



## Stiffness-strain relationship of Singapore residual soils

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**ABSTRACT:** Residual soils are formed from the in-situ weathering of rock formations. In Singapore, residual soils occupy about two-thirds of the total land area. Cyclic triaxial tests were performed on three saturated residual soil samples from the Singapore Jurong formation. The soil samples were tested at strain levels from  $\pm 0.005\%$  to  $\pm 1\%$  for 40 loading cycles at a frequency of 0.5Hz and then sheared to failure. From the stress-strain loading curves, shear modulus and damping ratio at each strain level can be determined. The shear moduli increase slightly and the damping ratios decrease slightly with increasing number of loading cycles. The shear moduli and damping ratios reported in the literature are mainly for saturated sands and clays. The trends of the shear modulus and damping ratio with shear strain amplitude for the Jurong formation residual soils are similar to those reported in the literature. However, the stiffness-strain relationships suggested for saturated sands and clays in the literature were found to overestimate the shear moduli and damping ratios of the residual soils. A general stiffness-strain relationship was found to provide a reasonable fit to the shear modulus curve of the residual soils.

### 1 INTRODUCTION

Current understanding of “dynamic soil properties” and “static soil properties” is that they form a continuum on the strain axis (Woods 1994). The importance of recognising that the stress-strain behaviour is non-linear and that the appropriate stiffness is used for certain problems has been highlighted by Jardine et al. (1986), Burland (1989) and Tatsuoka (2000). To date, a number of laboratory tests are available for measuring the stiffness of soils at different strain ranges. These include the resonant column tests, triaxial tests, simple shear tests and torsional shear tests. Usually the strain range does not straddle from the very small strain to the very large strain. Two or more test methods are normally combined to provide soil stiffness over the whole strain range. The triaxial test is one of the most commonly used tests for characterising soil behaviour. With a servo-controlled loading system, the triaxial test can be used to perform cyclic triaxial tests at various strain levels. The procedure for the cyclic triaxial test has been standardised in ASTM D3999-91.

The shear moduli and damping ratios for saturated sands and clays over a wide strain range are plentiful in the literature. However, shear moduli and damping ratios for residual soils are scarce. In this paper, the shear moduli and damping ratios for four residual soils from the Singapore Jurong formation are investigated using the cyclic triaxial test. The soil samples were tested at strain levels from  $\pm 0.001\%$  to  $\pm 1\%$  for 40 cycles each and then sheared to failure. The trend of the shear moduli and damping ratios for the residual soils are compared with those established for sands and clays. The relationships suggested for the stiffness-strain relationships for sands and clays are also examined for the residual soils.

### 2 SOIL SAMPLES

The soil samples were residual soils derived from the Jurong formation. The Jurong formation is a

severely folded and faulted sedimentary deposit consisting of mudstones, siltstone, sandstone, conglomerate and limestone (PWD 1976). The sedimentary deposit has been subjected to intensive weathering leading to a thick layer of residual soils above the rock formation. The thickness of the residual soil varies from a few metres to several tens of metres. The residual soil is usually present in the form of inter-bedded layers of predominantly medium plastic clayey silt, sandy clay and clayey to sandy materials. The interest in the study was to study the seismic response of the soils at shallow depths. Several soil samples were collected at shallow depths of 1.5 to 1.8m from various locations of a site. In this paper, the results of triaxial tests on three of these samples are reported. The basic soil properties of the three residual soil samples, JF1, JF2 and JF3, are shown in Table 1.

**Table 1. Properties of residual soil samples.**

Soil Sample	JF1	JF2	JF3
Void ratio	0.71	0.48	0.44
Specific gravity	2.72	2.74	2.72
Bulk density (Mg/m <sup>3</sup> )	2.01	2.05	2.06
Natural water content (%)	26.6	10.4	8.7
LL (%)	48	44	35
PL (%)	28	20	19
PI (%)	20	24	16
Sand (%)	25	36	33
Silt and clay (%)	75	64	67
USCS	ML	CL	CL

### 3 TRIAXIAL TESTS

Soil specimens, 72 mm in diameter with a height to diameter ratio of 2, were trimmed from the 100 mm diameter tube samples. The soil specimens were then saturated in the triaxial cell under high confining pressure and back pressure in the region of 400 kPa. All the soil specimens have a B value greater than 0.95 and thus they were assumed to be fully saturated. The soil specimens were then isotropically consolidated to its estimated initial effective vertical stress. Each soil specimen was then subjected to a cyclic axial strain ranging from  $\pm 0.005\%$  to  $\pm 1.0\%$  under undrained condition in accordance to ASTM D3999-91 at a frequency of 0.5Hz. The axial strain was controlled via an internal submersible LVDT and local radial strain was measured using a pair of Hall-effects transducers. The shear modulus G, and damping ratio, D, at 1 to 5, 10, 20, 40 cycles were computed as shown in Figure 1. Poisson's ratios computed from the axial and radial strain measurements were approximately 0.5.

Subsequent to the cyclic triaxial tests, the soil specimens were sheared to failure in the undrained condition at a strain rate of 1.44 mm/h in accordance to ASTM D4765-95. The tangent moduli at axial strains from 0.5% to 10% were calculated from the load-deformation plots.

### 4 RESULTS AND DISCUSSIONS

The stiffness of soil is affected by a number of factors. Amongst these factors are shear strain amplitude, void ratio, confining pressure, number of loading cycles, frequency, plasticity index, overconsolidation ratio, degree of saturation and stress history (Hardin & Black 1968, Hardin & Drnevich 1972a, Seed et al. 1986, Kramer 1996). In this paper, the effects of shear strain amplitude and number of loading cycles were investigated. The effects of void ratio, confining pressure, plasticity index and degree of saturation were examined in the light of generalisations by other

researchers.

#### 4.1 Effects of Shear Strain Amplitude and Loading Cycles

The effects of shear strain amplitude and loading cycles on the shear modulus and damping ratio of the three soil specimens are shown in Figure 2. For the damping ratio, the hysteresis loops were almost non-existent at very low strain levels of loading (i.e. for  $\gamma < 0.03\%$ ) and therefore, these data are not available in Figure 2. As shear strain amplitude increases, shear modulus decreases whereas damping ratio increases. This is the normal trend expected for shear modulus and damping ratio for both cohesionless and cohesive soils (e.g. Hardin and Drnevich 1972a). The shear modulus and damping ratio are practically independent of the number of loading cycles, except in the first few loading cycles. For the shear modulus, there is a slight decrease from the first loading cycle to the fifth loading cycle. For the damping ratio, there is a slight increase from the first loading cycle to the fifth loading cycle. Hardin & Drnevich (1972a) found that the shear modulus increases slightly with the number of loading cycles for sands and decreases with the number of loading cycles for cohesive soils. The damping ratios for cohesive and cohesionless soils were found to decrease with the number of loading cycles and some evidence indicates that they begin to increase when the number of loading cycles exceeds 50000 (Hardin & Drnevich 1972a). However, the shear modulus and damping ratio trends for JF1, JF2 and JF3 were found to be in general agreement with the observations on Piedmont residual soils reported by Borden et al. (1996).

#### 4.2 Comparison with Piedmont residual soils

The average shear modulus and damping ratio for JF1, JF2 and JF3 are shown in Figure 3. The relationship of shear modulus with shear strain is usually expressed as a normalised shear modulus in terms of a maximum shear modulus, i.e.  $G/G_{\max}$ . Unfortunately,  $G_{\max}$  was not measured for JF1, JF2 and JF3.  $G_{\max}$  is a function of void ratio, confining pressure, overconsolidation ratio and plasticity index. For angular-grained cohesionless soils, Hardin and Black (1968) suggested the following equation for  $G_{\max}$ :

$$G_{\max} = \frac{3230(2.97 - e)^2}{1 + e} \bar{\sigma}_0^{1/2} \quad (1)$$

where  $e$  is void ratio and  $\bar{\sigma}_0^{1/2}$  is the effective octahedral stress. For cohesive soils, Hardin & Drnevich (1972b) suggested the following equation for  $G_{\max}$ :

$$G_{\max} = \frac{3230(2.97 - e)^2}{1 + e} (\text{OCR})^K \bar{\sigma}_0^{1/2} \quad (2)$$

where OCR is the overconsolidation ratio and  $K$  is a function of plasticity index. Both Equations 1 and 2 are identical for  $\text{OCR}=1$ .

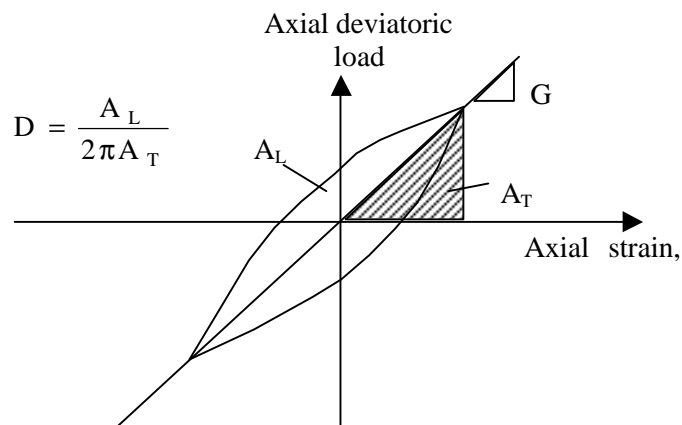
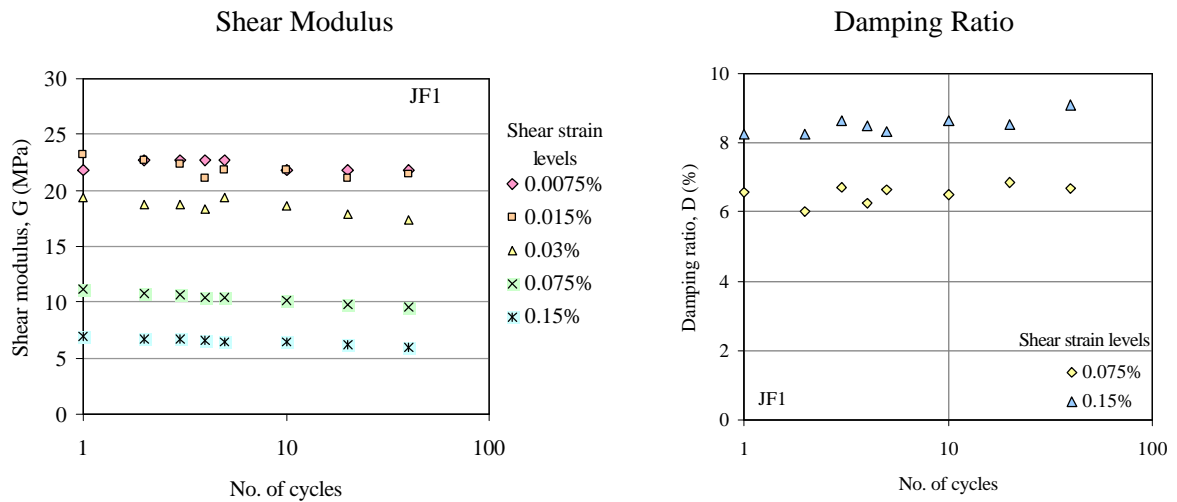
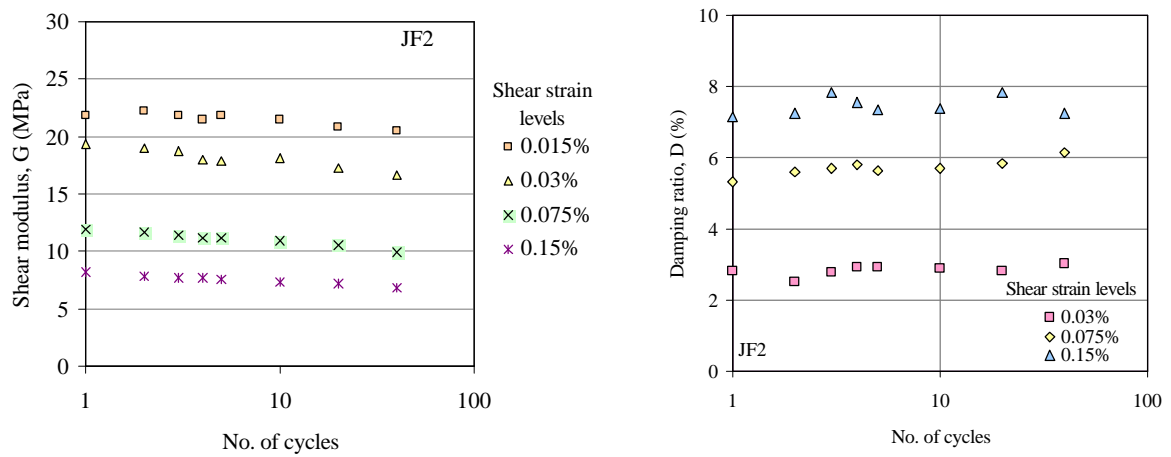


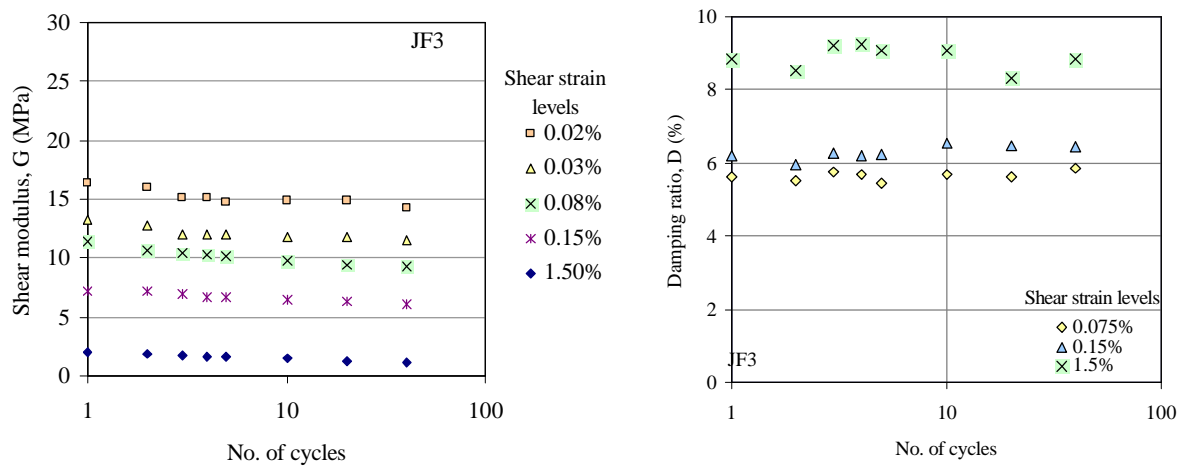
Figure 1. Schematic of a hysteresis loop of one loading cycle in the cyclic triaxial test.



(a) Soil specimen JF1



(b) Soil specimen JF2



(c) Soil Specimen JF3

Figure 2. Shear modulus and damping ratio with number of loading cycles for JF1, JF2 and JF3.

Borden et al. (1996) reported the dynamic properties for 32 specimens of Piedmont residual soils. They had classified the 32 specimens into MH, ML, SM-ML and SM according to the Unified Soil Classification System (USCS). If  $G_{max}$  of all the Piedmont residual soil specimens classified as ML are

computed using Equation 1 and plotted against the measured  $G_{max}$ , the reasonableness of Equation 1 can be evaluated. The plot of the estimated  $G_{max}$  and the measured  $G_{max}$  is shown in Figure 4. Most of the data points are below the equality line. The underestimation in  $G_{max}$  using Equation 1 can be as much as 50% as indicated by the dashed line. The differences can be reconciled by the fact that the Piedmont residual soils are unsaturated with degree of saturation varying from 42% to 70%. An unsaturated soil is expected to have a higher stiffness. Using Equation 1,  $G_{max}$  for JF1, JF2 and JF3 were computed as 39.6 MPa, 52.7 MPa and 64.7 MPa, respectively. The normalised average  $G$  i.e.  $G/G_{max}$  and average damping ratio in comparison with those of the Piedmont ML residual soils are shown in Figure 5 and Figure 3b, respectively. The normalised  $G$  for JF1, JF2 and JF3 are lower than the average normalised  $G$  of the Piedmont ML residual soils. This cannot be due to the underestimation of  $G_{max}$  as a higher  $G_{max}$  will plot the normalised  $G$  even lower down the plot. The damping ratios for JF1, JF2 and JF3 are within the lower portion of  $\pm 50\%$  band of the Piedmont ML residual soils' damping ratios.

### 4.3 Comparison with Cohesionless and Fine-grained soils

The normalised shear modulus and damping ratio for JF1, JF2 and JF3 are compared with the shear modulus and damping ratio bands established by Seed & Idriss (1970) for saturated sands in Figure 6 and the curves by Vucetic and Dobry (1991) for saturated fined-grained soils in Figure 7.

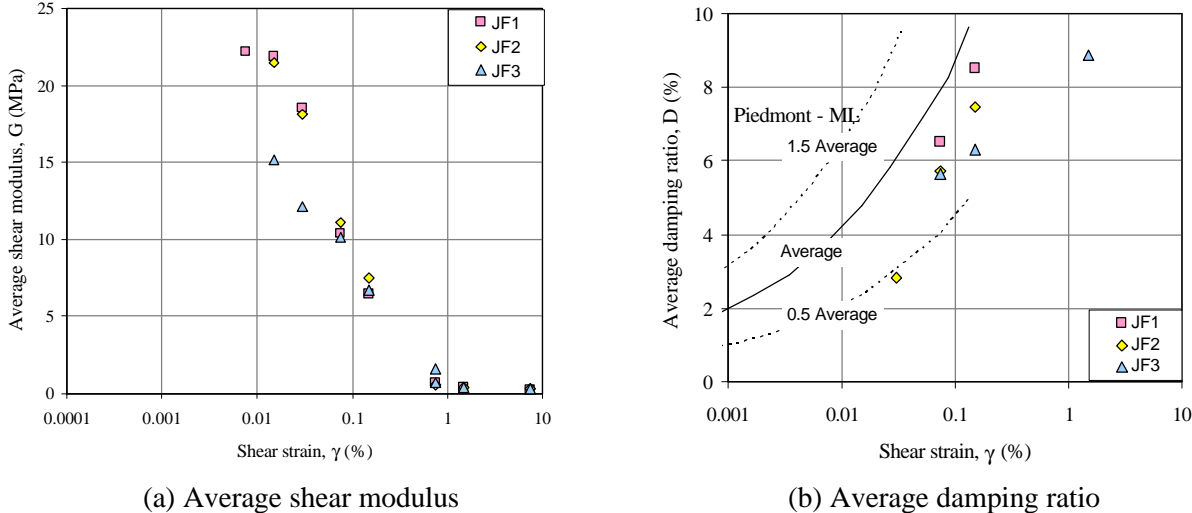


Figure 3. Average shear modulus and damping ratio with shear strain amplitude.

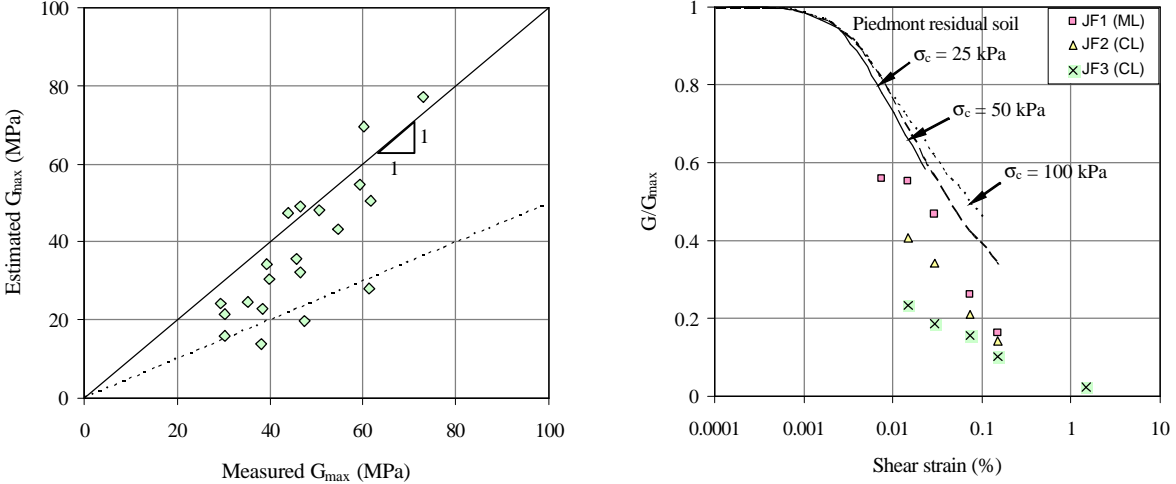
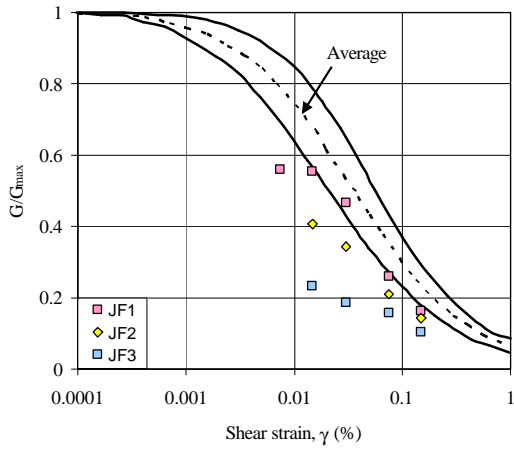
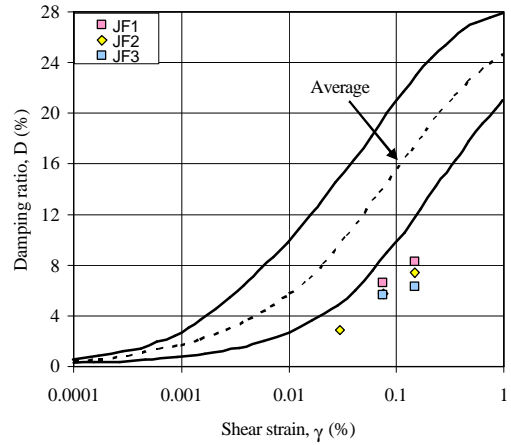


Figure 4. Plot of estimated  $G_{max}$  using Equation 1 and measured  $G_{max}$  for Piedmont ML residual soils.



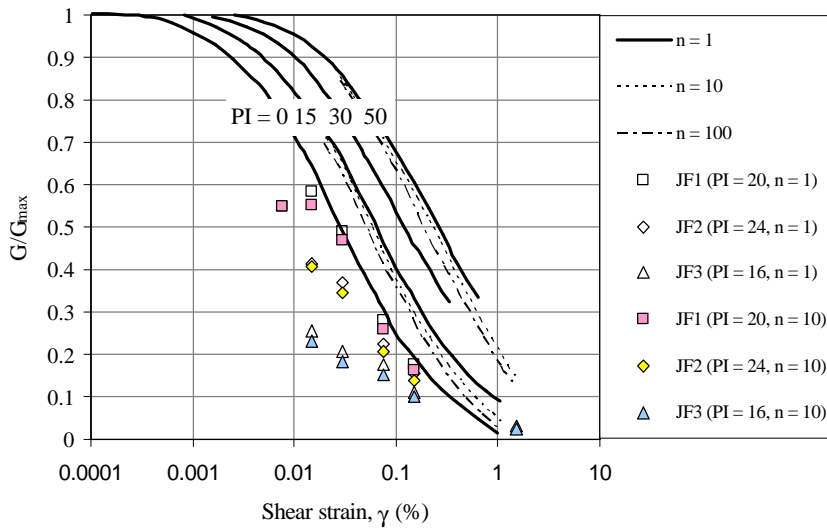
(a) Normalised shear modulus

Figure 5. Comparison of normalised G with Piedmont ML residual soils.

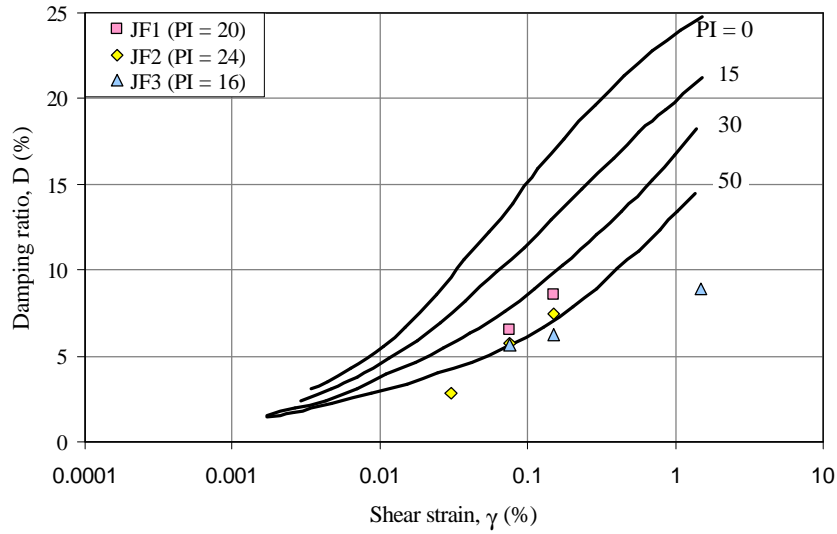


(b) Damping ratio

Figure 6. Comparison of normalised shear modulus and damping ratio with those of Seed & Idriss (1970).



(a) Normalised shear modulus



(b) Damping ratio

Figure 7. Comparison of normalised shear modulus and damping ratio with those of Vucetic and Dobry (1991).

The normalised shear modulus and damping ratio of JF1, JF2 and JF3 plot below the band established for sands by Seed & Idriss (1970). The spread in the normalised shear modulus of JF1, JF2 and JF3 are wider than the band of Seed & Idriss (1970). When compared with the curves of Vucetic and Dobry (1991) established for fine-grained soils, the normalised shear modulus plots below the curve of PI = 0. However, a similar trend of the shear modulus being lower with increasing number of loading cycles is observed for JF1, JF2 and JF3. The damping ratios of JF1, JF2 and JF3 plots in the region of the damping curve for PI = 50. Therefore if the normal range of normalised shear modulus and damping ratio established for saturated sands and fine-grained soils were applied to the Singapore residual soils, there is a tendency to overestimate the shear modulus and damping ratio.

#### 4.4 Stiffness-strain Relationship

A general equation used to describe the stiffness-strain relationship of a soil (Hardin & Drnevich 1972b) is given by:

$$\frac{G}{G_{\max}} = \frac{1}{1 + \frac{\gamma}{\gamma_r}} \quad (3)$$

where  $\gamma_r$  is a reference strain. A more general form of Equation 3 (Borden et al. 1996) is given by:

$$\frac{G}{G_{\max}} = \frac{1}{(1 + a\gamma^b)^c} \quad (4)$$

where  $a$  and  $b$  are constants. Borden et al. (1996) found Equation 4 to fit the normalised shear modulus data for Piedmont residual soils reasonable well. Soga et al. (1995) found Equation 4 with  $c = 1$  fitted the normalised shear modulus of clays very well. Equation 4 was used to fit the normalised shear modulus of JF1, JF2 and JF3 with  $c = 1$  and  $c$  as a variable. It was found that using  $c = 1$  gives a smaller residual for JF1, JF2 and JF3. The fitted curves together with their respective  $a$  and  $b$  values are shown in Figure 8.

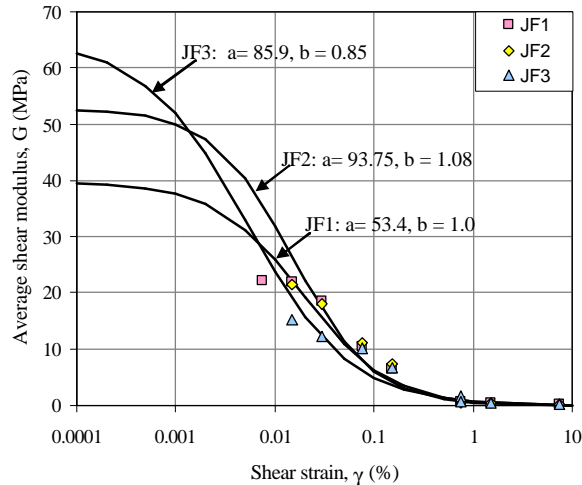


Figure 8. Fitted stiffness-strain curves using Equation 4 with  $c = 1$ .

## 5 CONCLUSION

The dynamic properties of three Singapore residual soil samples were investigated. These samples were fully saturated before the cyclic triaxial tests. The shear moduli of the residual soils were found to decrease slightly with increasing number of loading cycles and the damping ratios were found to increase slightly with the number of loading cycles. The shear moduli of the residual soils were found to plot below the average shear modulus curves for the Piedmont ML residual soils by Borden et al. (1996). The damping ratios of the residual soils however were within the lower portion of the damping curves for the Piedmont ML residual soils. The shear moduli of the residual soils were also found to be lower than the shear modulus band for sands by Seed & Idriss (1970) and the shear modulus curves for fine-grained soils by Vucetic & Dobry (1991). Similarly, the damping ratios of the residual soils were below the damping ratio band by Seed & Idriss (1970) and the damping ratio curves of Vucetic & Dobry (1991). Therefore, using the normal stiffness–strain relationships and the normal damping ratio–strain relationships established for sands and fine-grained soils will overestimate the shear moduli and damping ratios of the residual soils. There is a need to investigate the dynamic properties of the residual soils further to establish the effects of confining pressure, frequency of loading and the degree of saturation. The effect of degree of saturation is especially important as the residual soils at shallow depths usually exist in an unsaturated state.

## ACKNOWLEDGEMENT

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