2003 Pacific Conference on Earthquake Engineering

A general model for analysing response of slender masonry structures under multi-component earthquake excitations

B.L. Pintucchi

Dipartimento di Costruzioni, Università di Firenze, Firenze, Italia.

ABSTRACT: In this paper, a numerical method is presented to perform non-linear dynamic analysis of coupled transverse and longitudinal oscillations of masonry beams with different constrains when subjected to any dynamic loading type, including vertical and transverse motion imposed at their support. In developing the model, a non-linear constitutive equation giving stress characteristics as a function of strain ones, for materials with no tensile strength and limited compressive strength, has been used. The model allows to analyse dynamic behaviour of masonry slender structures with primarily flexural behaviour - such as towers - under multi-component earthquake excitations: it seems a useful model since it requires short computational time, while accounting for the material non-linear behaviour. Results from a case study evidence the difference in dynamic response with respect to responses obtained both from a linear elastic analysis and from a simpler non-linear model, where no allowance for a limited compressive strength was made. Subsequently, preliminary results on dynamic response of slender towers under real input ground motion evidence the ability of the model to supply information which may be used to evaluate indices of global and local structural damage.

1 INTRODUCTION

The problem of structural analysis of masonry buildings and monuments, subjected to static and dynamic loads, appears very interesting because of both their historical values and structural peculiarities. Moreover, some recent events as the sudden collapse of the Civic Tower of Pavia (Italy) in 1989 as well as severe damage observed in ancient isolate towers following past earthquakes have emphasized interest for this class of structures.

In recent years, large efforts have been devoted to analyse the behaviour of masonry structures and, among them, of ancient isolate towers. Nevertheless, in many cases, investigations have been conducted by means of linear elastic models, incapable of realistic description of these structures, made of a heterogeneous material whose response to tension is fundamentally different from that in compression.

In a previous work, a model for analysing the dynamic behaviour of masonry beams, columns and slender towers has been developed taking into account the afore-mentioned material mechanical characteristic (Pintucchi 2000). In fact, recent studies have led to a better understanding of the so-called no-tension materials (*masonry-like* materials), having constitutive equations which can, at least in certain aspects, describe mechanical behaviour of masonry (Del Piero 1989, Gennai and Padovani 1989).

Implementation of the above-mentioned constitutive equation into a finite-element code has been successfully applied to study static behaviour of some buildings of historical and architectural interest (Lucchesi, Padovani, Pagni 1994). Major difficulties have instead been found when trying to extend application of this constitutive relation to dynamic analyses of complex structures. However, masonry beams, columns and slender towers, for which geometry is simple and flexural behaviour is prevailing, can be represented by a simplified model that, while being able to account for the main mechanical characteristics of the material, is easier to be used.

Going more into detail, the model has been developed using a constitutive equation for rectangular cross-section beams that expresses the stress characteristics (axial force and bending moment) as a function of the strain characteristics (the extensional strain and the curvature of the beam longitudinal axis) for no-tension materials with infinite compressive strength (Orlandi 1998). Moreover, the constitutive relations have been generalized to the case of hollow rectangular cross section (Pintucchi 2000).

Some preliminary results have evidenced the main aspects of the dynamic behaviour of masonry slender structures and have clarified significant differences in response with respect to the corresponding linear-elastic structures. Firstly, transverse oscillation problems have been analysed neglecting longitudinal component of motion, on the basis of an approximation about the normal force, for which a constant and known value has been assumed. Subsequently, in (Lucchesi and Pintucchi 2002), the coupling between axial and transverse vibrations occurring in the non linear field, due to the constitutive relation used, has been introduced. As expected, coupling phenomena together with the presence of vertical earthquake excitations have been confirmed to be sources of additional damage.

In this paper, contrary to the previous model assuming masonry with no resistance to tension and infinite resistance to compression, material limited compressive strength has been taken into account using the constitutive relation presented in (Zani 2001).

A case study of an isolate slender tower subjected to an harmonic loading at its base shows the different response trends with respect to that obtained assuming the material compressive strength to be infinite and to that from linear elastic analysis. It is also evidenced, by some preliminary analyses of towers subjected to real earthquake excitations, that the model, allowing to evaluate both global and local parameters of structural damage, is suitable for analysing this type of structures for which past earthquakes showed that damage pattern tended to be distributed along the height, without concentrating at their base only.

2 MODEL SPECIFICATION

This section describes the main characteristic of the proposed numerical model. Slender structures have been modelled as continuous cantilever beams with rectangular cross-section, though an extension to hollow rectangular section beams, as developed in (Pintucchi 2000), should be conducted easily.

As regards masonry behaviour, a non-linear elastic constitutive relation for beams made of no-tension materials, with limited compressive strength, has been used. By making the usual assumption of plane sections, according to the classical Euler-Bernoulli hypothesis, and accounting for axial stresses only, the beam strain is described by the extensional strain e and the curvature κ of the longitudinal axis, whereas the stress state is represented by axial force N and bending moment M.

Since it is assumed that material has no resistance to tension and limited compressive strength, under the condition of uniaxial stress, the considered constitutive equations is represented by the diagram of Figure 1, where σ_o denotes the maximum compressive stress. The pattern of the axial stress component σ in any given transverse section of the beam can be one of those described in the same Figure 1. Each illustrated case corresponds to a different region in the plane (e, κ), in which different expression of the constitutive relation holds. In the region corresponding to cases *a* and *b*, where the section is uncracked and σ does not reach the limit value σ_o in any point, the constitutive relation is linear elastic. In the remaining regions, the constitutive law is not linear and the extensional and flexural problems are coupled. A detailed description of the constitutive equations is presented in (Zani 2001). It should only be noted here that in cases *c* and *d* of Figure 1, cracking of section occurs; in cases *e* and *f* a portion of section has reached its maximum compressive strength; finally, in case *g* and *h*, both previous situations occur at the same time.

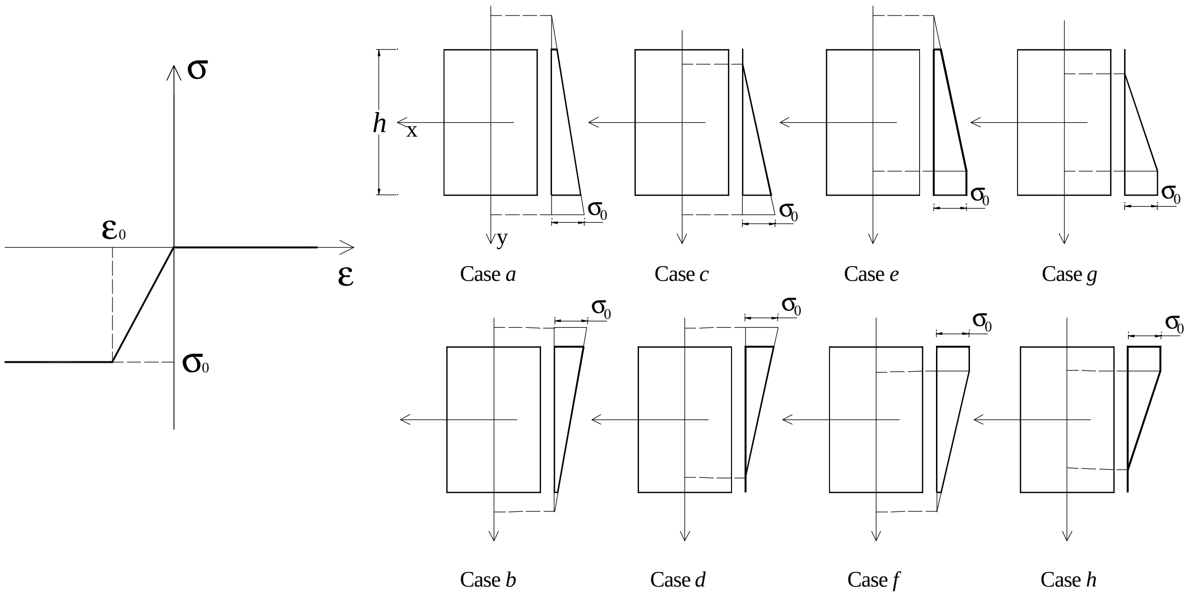

Figure 1: Stress σ- strain ε diagram and σ patterns over the cross-section.

The described numerical method has been implemented into a computer code that allow to study dynamic behaviour of beams with general constrain conditions subjected to both axial and horizontal dynamic excitations applied at their base and different loading conditions including their own weight. Using the equation of motion in its weak formulation, the finite-element method has been used to discretize the beam, by assuming three degrees of freedom at each node. In order to guarantee the continuity of both transverse displacement of beam axis and rotation, conforming elements and Hermite shape functions have been selected, while linear shape function have been used for axial displacement. As shown elsewhere (Lucchesi and Pintucchi, 2002), the consistent mass and stiffness matrixes as well as the loading vector have been obtained and standard numerical techniques have been used to integrate the coupled transverse and longitudinal equations of motion obtained through the discretization.

Numerical analyses performed comparing the described model and finite element code NOSA (Degl'Innocenti, Lucchesi, Padovani, Pagni, Pasquinelli and Zani) show the suitability of the former: a good agreement of results and a significant reduction in computational time have been found. Moreover, since the model takes into account material non linear behaviour in all sections along the height, it allows to obtain information closely correlated with both local and global damage of the structure. Namely, maximum deformation reached in compression over the entire volume of the tower, the global cracked portion of structure volume over the entire height and the global portion of the volume where the maximum compressive strength is reached. It should be noted, however, that no allowance for degradation due to load reversals can be made by the model presented, as it is a non-linear elastic one.

3 CASE STUDY

Referring to some structural characteristics that can be considered typical of ancient tower in Italy, a slender tower 40 m height with a square section 5.5 wide has been analysed. A value of 2800 MPa has been assumed for Young's compression modulus *E*, while the mass density ρ is taken to be 1900 kg/m^3. For compression strength a value of σ_o= -2.0 MPa has been set forth (Casolo 1998). Regarding the viscous damping, in the following examples the widely assumed value of 0.05 for the damping ratio of the tower first two elastic flexural modes, has been considered, being difficult to define an alternative value.

The tower has been subjected at its base to an horizontal acceleration varying according to a sine law, with period equal to the fundamental elastic transverse period of the structure and amplitude equal to 0.2g. The harmonic loading has been applied for 1s, thereafter oscillations are free.

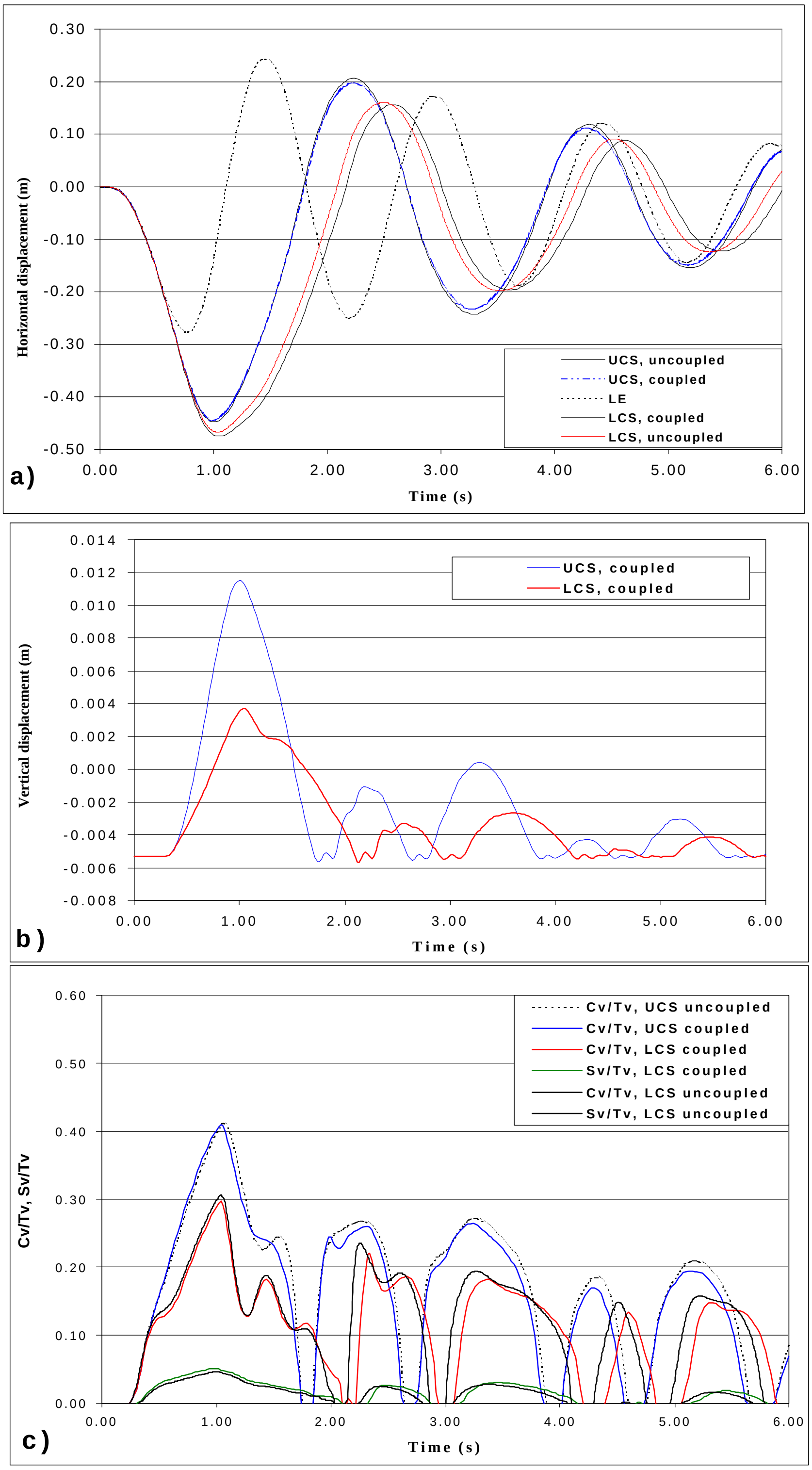

Figure 2: Results from case study: a) horizontal displacement; b) vertical displacement; c) parameters of damage *Cv/Tv* and *Sv/Tv*.

Results obtained with the proposed model (denoted as LCS in graphs) are compared to both the linear elastic response (LE) and results yielded considering the no tension material but neglecting its limited compressive strength (UCS). For the two non-linear cases, results are also compared with those obtained if no allowance for axial coupling is made.

Comparisons have been carried out for transverse and longitudinal displacements as well as variations of two parameters of global structural damage: the volume ratio of the cracked portions of the structure, *Cv*, to the overall volume of the tower, *Tv*, and the volume ratio of the portion of the structure where the maximum compressive strength is reached, denoted as *Sv*, to *Tv*. It must be underlined that the above-defined damage parameters have not been validated by means of experimental evidences and, therefore, no limit values corresponding to performance levels are available. However their arbitrary definition appears to be suitable since damage levels are certainly correlated to the spread of cracking and crushing throughout the tower volume.

Figure 2 shows time histories of the transverse and axial displacements at the top of the tower, as well as of the afore-mentioned damage parameters.

First of all, results obtained by this example confirm the consistent difference in response with respect to the linear elastic behaviour, if material no-resistance to tension is taken into account as it leads to amplifications in horizontal displacement. Moreover, as expected, with formation of the first cracked zones and the consequent loss of stiffness, the system lateral period becomes longer. When limited compressive strength is also considered, a further amplification in displacements occurs and an elongation in axial and transverse periods of vibration. Moreover, limited compressive strength reduces vertical displacements due to coupling phenomena and values of the parameter *Cv/Tv* with respect to the case considering no-tension material only. Furthermore, as regards horizontal displacements, this refined model (LCS) seems to be more sensitive to coupling phenomena than the previous one (UCS).

4 PRELIMINARY RESULTS FOR REAL EARTHQUAKES

Some preliminary results have been obtained varying few of the main structural parameters of slender towers, considering real earthquake excitations. The height of the tower has been varied in a range going from *35* m to *45* m, being the case study presented before an intermediate case. Values of the other parameters defined in the previous case study have been kept unchanged. A further analysis has been conducted varying the maximum compressive strength σ_o from *1.8* MPa to *2.5* MPa. As input ground motions, the horizontal component of Newhall record from Northridge earthquake (Newhall, 34.21N-118W, 17/1/94) and El Centro record (El Centro, S00E, 18/5/40) have been used.

Figure 3 represents, for both applied input ground motions, maximum values of *Cv/Tv*, as the height and, consequently, the tower slenderness varies, for the following cases: 1. neglecting the material's limited compressive strength and coupling phenomena (UCS uncoupled); 2. considering coupling between transversal and axial motions but assuming material compressive strength unlimited (UCS coupled); 3. considering both limited compressive strength and coupling phenomena (LCS coupled). In the same graph, the maximum of the parameter *Sv/Tv*, computed for the third case only, is also shown. In addition, Figure 3 reports maximum deformation $\varepsilon_{c,\ max}$ reached in compression over the entire volume of the tower and maximum horizontal displacements at the top of the tower, as evaluated in the three afore-mentioned cases. In the latter graph, horizontal displacements from linear elastic analysis (LE) are also represented. Finally, vertical displacements due to coupling phenomena for UCS and LCS model have been plotted. Figure 4 shows the same results as Figure 3 for different values of material maximum compressive strength σ_o.

Results seem to indicate that considering material limited compressive strength induces a consistent amplification of $\varepsilon_{c,\ max}$, whereas differences in values of parameter *Cv/Tv* are slight. Moreover, *Cv/Tv* seems not to be very sensitive to structural characteristic varied in the analysis, contrary to both the maximum deformation $\varepsilon_{c,\ max}$ and the dimensionless portion of the structure where the maximum compressive strength is reached, *Sv/Tv*. As regards horizontal displacements, comparison between results obtained by UCS and LCS models shows slight variations, whereas they differ, also qualitatively, from the linear elastic case.

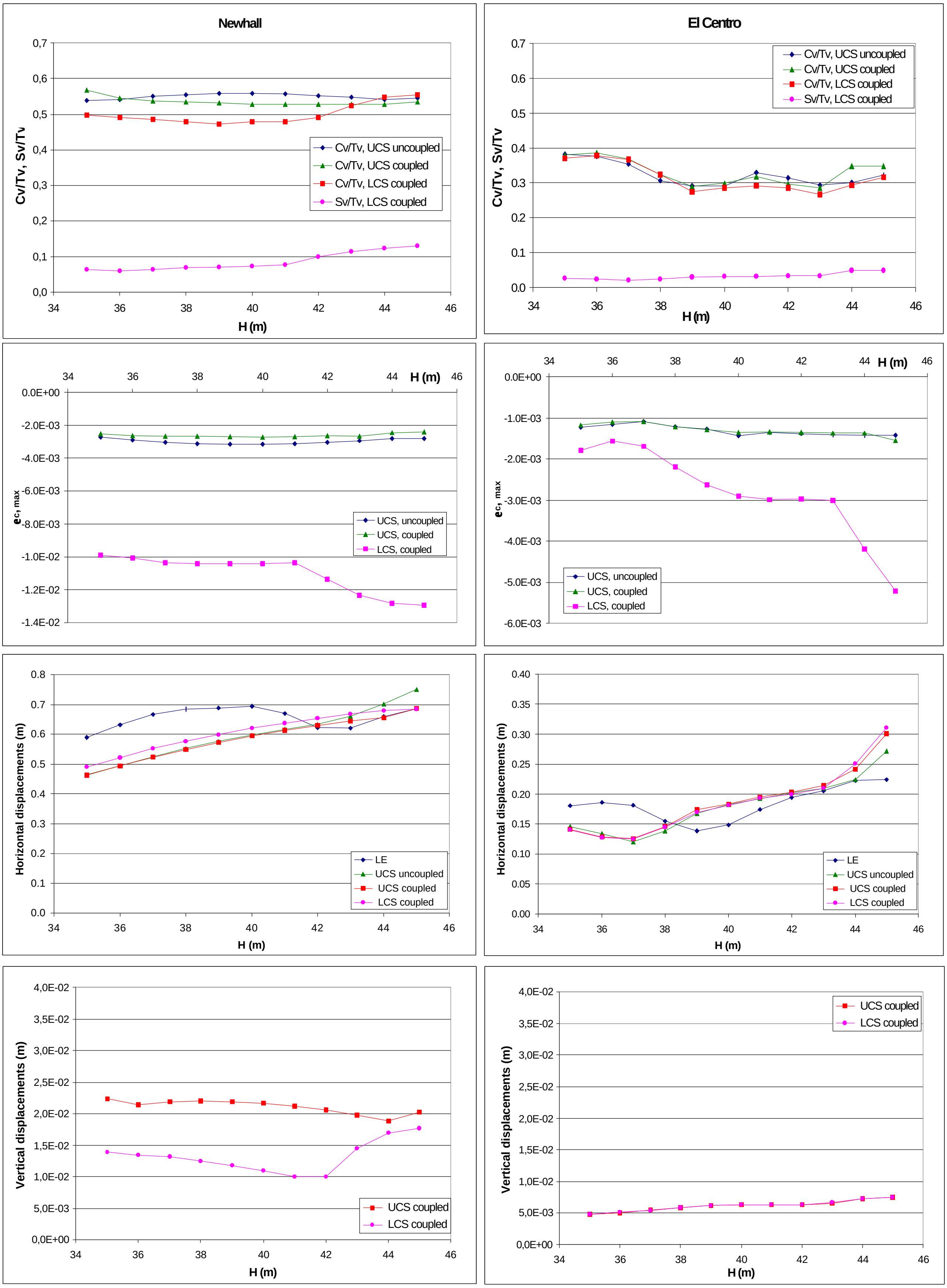

Figure 3: Maximum damage parameters *Cv/Tv*, *Sv/Tv*, $\varepsilon_{c,\ max}$ and displacements at the top, as a function of tower height.

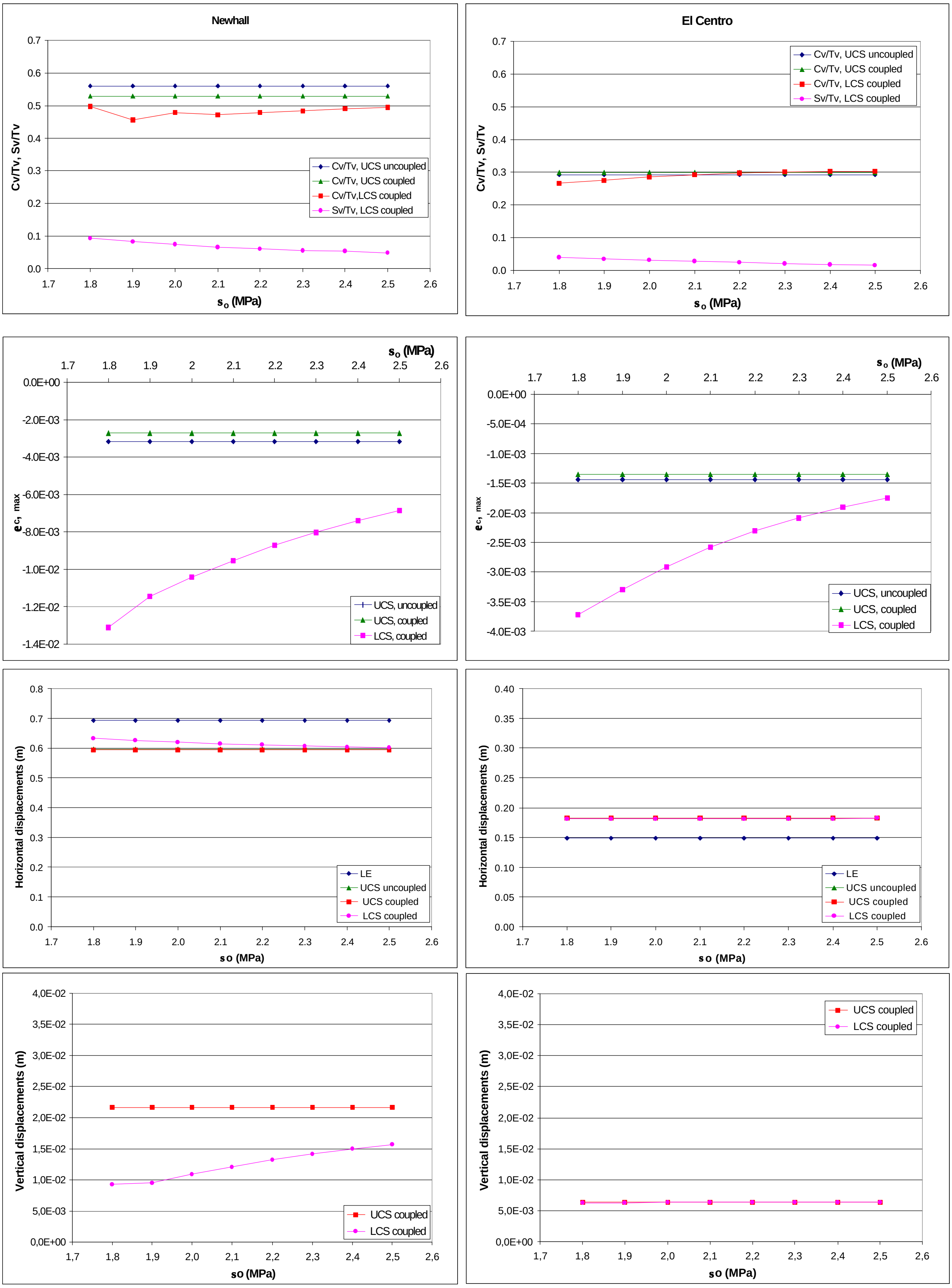

Figure 4: Maximum damage parameters *Cv/Tv*, *Sv/Tv*, $\varepsilon_{c,\,max}$ and displacements at the top of the tower, as a function of σ_{o}.

5 CONCLUSIONS

In this paper, a numerical model allowing for non-linear analyses of masonry beams with primarily flexural behaviour - such as towers - under multi-component earthquake excitations, has been presented. The non-linear elastic model takes into account both the no-resistance to tension of the masonry and its limited compressive strength, by means of a constitutive relation for beams made of *masonry like* materials. The model also accounts for coupling phenomena between transverse and axial vibrations arising in the non-linear field of behaviour, which are widely recognized as very influencing seismic behaviour of such slender structures.

Results from a case study evidence qualitative differences in dynamic response with respect to those obtained both by means of a linear elastic analysis and from a simpler non-linear model, where no allowance for a limited compressive strength was made. Moreover, since the model takes into account material non linear behaviour in all sections along the height, it may be useful, to obtain some measures of local and global damage and to predict seismic behaviour of these structures, whose behaviour is often affected by higher modes.

Few preliminary analyses of slender towers, with different values of key structural parameters, has been conducted using two real earthquake excitations. The principal aim was to highlight the influence of different modelling assumptions and to evidence information supplied by the model on seismic behaviour and structural damage. In order to obtain some general conclusions as well as to verify the sensitivity of the chosen damage parameter and their capability to represent real mechanisms of collapse and damage, parametric investigations and comparisons with experimental behaviour should be conducted in further research.

REFERENCES:

Bennati, S. & Barsotti R. 2001. Optimum radii of circular masonry arches. *Proc. of the Third international arch bridges conference, Paris, 19-21 September 2001*.

Casolo, S. 1998. A Three-dimensional model for vulnerability analysis of slender medieval masonry towers. *Journal of Earthquake Engineering*, vol.2, 4, 487-512.

Del Piero, G. 1989. Constitutive equation and compatibility of the external loads for linear elastic masonry-like materials. *Meccanica*, 24, 150-162.

Degl’Innocenti, S. Lucchesi, M. Padovani, C. Pagni, A. Pasquinelli, G. & Zani, N. 2001. Dynamical analysis of masonry pillars. *Third International Congress on Science and Technology for the Safeguard of Cultural Heritage in the Mediterranean Basin, Alcalá de Henares, 9-14 July 2001.*

Gennai, A. M. & Padovani, C. 1989. Constitutive equations for masonry-like materials. In Boffi, V. & Neunzert, H. (Eds), *Applications of Mathematics in Industry and Technology*, B.G. Teubner, Stuttgart: 229-238.

Lucchesi, M. Padovani, C. Pagni, A. 1994. A numerical method for solving equilibrium problems of masonry-like solids. *Meccanica,* 29, 175-193.

Lucchesi, M. & Pintucchi, B.L. 2002. Coupled Longitudinal and Transverse Vibrations of Beams Made of No-Tension Material. *Proc. of the VII International Seminar on Structural Masonry for Developing Countries, Belo Horizonte, 18-20 September 2002*.

Orlandi, D. 1998. Analisi non lineare di strutture ad arco in muratura. (in Italian) *Dottorato di Ricerca in Ingegneria delle Strutture*. VII ciclo, Università degli Studi di Firenze.

Pintucchi, B.L. 2000. Vibrazioni trasversali di elementi monodimensionali non resistenti a trazione in direzione longitudinale. (in Italian) *Dottorato di Ricerca in Storia delle scienze e delle tecniche costruttive*. XIII ciclo, Università degli Studi di Firenze.

Zani, N. 2001. A constitutive equation for beams made of no-tension materials with limited compressive strength. *Proc. of the Third international arch bridges conference, Paris, 19-21 September 2001*.