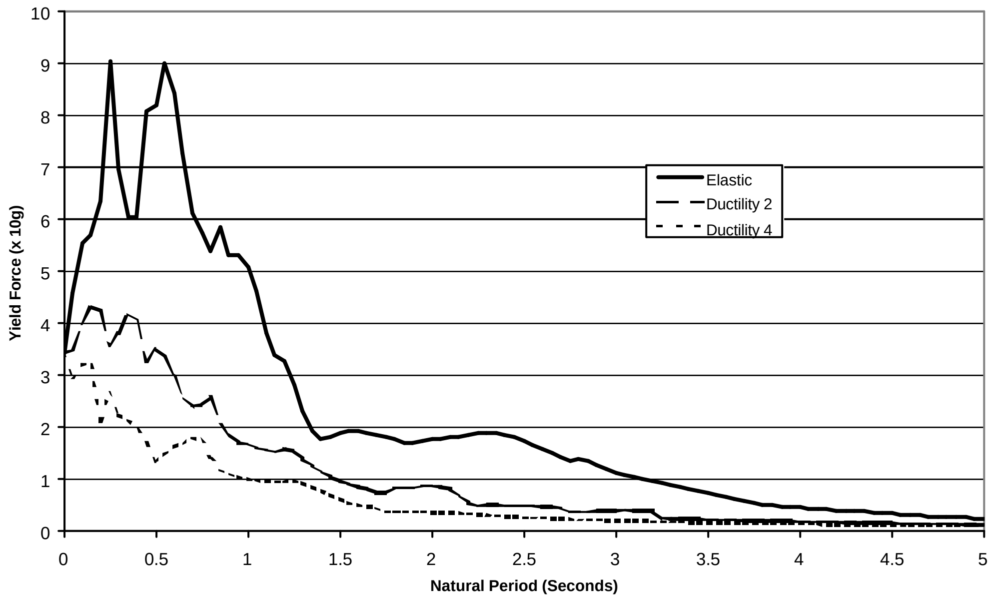

Figure 5. Spectral Yield Force – Elasto-plastic

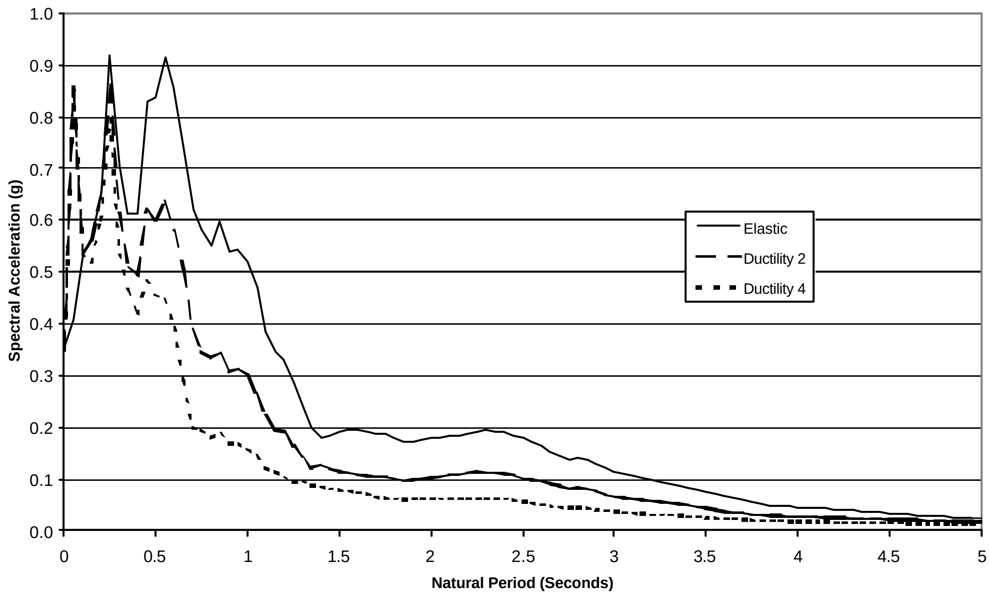

Figure 6. Spectral Acceleration – Takeda

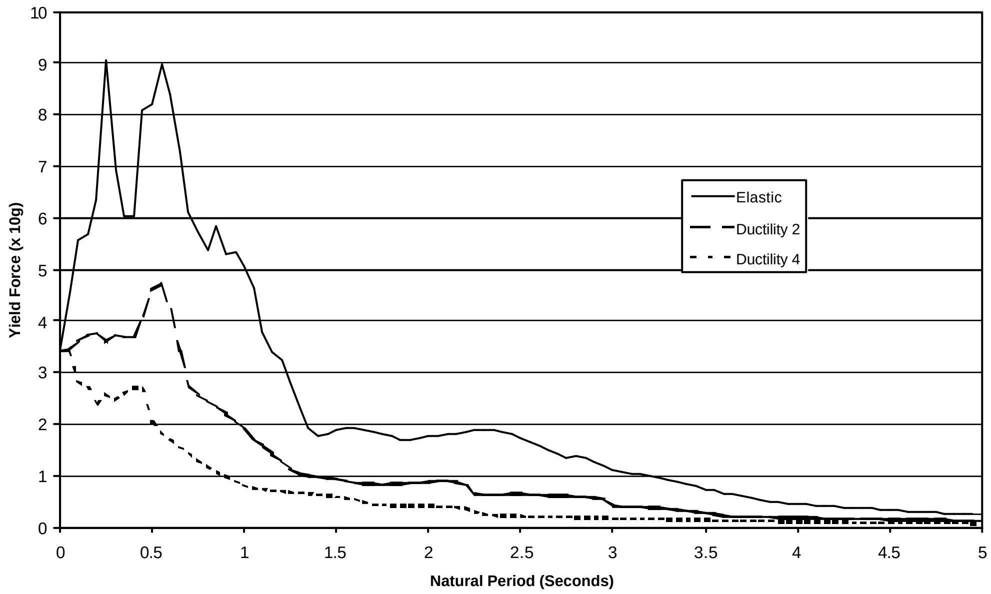

Figure 7. Spectral Yield Force – Takeda

The analysis is also able to compute the residual displacement in the structure that is associated with the inelastic displacement. Some designers are now regarding the residual displacements an important design parameter. It does represent, in a way, the state of the structure at the end of the earthquake, a large residual displacement implying that considerable effort may be required to restore the structure to a fully operational state as it is a measure of the permanent deformation in the structure. In the example shown in Figure 8 it is achieved by continuing the analysis for a further 10 seconds of free vibration following the 20 seconds of earthquake excitation. For the elastic structure this is sufficient to return the structure to its *at rest* position. For the ductile structures the inelastic displacements have left the structure with the permanent deformation shown. The effect is very dependant on the choice of hysteresis loop which the more pronounced effects being shown with loops such as the Elasto-plastic loop while the loops showing a degree of stiffness degradation do not exhibit the same sideways ratcheting effect during their inelastic excursions.

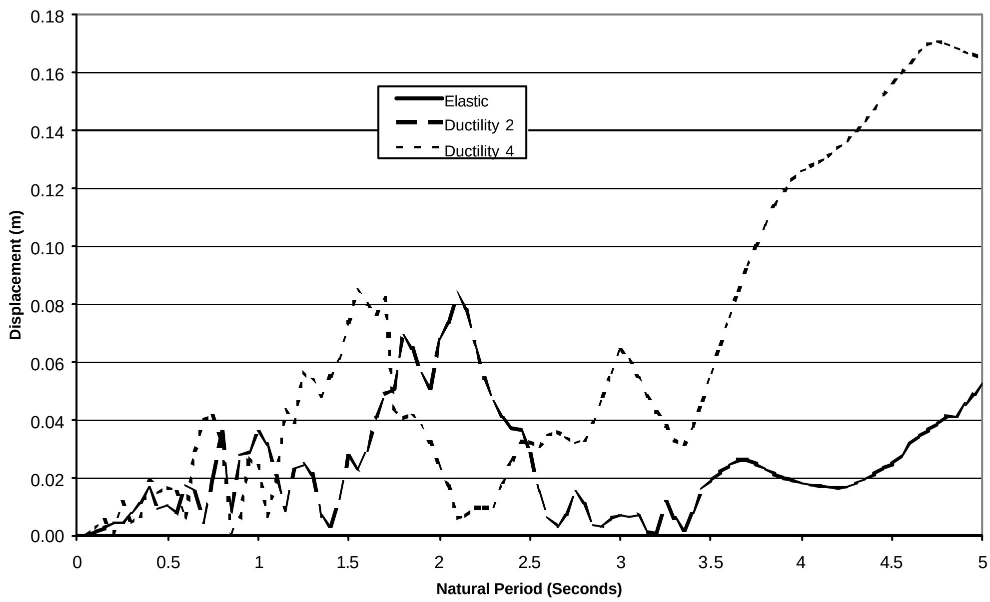

Figure 8. Residual Displacement – Elasto-plastic

Figure 9 shows the achieved ductility spectra where the targets were ductility 1, ductility 2 and ductility 4. It can be seen that the agreement is very good, for the Elasto-plastic hysteresis the agreement was to within 1% of the target for all but 1 natural period of free vibration. There were about 3 natural periods that did not achieve the target for the Modified Takeda hysteresis.

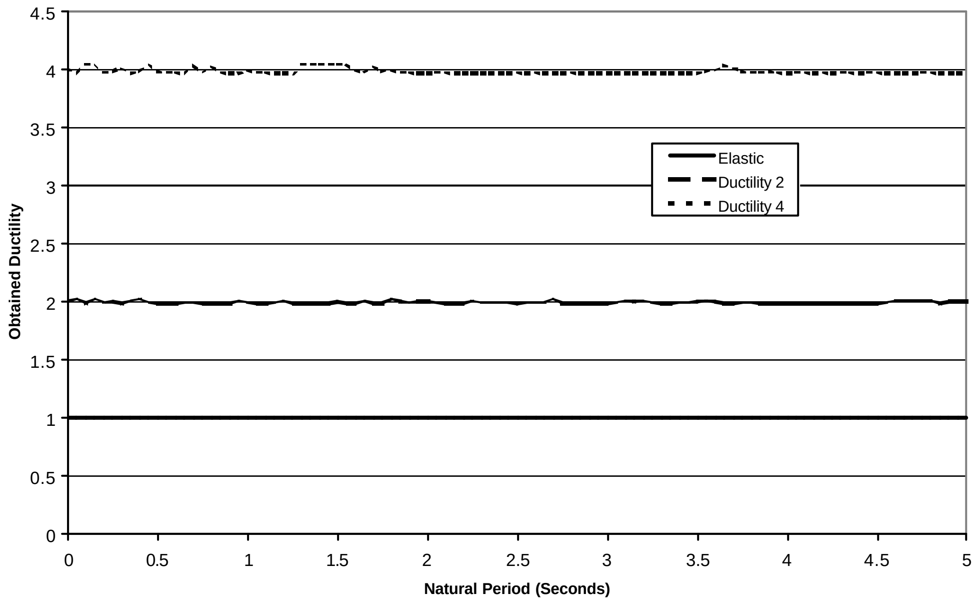

Figure 9. Ductility – Elasto-plastic

Figures 10 and 11 show the Force Reduction factors required for different ductility ratios for the various natural periods of free vibration. The equal displacement concept would imply that the reduction factors would be 2 and 4 for ductilities 2 and 4 for natural periods greater than about 0.7 seconds.

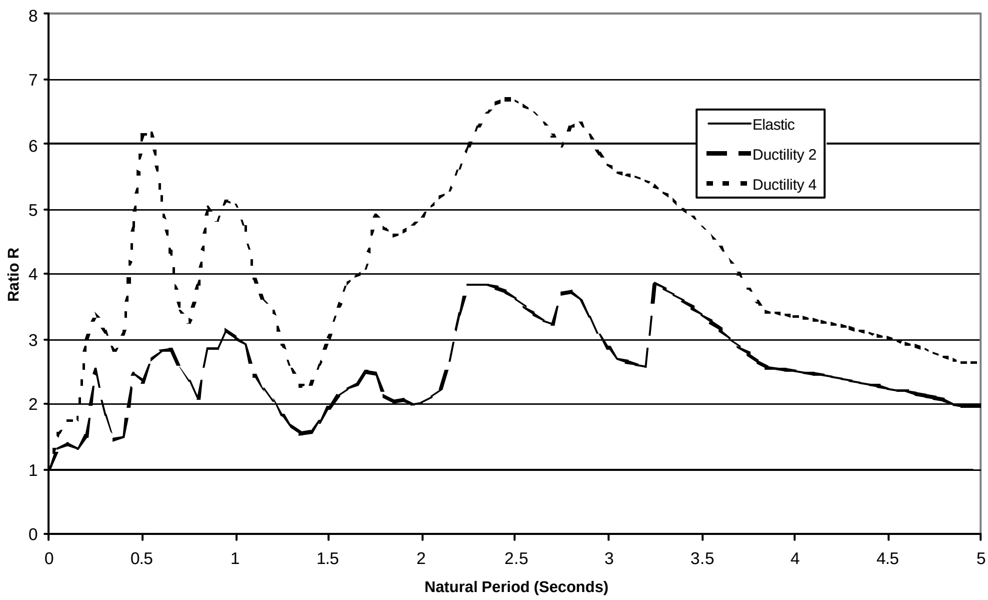

Figure 10. Force Reduction Ratio – Elasto-plastic

The Inspect program has been in use by Ph.D students in the Civil Engineering Department at the University of Canterbury for the past 2 years during which time it has undergone a fairly rigorous testing program. In future it is intended that it will be included as part of the Ruaumoko suite of programs.

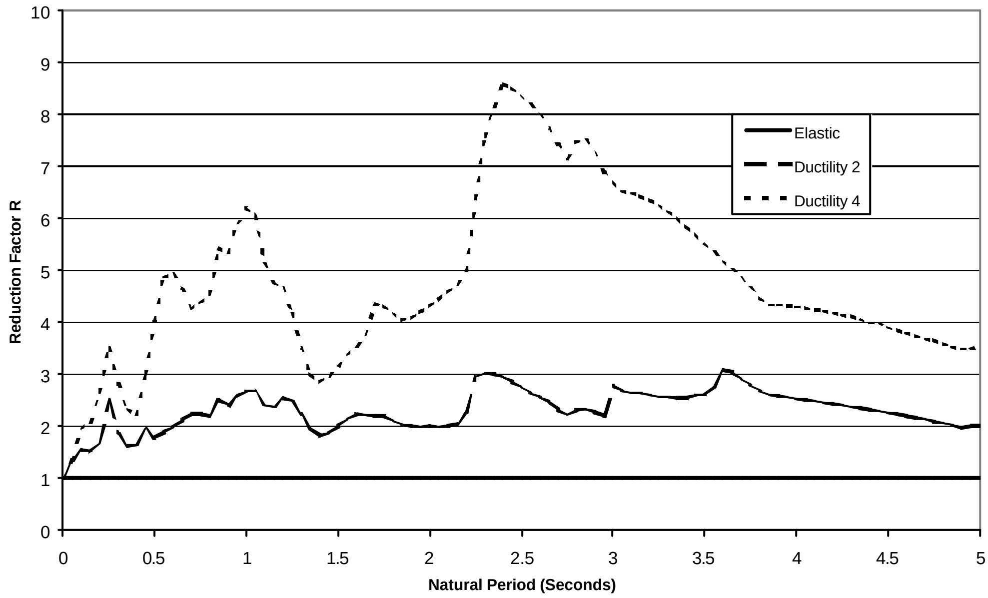

Figure 11. Force Reduction Ratio - Takeda

4 CONCLUSIONS

This paper has shown that the assumptions made in the selection of inelastic design spectra are not valid. The *equal displacement concept* of Newmark does not appear to hold even for the earthquake which was always described as one which Newmark used as part of the analyses from which he drew his *similar displacement observation*. Possibly, the real problem has been the code writers who interpreted Newmark's *similar displacement* to give them their *equal displacement*. This may have served a useful purpose in the days of limited computational capabilities but it seems appropriate, forty years on, to revisit the basis of the inelastic design force spectra. This analysis program could be one of the tools that may be helpful in developing a more rational basis for the design. It also shows that the inelastic displacement spectra are also a function of the ductility levels and of the shape of the hysteresis rule. This will have implications for those designers using the newer displacement based design.

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