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A Constitutive Model for Concrete Cylinder Confined by Steel Reinforcement and Carbon Fibre Sheet

By Yeou-Fong Li[1] Tsang-Sheng Fang[2] and Ching-Churn Chern[3]

ABSTRACT: In this paper, we modify the L-L model (Li et al., 2002) and extend the application of this model to concrete cylinders confined, respectively, by steel reinforcement only, by carbon fibre reinforced plastic (CFRP) only, and by both steel reinforcement and CFRP. Thirty-six concrete cylinders with dimensions of f 30×60 cm were tested to verify the effectiveness of the Modified L-L model. The design parameters of the concrete cylinders include the different confinement types of the steel reinforcement (such as spiral and circular hoop) and the number of layers of CFRP. The experimental test results show that different types of steel reinforcement have a great effect on the compressive strength of concrete cylinders confined by steel reinforcement, but the different types of steel reinforcement have very little effect on concrete cylinders confined by both steel reinforcement and CFRP. Compared with the stress-strain curves of confined concrete cylinders, we can conclude that the Modified L-L model can provide more effective prediction than Kawashima models.

1. INTRODUCTION

Columns are an important structural member, and their strength and ductility affect the seismic performance of the entire structure significantly. Therefore, the seismic retrofit of columns has become a very important issue in areas of high seismicity. Although retrofitting materials and methods have been around for a long time, CFRP composite materials applied to seismic retrofit projects have became a very popular method in the last decade. In this paper, the Modified L-L model was proposed and verified by the stress-strain relationships of thirty-six concrete cylinders with three different types of steel reinforcements (circular hoop, circular lap spliced hoop, and spiral) confined by CFRP.

Confined concrete constitutive models have been researched extensively since the early 20th century.

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[1] Associated Professor, Department of Civil Engineering, National Taipei University of Technology, Taiwan, R.O.C.
[2] Graduate student, Department of Civil Engineering, National Taiwan University, Taiwan, R.O.C.
[3] Professor, Department of Civil Engineering, National Taiwan University, Taiwan, R.O.C.

Kawashima et al. (1997, 1998, and 1999) proposed a series of stress-strain models for concrete confined by steel reinforcement, CFRP, and both steel reinforcement and CFRP. In the constitutive model of concrete confined by steel reinforcement proposed in 1997, the ascending branch was idealized by an n^{th}-order polynomial equation. Kawashima et al. subsequently used the regression analysis of experimental results and modified the above-mentioned 1997 model to extend the application to differently confined materials by adjusting the coefficients. The model for concrete confined with both steel reinforcement and CFRP proposed by Kawashima et al. (1999) will be called the Kawashima model in this paper. Li et al. (2002) proposed an effective constitutive model for concrete confined with CFRP. More details about this model will be discussed in the next section.

2. CONSTITUTIVE MODEL

The confined concrete constitutive model proposed by Li and Lin (2002) (L-L model) was originally developed for concrete confined by CFRP. In this paper, we modify the L-L model (Modified L-L model) and extend the application of this model to concrete cylinders confined, respectively, by steel reinforcement only, by CFRP only, and by steel reinforcement and CFRP together. In the Modified L-L model, the equation of the lateral confining stress due to steel reinforcement was adopted from the Mander model (1988), and the equation of the lateral confining stress due to CFRP was adopted from the L-L model. In this section, the ascending branch of the Modified L-L model stress-strain curve for concrete confined by both steel reinforcement and CFRP will be introduced and detailed. Fig. 1. illustrates a steel reinforcement and CFRP confining concrete cylinder.

Because the mechanism of confined concrete is similar to the mechanism of soil under tri-axial loading, the stress relationships of confined concrete can be derived from tri-axial stress relationships. According to the Mohr-Coulomb failure envelope of the soil under confined stresses (s_3) and axial stress (s_1) could be expressed as follows:

$$s_1 = 2c\tan\left(45^\circ + f/2\right) + s_3\tan^2\left(45^\circ + f/2\right) \qquad (1)$$

In Eq. (1), s_1 is the axial stress, c is the cohesion of the soil or rock, s_3 is the lateral confining stress, and f is the angle of internal friction of the material. If Eq. (1) is the tri-axial stress relationship equation for confined concrete, then s_3 is the effective confining stress and $f_l^{'} = s_3$, while s_1 is the maximum axial strength and $f_{cc}^{'} = s_1$. When $s_3 = 0$ (i.e. the unconfined situation), the plain concrete strength can be expressed as $s_1 = 2c\tan(45^\circ + f/2) = f_c^{'}$. By using the above physical-based constitutive model for confined concrete, the compressive strength of confined concrete ($f_{cc}^{'}$) can be calculated as follows:

$$f_{cc}^{'} = f_c^{'} + f_l^{'} \times \tan^2\left(45^\circ + \frac{f}{2}\right) \tag{2}$$

In Eq. (2), $f_c^{'}$ is the compressive strength of the unconfined concrete and $f_l^{'}$ is the effective lateral confining strength. The effective lateral confining strength might come from steel reinforcement, CFRP, or both steel reinforcement and CFRP together. The Modified L-L model can be calculated as follows:

$$f_{cc}^{'} = f_c^{'} + \left(f_{l1}^{'} + f_{l2}^{'}\right) \times \tan^2\left(45^\circ + \frac{f}{2}\right) \tag{3}$$

In the above equation, $f_{cc}^{'}$ is the peak compressive strength of the confined concrete, and $f_c^{'}$ is the compressive strength of the unconfined concrete. In Eq. (3), $f_{l1}^{'}$, $f_{l2}^{'}$, and f can be represented as follows:

$$f_{l1}^{'} = \frac{1}{2} k_e r_s f_{yh} \quad \text{(From Mander's model)} \tag{4}$$

$$f_{l2}^{'} = \frac{2 \times k_c \times n \times t \times E_{cf} \times e_{cf}}{D} \tag{5}$$

$$f = 36^\circ + 1^\circ \times \left(\frac{f_c^{'}}{35}\right) \leq 45^\circ \tag{6}$$

In Eq. (4), $f_{l1}^{'}$ is the effective lateral confining strength due to steel reinforcement, k_e is the confinement effectiveness coefficient, and k_e depends on the type of lateral steel reinforcement, shown as follows:

$$k_e = \frac{A_e}{A_{cc}} = \frac{\left(1 - \frac{s^{'}}{2d_s}\right)^2}{1 - r_{cc}} \quad \text{(For circular hoop)} \tag{7}$$

$$k_e = \frac{1 - \frac{s^{'}}{2d_s}}{1 - r_{cc}} \quad \text{(For circular spiral)} \tag{8}$$

Also in Eq. (4), r_s is the ratio of the volume of transverse confining steel to the volume of confined concrete core, and f_{yh} is the yield strength of the transverse reinforcement.

In Eq. (5), k_c is the coefficient of section shape (Priestley et al., 1996), n is the jacket layer of CFRP, t is the thickness of CFRP per layer, E_{cf} is the elastic modulus of CFRP, D is the diameter of the cylinder, and e_{cf} is the ultimate strain of CFRP.

In Eq. (6), f is the angle of internal friction of concrete, which is in proportion with the compressive strength of concrete, and usually varies from 36^{o} to 45^{o} (Goodman, 1989). The angle of internal friction "f" can be expressed as a function of concrete strength (Li et al., 2002) as shown in Eq. (6).

In Eq. (7) and Eq. (8), A_e is the area of the effectively confined core concrete, A_{cc} is the area of the core of column section within center lines of perimeter spiral, $s^{'}$ is the clear spacing between spiral or hoop bars, d_s is the diameter of spiral, and r_{cc} is the ratio of the area of axial steel to the area of the core of section.

When the axial stress reaches the peak compressive strength "$f_{cc}^{'}$", the CFRP breaks and its strain reaches the ultimate strain $e_{cc}^{'}$. For the compatibility condition of concrete cylinder and CFRP deformation, the ultimate strain is mainly controlled by the strength of CFRP composite materials. Therefore, $e_{cc}^{'}$ can be expressed as the following equation:

$$e_{cc}^{'} = e_c^{'}\left[1 + a \tan^2\left(45^{\circ} + \frac{f}{2}\right)\frac{f_{l2}^{'}}{f_c^{'}}\right] \tag{9}$$

In Eq. (9), $e_c^{'}$ is the strain at the compressive strength of the unconfined concrete ($f_c^{'}$), usually set at $e_c^{'}$=0.002. Parameter "a" is related to the material properties of confinement material; a equals to 2.24 (Li et al., 2002) in this paper and its corresponding CFRP material properties are listed in Table 1.

As the strain "e_c" falls between 0 to $e_{cc}^{'}$, the ascending branch stress-strain relation can be simulated by using the second-order parabolic equation. Substituting three boundary conditions $f_c = 0$ (at $e_c = 0$), $f_c = f_{cc}^{'}$ (at $e_c = e_{cc}^{'}$), and $df_c / de_c = 0$ (at $e_c = e_{cc}^{'}$) into the second-order parabolic curve, the stress-strain relation of confined concrete are shown as follows:

$$f_c = f_{cc}^{'}\left[-\left(\frac{e_c}{e_{cc}^{'}}\right)^2 + 2\left(\frac{e_c}{e_{cc}^{'}}\right)\right] \qquad (10)$$

where $f_{cc}^{'}$ and $e_{cc}^{'}$ are calculated from Eq. (2) and Eq. (9).

3. EXPERIMENTAL PROGRAM

Thirty-six concrete cylinders with a dimensions of f 30×60 cm were designed and tested to verify the effectiveness of the Modified L-L model. In this section, the design and fabrication of concrete cylinders and the related uni-axial test programs will be discussed.

3.1 Design of Concrete Cylinders

The thirty-six concrete cylinders were divided into 4 groups, and each group was applied with no, one or 2 layers of CFRP composite material. For each design parameter, three concrete cylinders were needed. Groups A, B, C, and D represent different types of steel reinforcement, such as circular hoop, two C-shaped lap-splice hoops, circular spiral, and without steel reinforcement, respectively. The illustration configuration of the concrete cylinder is shown in Fig. 2.

Table 2 introduces the nomenclature principles used for the thirty-six concrete cylinders. The first letter means the type of steel reinforcement. “A” means circular hoop, “B” means two C-shaped lap-splice hoops, “C” means circular spiral, and “D” means no steel reinforcement. The number following the first letter means the number of layers of CFRP applied on the concrete cylinder. The second number means the serial number of the concrete cylinder. For example, “A-1-2” represents the specimen confined by circular hoop with 1-layer CFRP, and its serial number is 2.

The design concrete strength was 17.2 MPa (175 kgf/cm^2), and the design concrete slump is 12 cm. The steel reinforcement used in the concrete cylinders is No. 3 steel with a yield strength of 274.7 MPa. The steel reinforcement was placed at 10 cm spacings, and the concrete cover thickness was 2.5 cm. For the circular hoop and two C-shaped lap-splice hoops, the lap length of steel reinforcement was 11.5 cm. The longitudinal rebars are used to hold the horizontal steel reinforcements in position. A total of thirty-six plastic pipes, with an internal diameter of 30 cm and a height of 60 cm, were used as the formwork for the concrete cylinders. Premixed concrete is used in the experiment.

3.2 Instrumentation of the Compression Test

This test program was undertaken using a 4900 kN universal-testing machine, which is load controlled, at the structural laboratory of the National Taipei University of Technology. The experimental equipment includes load cells, a linear voltage displacement transformer, an analog/digital converter with a signal amplifier, and a personal computer.

3.3 Experimental Observations

The failure mode shape of the concrete cylinders confined by CFRP is approximately conical. The failure mechanism of cylinders confined by steel reinforcement and 2-layer CFRP is described as follows. The concrete between CFRP and steel reinforcement spalled, and the CFRP was broken in the center of the cylinder. This shows that the concrete inside the steel reinforcement was still confined by steel reinforcement when the CFRP was broken.

The experimental observations of the rest of the specimens confined with CFRP and with or without steel reinforcement are described as follows. When the stress of the actuator reached the peak strength of the confined concrete, breaking sounds of the CFRP was heard continuously, the CFRP subsequently broke in the middle of the cylinder and the concrete was crushed. The breaking position of the CFRP was not necessarily at the overlaying position.

The relationships of the average peak compressive strength of confined concrete and the number of layers of CFRP are drawn in Fig. 3. As seen from Fig. 3, we can obtain the following conclusions:

1. When the concrete cylinder is confined by steel reinforcement (Groups A, B, and C), its compressive strength is higher than those without steel reinforcement (Group D) and the compressive strength is highly dependent on the types of steel reinforcement. The compressive strength due to spiral is larger than the 2-C-shaped lap-splice and circular hoop reinforcement.

2. When concrete cylinders are confined by steel reinforcement and CFRP together (Groups A, B, and C), their compressive strengths are very close to each other. This indicates that the compressive strength of concrete cylinders confined by CFRP is irrelevant to the types of steel reinforcement. The reason is that when CFRP reaches its ultimate strain (usually 0.015), the strains of steel reinforcement of Groups A, B, and C are still within yielding and ultimate strains. The stresses of steel reinforcement of Groups A, B, and C are all at the yielding stresses.

4. DISCUSSION ON THE THEORETICAL/EXPERIMENTAL RESULTS

In this section, the theoretical (calculated from different models) and experimental results of the peak strength of confined concrete and the stress-strain curve are compared.

4.1 The Comparison of the Peak Strength

- The Modified L-L model

The peak strengths of Group D specimens are compared with the peak strengths calculated by the Modified L-L model, and the error analysis of the peak strengths are listed in Table 3. Since the unconfined concrete strength is the peak strength of the specimens, the average error of the D-0 specimens is zero. As for D-1 and D-2 specimens, the errors are 1.5% and 4.4% respectively. The average error of the Modified L-L model is 2.95%. It can therefore be said that the Modified L-L model can predict the peak strengths of concrete confined by CFRP very well.

- The Modified L-L and the Kawashima models

In the last century, most of the confined constitutive models were proposed specifically for concrete columns confined by either steel reinforcement or CFRP. Not until 1999 did Kawashima et al. (1999) propose a constitutive model for concrete confined by both steel reinforcement and CFRP*.

The peak strengths of A-1, A-2, B-1, B-2, C-1, and C-2 specimens are compared with the peak strengths calculated by the Modified L-L model and the Kawashima model, and the error analysis of the peak strengths are listed in Table 4. As seen in Table 4, the average absolute errors of the Modified L-L model and the Kawashima model are 2.8% and 10.1% respectively. We can conclude that the Modified L-L model is more accurate than the Kawashima model in predicting the peak strength.

4.2 The Comparison of the Stress-Strain Curve

- The Modified L-L model

The stress-strain curves of the experiment results of D-0, D-1, and D-2 specimens are compared with the stress-strain curves calculated by the Modified L-L model, shown in Fig. 4, Fig. 5, and Fig. 6, respectively. As seen in the above figures, we can conclude that the Modified L-L model can simulate the experimental results very well.

- The Modified L-L and the Kawashima models

The stress-strain curves of the experiment results of the A-1, B-1 and C-1 specimens, and the Modified L-L model and the Kawashima model are shown in Fig. 7~Fig. 9 respectively. Similarly, the stress-strain curves of the experimental results of the A-2, B-2 and C-2 specimens, and the Modified L-L model and the Kawashima model are shown in Fig. 10 to Fig. 12 respectively. As seen in Fig. 7 to Fig. 12, the stress-strain curves of the Modified L-L model can fit the experimental stress-strain curves very well. As for the Kawashima model, the prediction of stress is acceptable, but the prediction of strain is greatly overestimated.

From the observations and discussions on the experimental results of stress-strain curves among the three models, we can conclude that the Modified L-L model is more effective than the Kawashima model in the prediction of the peak stress, strain at the peak stress, and the stress-strain curves.

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* In the papers by Kawashima et al. (1997, 1998, 1999), the diameter to height ratio of the circular cylinder (column) is 1:3. But, the diameter to height ratio of the circular concrete cylinder used in this paper is 1:2. Therefore, we justify the peak strength proposed by the Kawashima model by raising it 8%.

5. CONCLUSIONS

From the observation of the experimental results and by comparing the experimental results to the constitutive models, we can arrive at the following conclusions:

1. The peak stress formula of the Modified L-L model is a theoretical equation, and it is derived from the Mohr-Coulomb failure envelope theory, which conforms to the fundamental theory of plasticity. The formula can be used in different levels of confining stress.

2. When concrete cylinders are confined by different types of steel reinforcement, the compressive strength is highly dependent on the types of steel reinforcement.

3. When concrete cylinders are confined by CFRP and different types of steel reinforcement, their compressive strengths are very close to each other. This indicates that the compressive strength of concrete cylinder confined by CFRP is irrelevant to the types of steel reinforcement.

4. Compared to the test results of the 36 concrete cylinders, the average absolute errors of the peak strength estimation of the Modified L-L model are less than 3 %, with the exclusion of the cylinders confined by steel reinforcement only (such as the A-0, and C-0 series). As for Kawashima's models, its average absolute errors are about 20 %.

5. Comparing the stress-strain curves of the experimental results with those of the Modified L-L and Kawashima's models, we can conclude that the Modified L-L model is more effective than Kawashima's models.

6. The Modified L-L model can be applied to concrete cylinders confined by steel reinforcement only, by CFRP only, and by both steel reinforcement and CFRP.

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Table 1. Material properties of CFRP

Material Specification	FAW 200 (g/m^2)
Young's Modulus, E_{cf}	230,535MPa (2.35 ×10^6 kgf/cm^2)
Tensile Strength	4120.2MPa (42000 kgf/cm^2)
Thickness	0.011 cm/layer
Ultimate Strain	0.018

Table 2. The naming of concrete cylinders

Group	Specimens	CFRP layers	Type of steel reinforcement
A	A-0	0	Circular hoop
	A-1	1	
	A-2	2	
B	B-0	0	Lap-splice
	B-1	1	
	B-2	2	
C	C-0	0	Spiral
	C-1	1	
	C-2	2	
D	D-0	0	Nil
	D-1	1	
	D-2	2	

Table 3. Error analyses of the peak strengths of the Modified L-L model

Specimens	Experiment MPa (kgf/cm^2)	Modified L-L MPa (kgf/cm^2)	Error (%)
D-0	16.68 (170.03)	16.68 (170.03)	0
D-1	25.52 (260.11)	25.90 (263.97)	1.5
D-2	33.64 (342.88)	35.11 (357.94)	4.4
Average absolute error (%)=		2.95	

Table 4. Error analyses of the peak strengths of the Modified L-L and Kawashima models

Specimen	Experiment MPa (kgf/cm^2)	Modified L-L MPa (kgf/cm^2)	Error (%)	Kawashima MPa (kgf/cm^2)	Error (%)
A-1	32.27 (328.94)	30.82 (314.15)	-4.5	35.35 (360.35)	9.5
A-2	39.89 (406.58)	40.04 (408.12)	0.4	44.70 (455.63)	12.1
B-1	31.85 (324.62)	30.82 (314.15)	-3.2	35.35 (360.35)	11.0
B-2	40.90 (416.94)	40.04 (408.12)	-2.1	44.70 (455.63)	9.3
C-1	33.13 (337.74)	31.86 (324.73)	-3.8	35.35 (360.35)	6.7
C-2	39.87 (406.45)	41.08 (418.71)	3.0	44.70 (455.63)	12.1
Average absolute error (%)=		2.8		10.1	

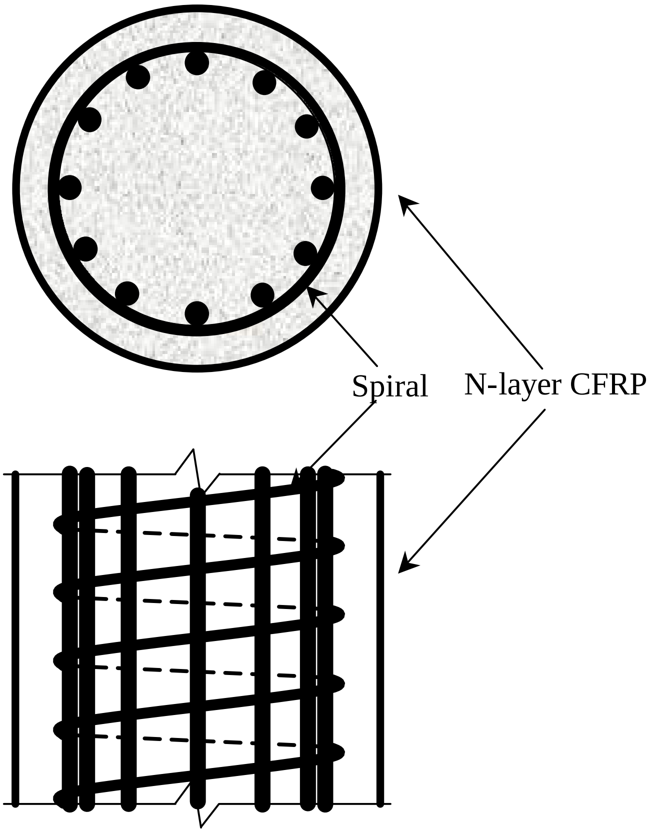

Fig. 1. The illustration of steel reinforcement and CFRP confining concrete cylinder

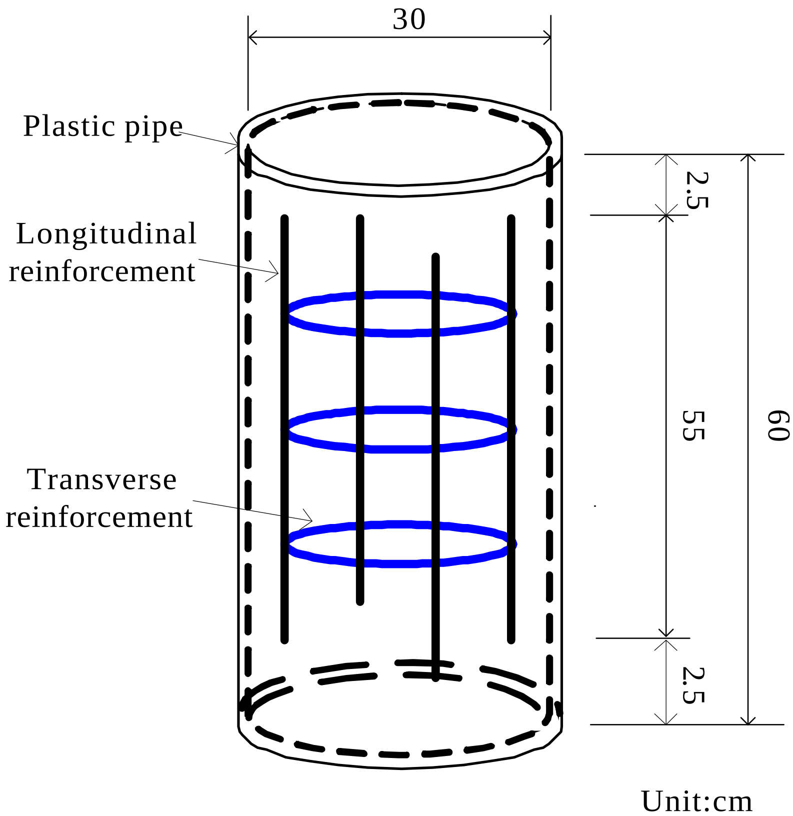

Fig. 2. The illustration configurations of the concrete cylinder

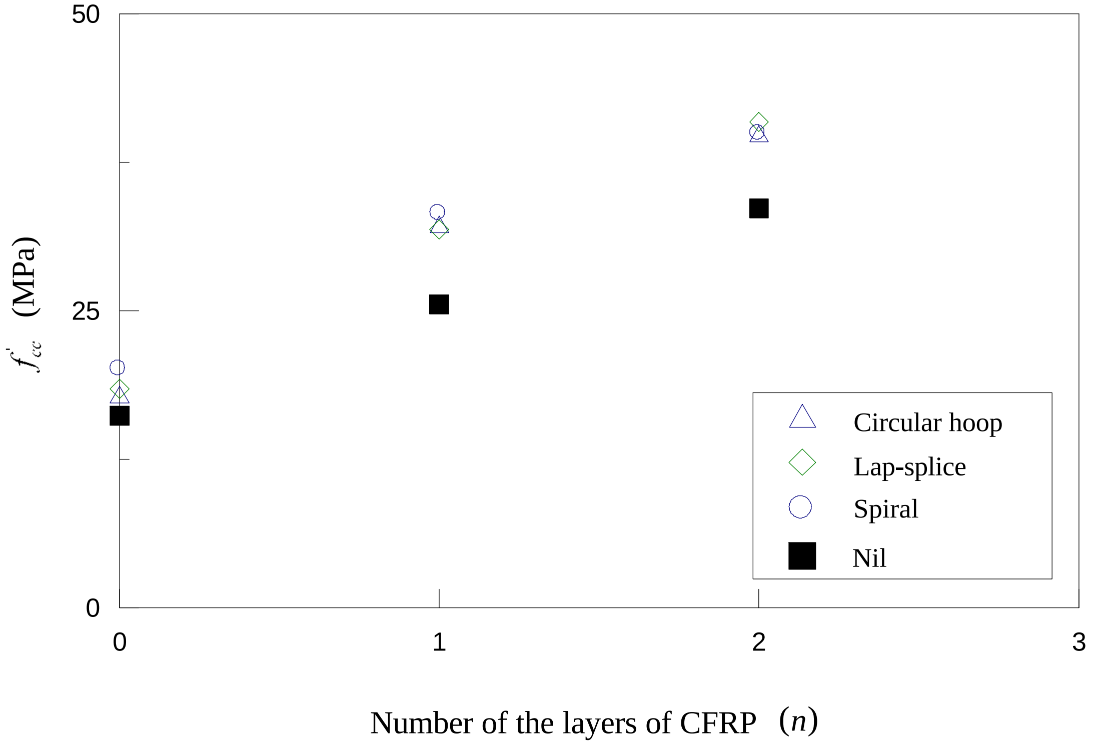

Fig. 3. The relationships of the average peak compressive strengths of confined concrete and numbers of layers of CFRP

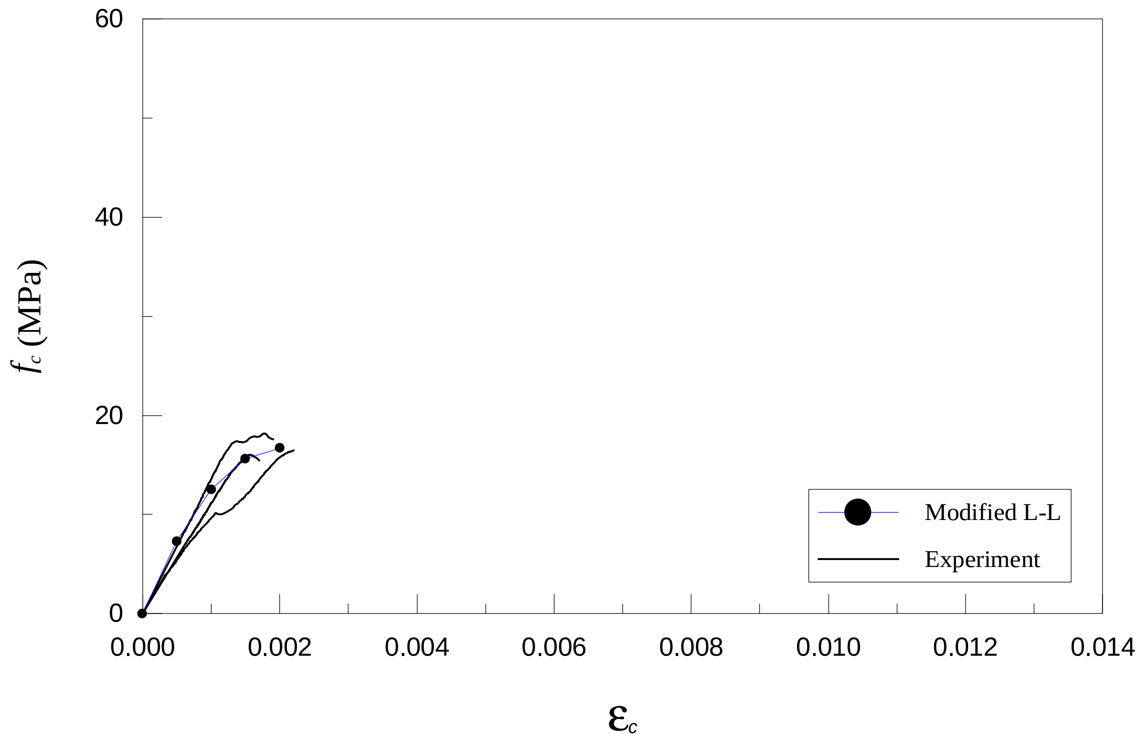

Fig. 4. The stress-strain curves of D-0 specimens and the M-L-L model

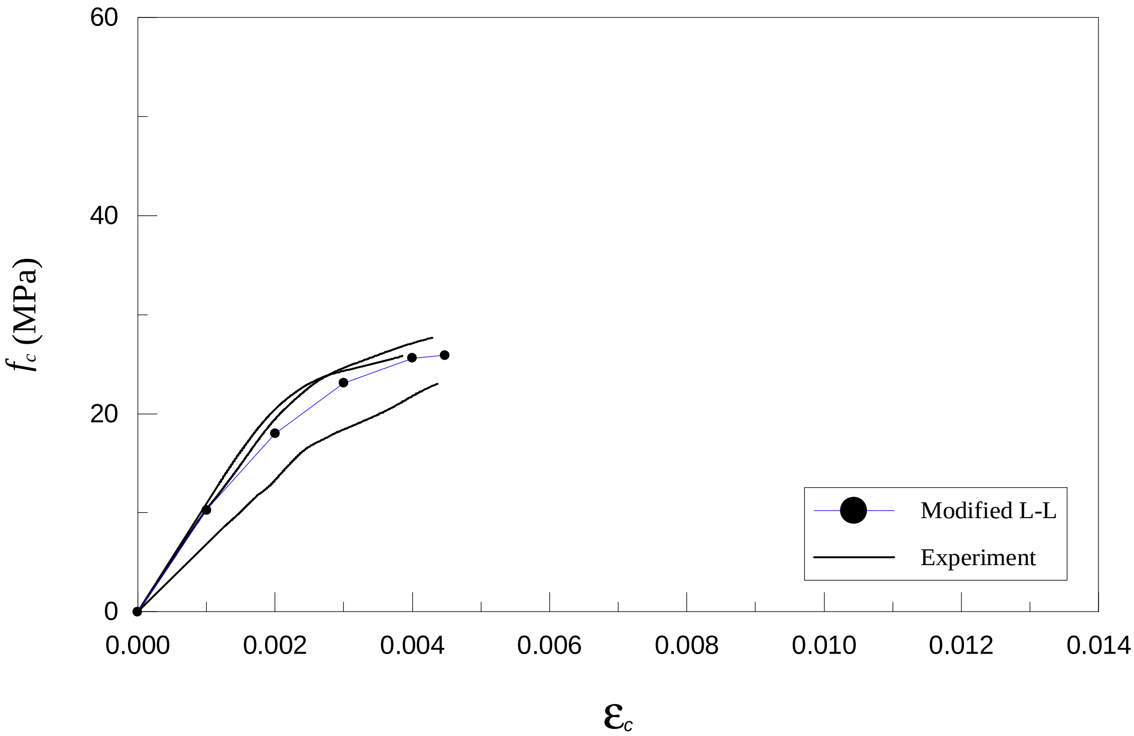

Fig. 5. The stress-strain curves of D-1 specimens and the M-L-L model

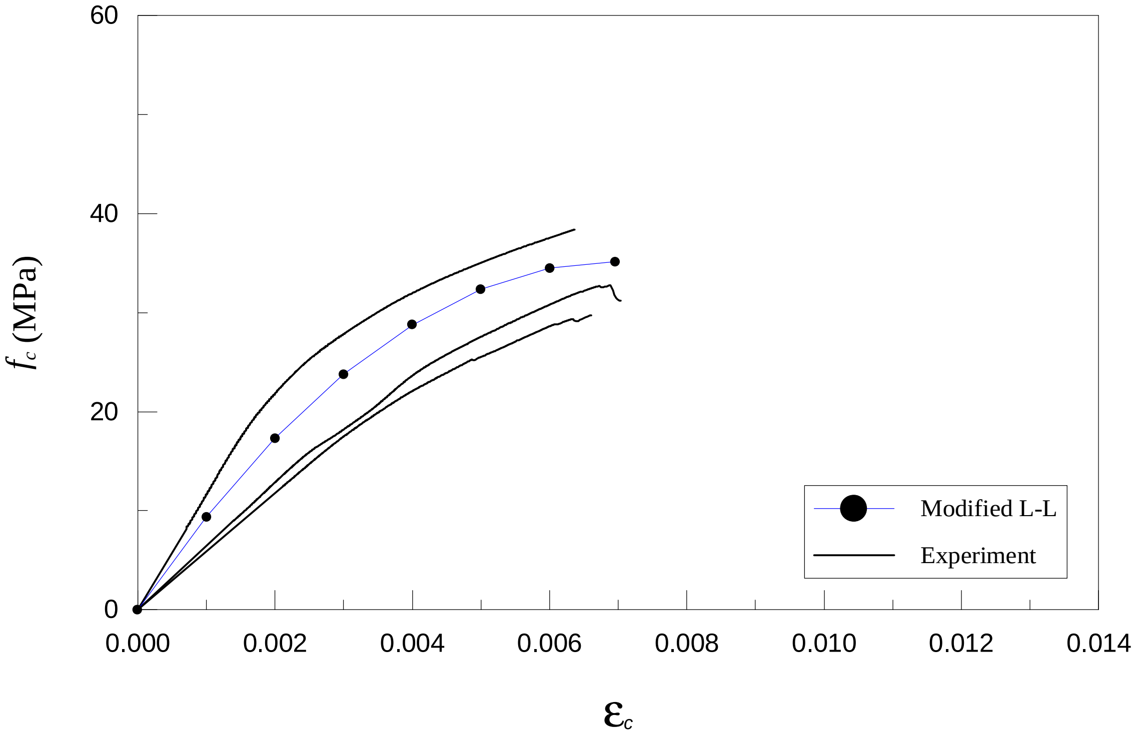

Fig. 6. The stress-strain curves of D-2 specimens and the M-L-L model

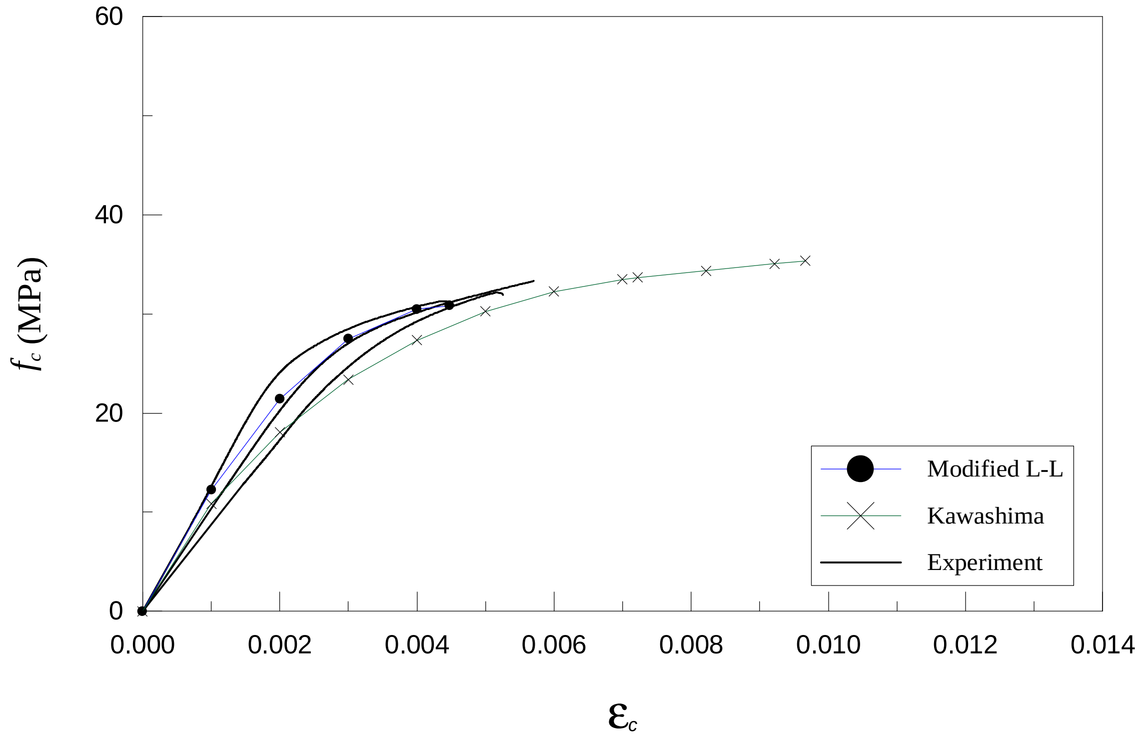

Fig. 7. The stress-strain curves of A-1 specimens, the Kawashima, and the M-L-L models

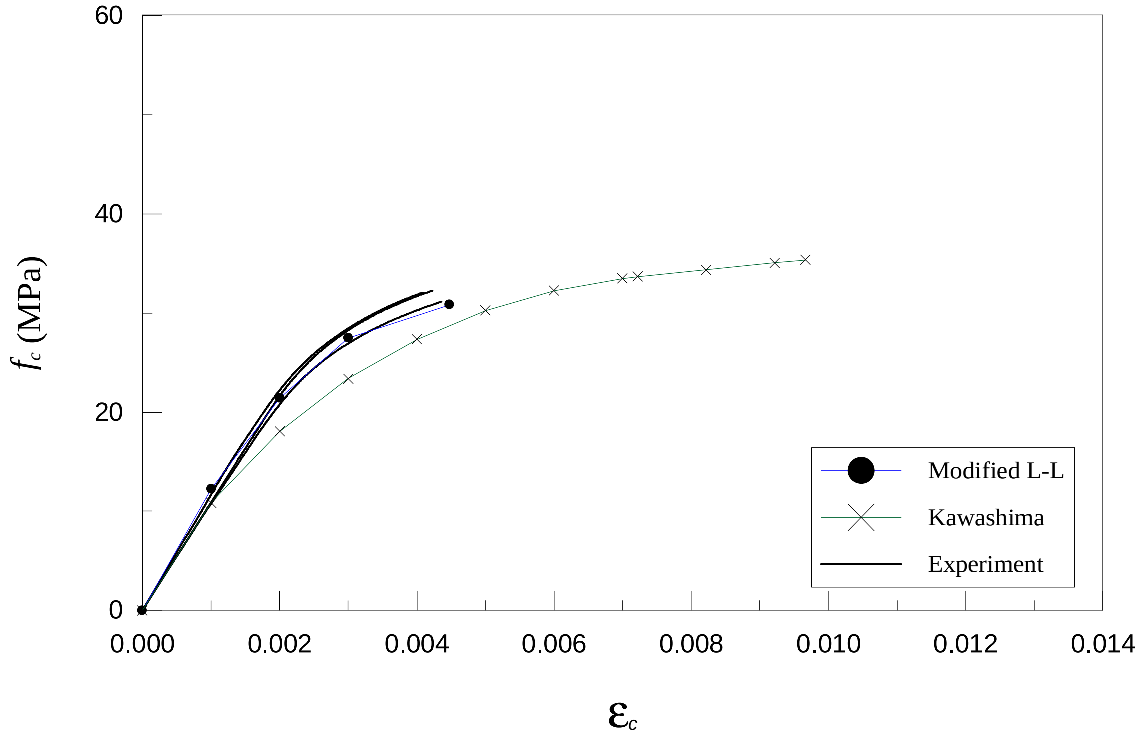

Fig. 8. The stress-strain curves of B-1 specimens, the Kawashima, and the M-L-L models

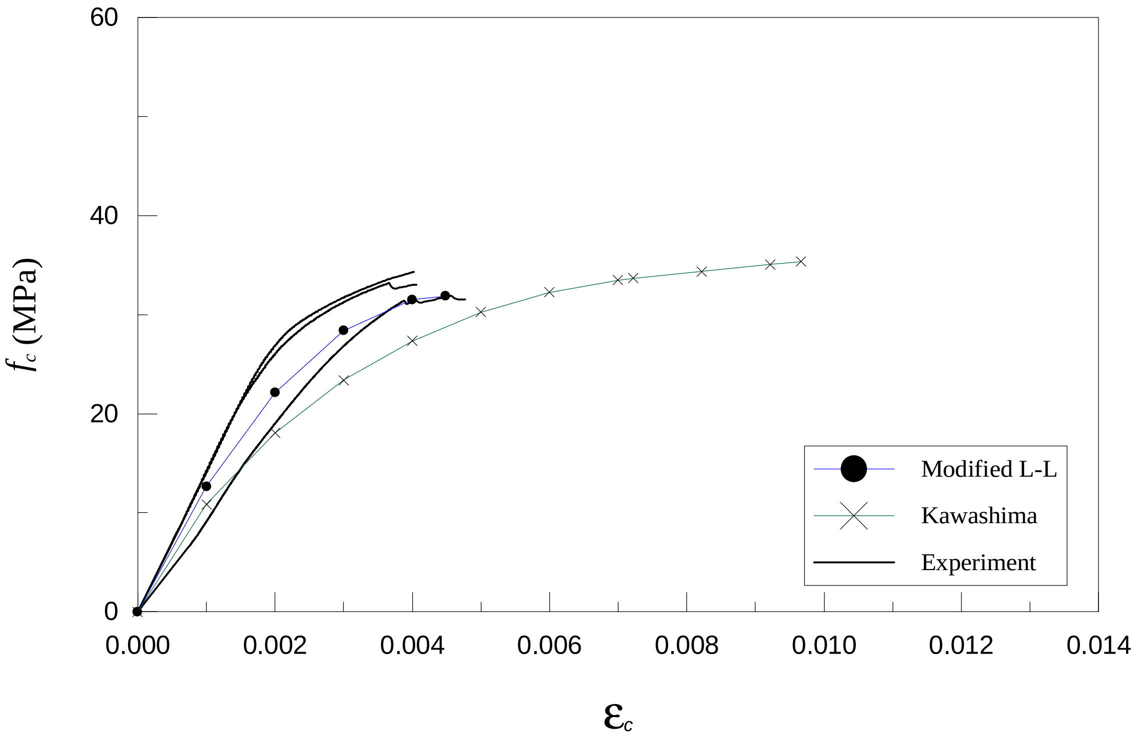

Fig. 9. The stress-strain curves of C-1 specimens, the Kawashima, and the M-L-L models

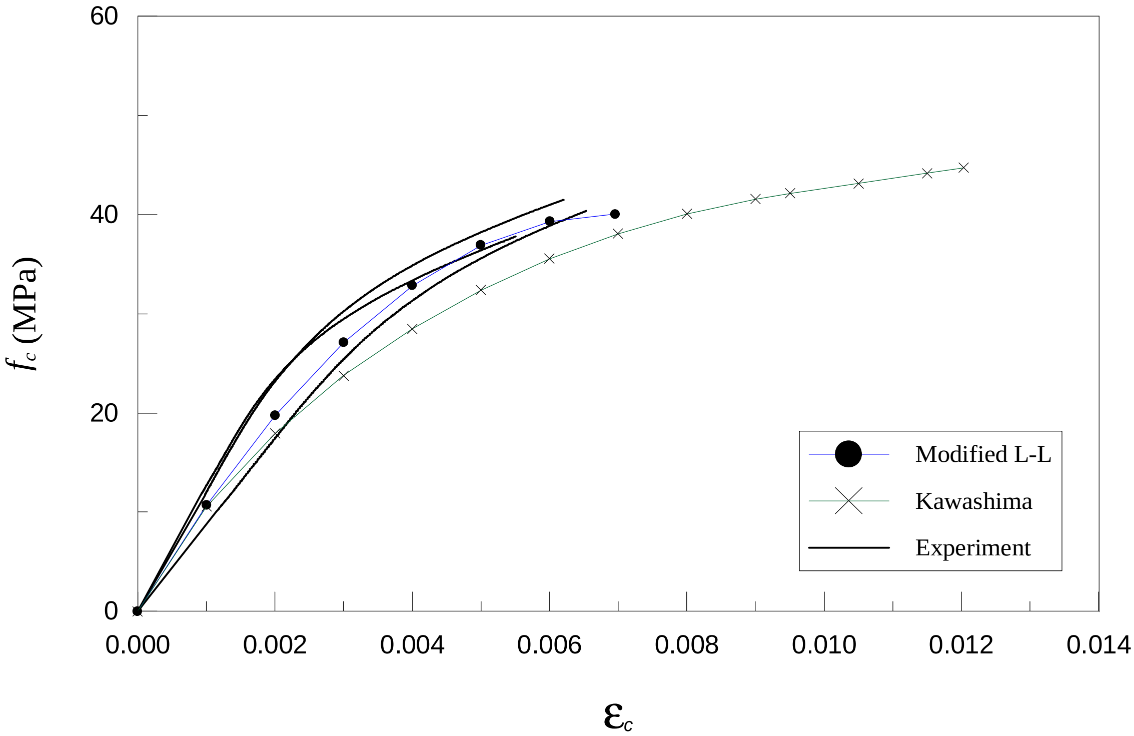

Fig. 10. The stress-strain curves of A-2 specimens, the Kawashima, and the M-L-L models

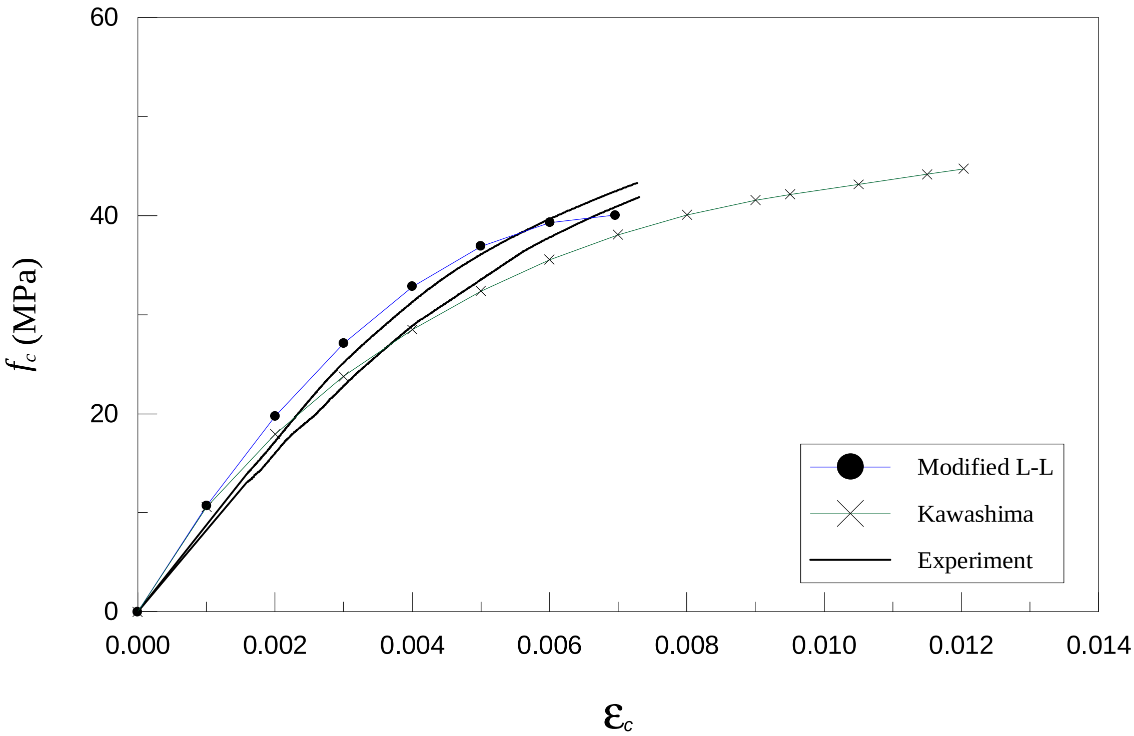

Fig. 11. The stress-strain curves of B-2 specimens, the Kawashima, and the M-L-L models

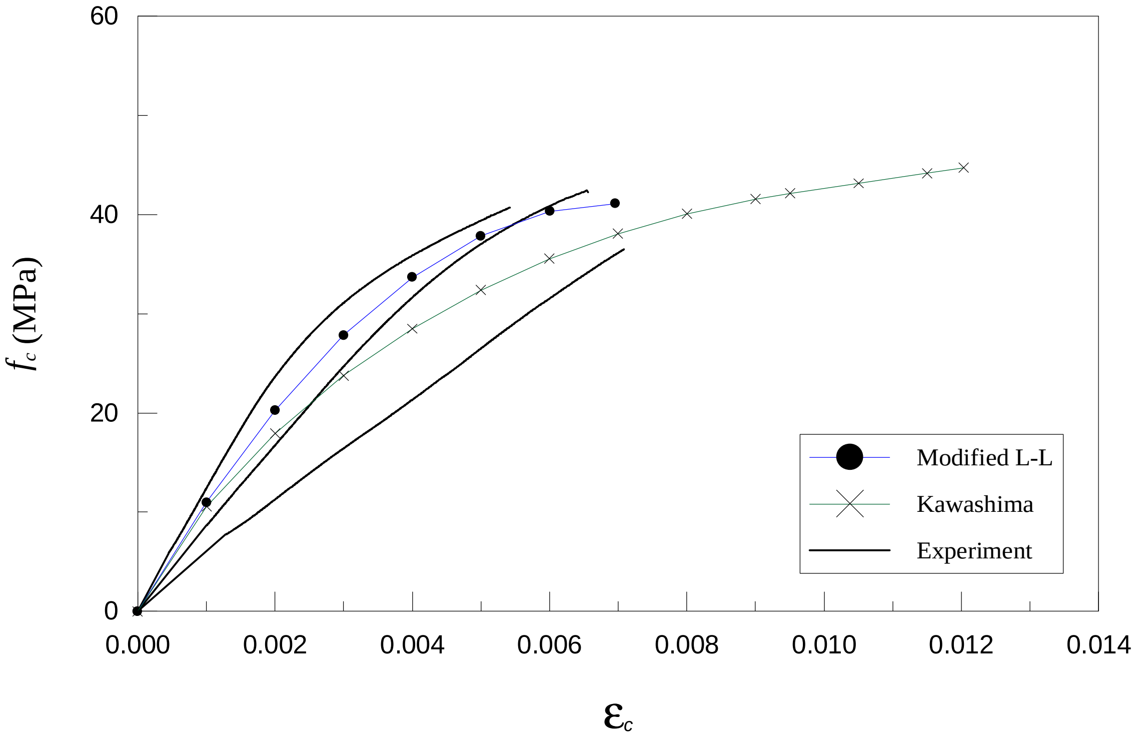

Fig. 12. The stress-strain curves of C-2 specimens, the Kawashima, and the M-L-L models