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Intergrain friction, contact density, and cyclic resistance of sands

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ABSTRACT: Soil liquefaction phenomenon involves progressive intergrain contact deformation, slip, reorganization of contacts, and eventual collapse of soil skeleton. During this process frictional energy is lost along contacts. Resistance to liquefaction depends on the density of active intergrain contacts, contact friction, and confining stress. Higher the density of active intergrain contacts (per grain) more resistant is the soil to liquefaction. Higher is the confining stress higher is the energy loss along contacts and higher is the resistance to liquefaction. This paper presents an analysis of the evolution of intergrain contact friction, slip, and energy loss during undrained cyclic loading in sand. A theoretical framework for estimation of an index of active contact density for sands is presented. Theoretical results for mobilized intergrain friction and frictional energy loss are compared with experimental data. A new expression is presented for porewater pressure generated during undrained cyclic loading as a function of energy loss.

1 INTRODUCTION

Liquefaction, lateral spreading due to temporary loss of strength, and post-liquefaction settlement are major causes of ground failures during earthquakes. Soil liquefaction is a process leading to structural collapse of the soil skeleton due to shear, with a concurrent frictional loss of energy along active intergrain contacts due to rolling and sliding friction and local yielding at contact zones. The net energy loss is equal to the frictional energy loss W_L plus energy released from the intergrain contact force network minus the energy gained due to increase in porewater pressure. The latter two components are small. There has been several studies on the use of W_L to study liquefaction potential of soils (Nemat-Nasser & Shokooh 1979, Berrill & Davis 1985, Okada & Nemat-Nasser 1994, Figureuoa et al. 1994, Kayen & Mitchell 1997, Thevanayagam 2000, and Thevanayagam et al. 2000). A complete quantitative understanding of the relationship for the cumulative energy loss W_L per unit volume of soil required to liquefy the soil is still far from grasp.

Consider a regular array of uniform spherical particles. Frictional energy loss per unit volume of this array subjected to shear is primarily a function of the coefficient of friction μ, intergrain contact normal force N, number of slipping contacts per grain n_g, number of grains per unit volume m_g, and the magnitude of slip between contacts $\Delta\delta$. N is a function of n_g and m_g, and effective confining stress σ'. m_g is a function of void ratio e of the soil and grain size D. n_g and m_g depend on soil structure. $\Delta\delta$ is a function of incremental shear strain $\Delta\gamma$ and D. The energy loss at small strains prior to full slip between particles is also dependent on the elastic properties of the grains (Mindlin & Deresiewicz 1953). However, liquefaction is a large strain problem and the energy loss prior to full slip between contacts is small, and hence the influence of the elastic properties is negligible.

During drained loading the soil volume changes and, if the volume change is disallowed (undrained), pore water pressure changes. During cyclic loading these processes continue with concurrent accumulation of frictional energy loss, eventually leading to cessation of further volume change or pore pressure changes. For loose soils subjected to undrained shear, such a state is called liquefaction. Intuitively, as a first order approximation, W_L may be expressed as a function of intergrain contact density, initial effective confining stress σ_c', and frictional characteristics of the soil.

What would be the form of the relationship for W_L, and what would be a suitable index of active intergrain contact density are difficult questions. For practical purposes, for rather uniform granular arrays, void ratio may be a useful index. Except for such regular arrays, there exist particles that do not actively participate in the internal contact force network within the medium. e is not a valid index. The relative density D_r may be useful in such cases. For a broadly graded granular mix containing coarse grains and fine grains of different sizes and shapes with large size disparity, like silty sands and gravelly sands, there exist many particles that do not actively participate in the intergrain contact force chain. Neither e nor D_r is useful. For such soils, a set of equivalent intergrain contact void ratios $(e_c)_{eq}$ (and equivalent relative density $(D_{rc})_{eq}$) and $(e_f)_{eq}$ (and equivalent relative density $(D_{rf})_{eq}$) have been introduced as useful indices of active intergrain contact density (Appendix I). This paper examines the form of W_L and the influence of intergrain contact density and confining stress on W_L for granular soils.

2 THEORY

2.1 External Energy E and E_L

The external energy dissipated per unit volume of soil (E Nm/m^3) is the cumulative area inside each hysteretic loop developed during cyclic loading, given by:

$$E = \sum_{i=1}^{n-1} \frac{1}{2} (t_i + t_{i+1})(g_{i+1} - g_i) \tag{1}$$

where τ = shear stress; γ = shear strain; n = number of points recorded up to γ. Definition of liquefaction in terms of pore pressure is difficult, as the excess pore pressure ratio r_u ($\Delta u/\sigma_c$') may reach unity only momentarily. It is customary to define liquefaction as a state where the soil experiences ±5% double amplitude axial strain during undrained shear. E at ±5% axial strain is termed as the energy spent E_L to cause liquefaction, in this paper.

2.2 Internal Frictional Energy Loss W and W_L

Consider a granular array (Table 1), initially at a confining stress σ_c'. The normal contact force is N. During shear, the magnitudes of confining stress σ', N, contact tangent force, and contact slip vary from contact to contact, except for very small strains. Initially slip occurs along an annular region of the contact area and grows to full-slip condition after a threshold magnitude strain (Mindlin and Deresiewicz 1953). Exact relationship for these parameters depends on loading history and the evolution of the granular skeleton. As a first order approximation, in this paper, neglecting the variations in the internal contact normal forces N and the magnitude of slip Δδ at each active contact in a grain, W (Nm/m^3) per unit volume of soil, for all packing, is:

$$W = \sum (Nm^*(\Delta d)(n_g m_g)/2) = 1.5m^* \sum (s'(\Delta g)) \tag{2}$$

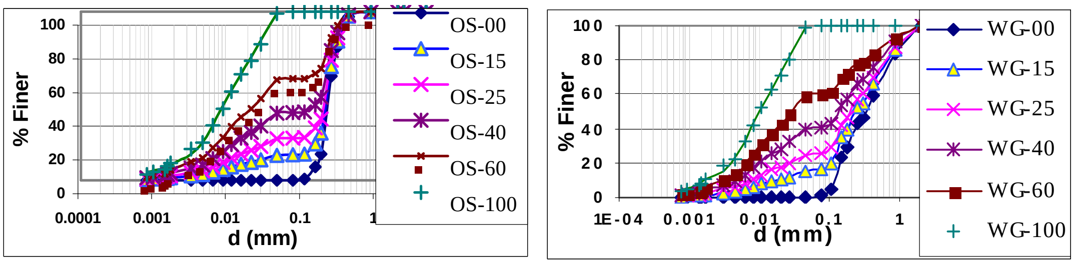
where Δγ= incremental shear strain amplitude, σ'= mean effective confining stress (σ_1'+σ_2'+σ_3')/3 during the incremental cyclic loading, and μ^* = effective coeff. of friction. W is given by 1.5μ^* times the cumulative area under the σ' versus γ diagram during cyclic loading. At small strains, however, due to lack of full slip, W given by Eq.2 would be higher than actual value. For small strains less than a threshold value, a magnitude of $\mu^*<\mu$ should be used to account for this error (μ=mobilized contact friction at full slip and $\mu^*=\mu$ at large strain). W at ±5% axial strain is termed as the internal frictional loss W_L required to cause liquefaction, in this paper. Neglecting other energy losses, E = W.

Table 1. Intergrain contact parameters – spherical array

Packing	Contact Normal, N	Active Contact Slips/grain, n_g	Active Grains/unit vol., m_g	Energy Loss/Unit vol. $Nm^*(Dd)(n_g m_g)/2$
Simple cubic	$D^2\sigma'$	6	$1/D^3$	$1.5\mu^*\sigma'\Delta\gamma$
Cubical-tetrahedral	$(D^2/3^{1/2})\sigma'$; $(3^{1/2}D^2/2)\sigma'$	2; 6	$2/(3^{1/2}D^3)$	$1.5\mu^*\sigma'\Delta\gamma$
Face centered cubic	$(D^2/8^{1/2})\sigma'$	12	$2^{1/2}/D^3$	$1.5\mu^*\sigma'\Delta\gamma$
Body centered cubic	$(D^2/3^{1/2})\sigma'$	8	$27^{1/2}/(4D^3)$	$1.5\mu^*\sigma'\Delta\gamma$

3 EXPERIMENTAL PROGRAM

The experimental program involved undrained cyclic triaxial tests on three sands (OS, FJ#80, and WG), and sand-silt mixes prepared by mixing the above sands with a non-plastic silt (Sil co sil #40, US Silica Company, Illinois). The OS sand-silt mixes at silt contents of 0%, 15%, 25%, 60, and 100% by dry weight are named OS-00, OS-15, OS-25, OS-60, and OS-100, respectively. The FJ#80 sand-silt mixes containing 0%, 25%, and 60% silt are termed FJ-00, FJ-25, and FJ-60, respectively. Similarly the WG sand-silt mixes are termed WG-00, WG-25, WG-60, respectively. Figure 3 shows the grain size data for two soils (Thevanayagam 2001).



(a) Ottawa sand (OS) – silt mix (b) Well-graded sand (WG) – silt mix

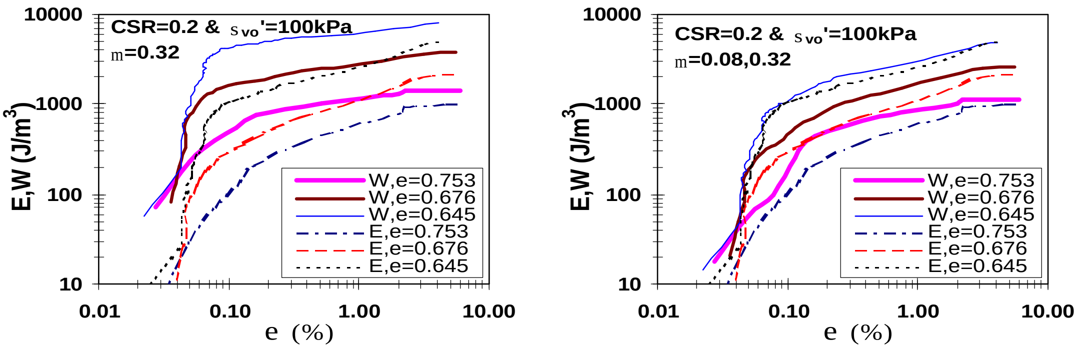
Figure 1 Grain Size Distribution

Cylindrical specimens (typically 75 mm diameter and 175 mm height) were prepared using moist-tamping or dry pluviation method. Following initial preparation, the specimens were percolated with carbon dioxide to displace the air in the void space of the specimens to facilitate full saturation by water. After that, deaired water was allowed to flow from the bottom of the specimen towards the top. The net volume of the water introduced into the specimen was measured accurately. Finally, the specimen was set up on the triaxial test apparatus to continue the saturation until the B-value ($=\Delta u/\Delta\sigma_c$) was typically greater than 0.95. Following this, the specimens were isotropically consolidated to σ_c'=100kPa or 400kPa. The final void ratio of each specimen was calculated based on the weight of the dry solid grains in the specimen, the net volume of water introduced into the specimen during saturation, and the measured volume change data during consolidation. Following consolidation, the drainage valves were closed and cyclic stress ratio (CSR=$[\pm\Delta\sigma_1/(2\sigma_c')]$=0.1, 0.2, and 0.3) controlled undrained cyclic triaxial tests were done at a frequency of 0.2 Hz or 1 Hz. The pore pressure, axial load, and axial deformation were recorded using a built-in data acquisition system.

4 ANALYSIS

4.1 E versus W

Figure 2a shows E and W versus axial strain amplitude reached during each cycle for OS-00 at 3 different void ratios. W was calculated using Eq.2 assuming $\mu^*=0.32$ (a reasonable μ for silica sand grains). At low strains less than about 0.1%, the calculated magnitude of W is consistently higher than E. The reason for this is that Eq.2 is based on the assumption that full-slip occurs along contacts whereas full-slip can occur only at large strains beyond a threshold slip-strain. This error in W was corrected by assigning a lower value for μ^* (=0.08) up to about 0.1% strain (typical range of threshold strain) and $\mu^*=0.32$ thereafter. The results are shown in Figure 2b. When such small-strain corrections are made, E and W agree well through out the cyclic loading up to liquefaction (±5% axial strain). Figure 3 shows E versus W from the beginning of cyclic loading up to liquefaction for OS-00 at 5 different e. W was calculated assuming $\mu^*=0.08$ for $\varepsilon<0.1\%$ and $\mu^*=0.32$ for $\varepsilon>0.1\%$. Results show a very close 1:1 relationship between E and W for all contact densities.



(a) μ=0.32 (b) μ=0.08 for ε<0.1%; μ=0.32 for ε>0.1%

Figure 2 E and W versus axial strain e

The same observation held for all specimens, including the granular mixes, although the magnitudes of μ differed slightly, depending on the soil grain type. This indicates the validity of the theory and that W_L is a function of contact density, confining stress, and intergrain friction. For spherical array (Table 1), $n_g m_g=5e^{-x}/D^3$, x=1±0.2. This and Eq.2 imply that, in general:

$$W_L = A\mu\sigma'_c(e)_{eq}^{-\alpha} \qquad (3)$$

where A, μ and α may depend on the gradation and other soil characteristics. $(e)_{eq}$ is defined in Appendix I.

4.2 Pore Pressure and Energy Loss

Figure 4 shows the pore pressure ratio r_u ($\Delta u/\sigma'_c$) versus normalized cumulative energy loss E/E_L for the OS sand. Results indicate that, except for the initial stage corresponding to very small strain level, r_u is log-linearly related to E/E_L, independent of contact density index, approximately following $r_u=0.5\log_{10}(100E/E_L)$, $E/E_L>0.05$. Such a relationship was found to hold for all specimens. The confining stress ratio (σ'/σ'_c) that can be sustained by the soil skeleton at any stage of cyclic loading depends on the cumulative energy loss ratio E/E_L up to that stage, where E_L is the cumulative energy loss required to fully reorganize the soil skeleton and liquefy the soil.

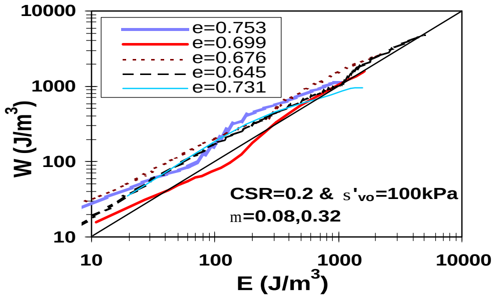


Figure 3 E versus W

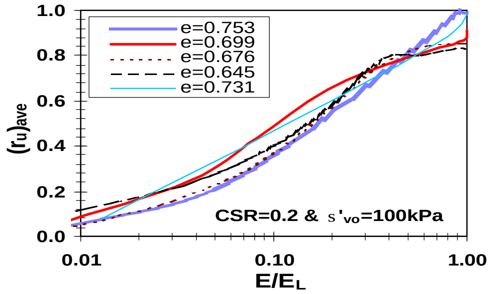
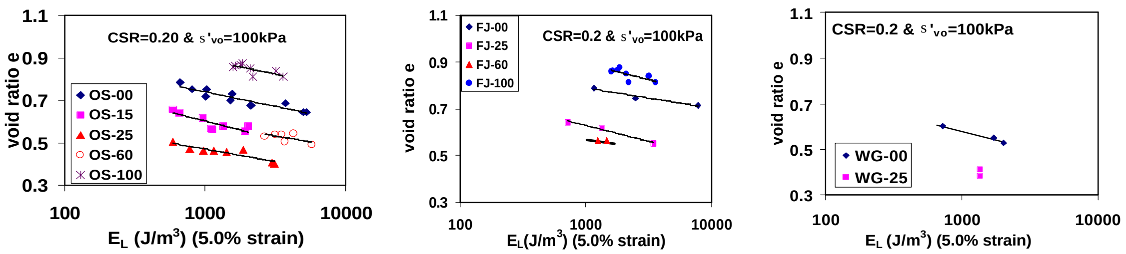


Figure 4 $(r_u)_{ave}$ $(=Du/s'_c)$ versus E/E_L

4.3 Effect of contact density on E_L

Figures 5a-c show E_L versus e for the three types of sand-silt mixes consolidated to σ'_{vo}=100kPa and sheared at CSR = 0.2. For each soil-mix, at the same silt content FC, there exists a log-linear relationship for E_L versus e ($E_L=A_1\mu e^{-\alpha}$). There is no unique relationship for E_L versus e for all mixes. The relationship depends on FC for a given sand-silt mix, and sand type for a given silt type and silt content. Even though e is a reasonably good intergrain contact index for a given soil mix at the same FC, it is not a valid contact index for all soils. For each type of sand-silt mix, at the same e, E_L decreases with increasing FC up to a threshold value (FC_{th}). Beyond FC_{th} the trend reverses. The reason for this is that at low FC, the intergrain contact force network is made of inter coarse grain contacts with only a secondary influence by the (confined) fine grains, and hence soil resistance is governed primarily by the inter coarse grain contacts (Figure I, Appendix I). Beyond FC_{th}, the intergrain force network is primarily made of the interfine contacts with only a secondary influence by the (dispersed) coarse grains, and hence soil resistance is governed primarily by the inter fine grain contacts. Void ratio is not a good index to account for such different roles by the coarse and fine grains. If such differing roles are accounted for using the indices $(e_c)_{eq}$ and $(e_f)_{eq}$, E_L correlates with $(e_c)_{eq}$ and $(e_f)_{eq}$ at $FC<FC_{th}$ and $FC>FC_{th}$, respectively, as shown in Figures 6a-c and 7, respectively ($E_L=A_1\mu(e)_{eq}^{-\alpha}$). A log-linear relationship is observed for E_L with $(e_c)_{eq}$ and $(e_f)_{eq}$. The relationships in Figures 6a-c depend on the sand type (gradation and shape effects). When the same data are plotted against $(D_{rc})_{eq}$, the shape and gradation effects of different sands is reduced; all data fall in a narrow band (Fig.8) and follows the form $E_L=B_1\mu(1-(D_{rc})_{eq}/100)^{-\beta}$.

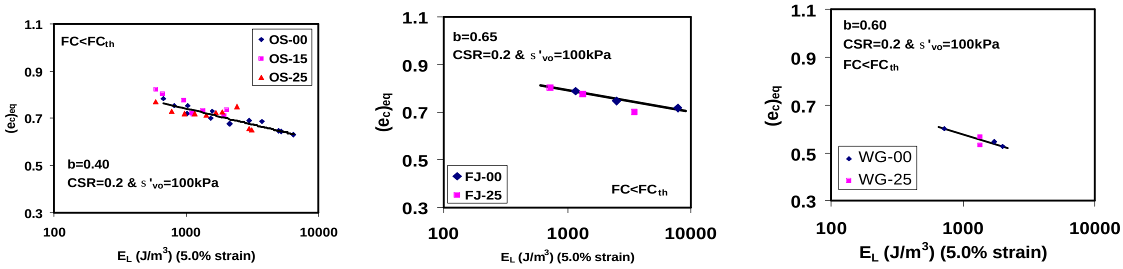


(a) OS-silt mix (b) FJ#80-silt mix (c) WG-silt mix

Figure 5 E_L at s_c'=100 kPa and CSR=0.2

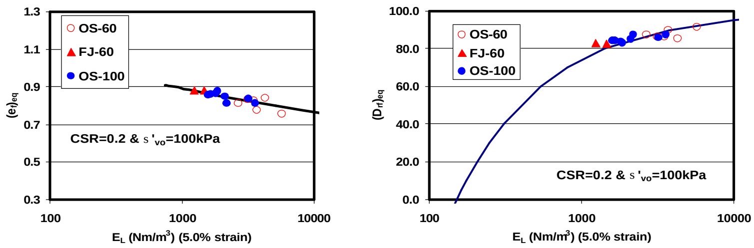
4.4 Effect of CSR and s'_c on E_L

The above test data (Figs 5-8) correspond to CSR=0.2. To investigate the influence of CSR on E_L versus contact density relationships, several additional cyclic tests were conducted at CSR=0.1 and 0.3 on specimens consolidated at σ'_c=100kPa. Figure 9 shows the data for the OS-silt mixes. The results indicate that E_L versus contact density index relationship is insensitive to CSR. Additional tests were conducted to study the effect of σ'_c on E_L. Figure10 shows the normalized energy E_{L1} ($=E_L*100/\sigma'_c$) versus $(e_c)_{eq}$ for $FC<FC_{th}$ for the OS-silt specimens tested at σ'_c=100kPa and 400kPa, and at CSR =1, 0.2 and 0.3. Results indicate that there exists a log-linear relationship for E_{L1} versus $(e_c)_{eq}$. E_L increases linearly with σ'_c, and follows the form $E_L=A\mu\sigma'_c(e)_{eq}^{-\alpha}$.



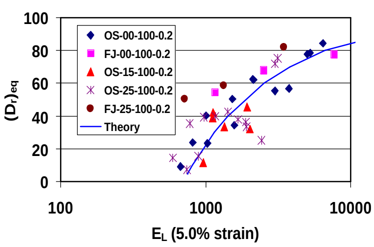
(a) OS-silt mix (b) FJ#80-silt mix (c) WG-silt mix

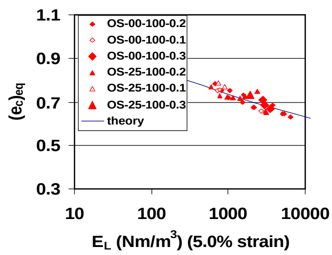
Figure 6 E_L versus $(e_c)_{eq}$: FC<FC$_{th}$



(a) E_L versus $(e_f)_{eq}$ (b) E_L versus $(D_{rf})_{eq}$

Figure 7 E_L versus $(e_f)_{eq}$ and $(D_{rf})_{eq}$: FC>FC$_{th}$





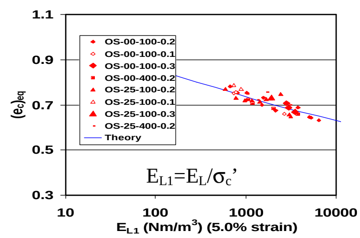


Figure 8 E_L versus $(D_{rc})_{eq}$ **Figure 9 Effect of CSR** **Figure 10 Effect of s$_c$' and CSR**

5 CONCLUSIONS

Semi-theoretical and experimental analyses of key factors that influence the frictional energy loss along active grain contacts during cyclic loading of granular soils indicate the following. The internal frictional energy loss W estimated based on intergrain friction at contacts agrees fairly well with the external energy input E. Cyclic pore pressure ratio increases log-linearly with E/E_L, independent of contact density. The energy E_L required to cause liquefaction increases linearly with initial effective confining stress (Fig. 10) and increases log-linearly with intergrain contact density. The recently proposed equivalent intergranular void ratios (or equivalent relative densities) are useful parameters to characterize the liquefaction resistance of granular mix soils (e.g. silty sands). Further research is needed to fully develop an understanding of the key factors affecting energy loss during cyclic loading, and better characterize liquefaction behavior of soil.

6 ACKNOWLEDGEMENTS

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REFERENCES:

Berrill, J.B. and Davis, R.O. 1985. Energy dissipation and seismic liquefaction of sands: revised model. *Soils and Foundations*. 25 (2): 106-118.

Kayen, R.E. & Mitchell, J.K. 1997. Assessment of liquefaction potential during earthquakes by arias intensity. *ASCE, J. Geotech. & Geoenv. Eng.* 123 (12): 1162-1174.

Mindlin, R.D. & Deresiewicz, H. 1953. Elastic spheres in contact under varying oblique forces. *ASME, J. Appl. Mech.* 74: 327-344.

Figueroa, J.L., Saada, A.S., Liang, L. & Dahisaria, M.N. 1994. Evaluation of soil liquefaction by energy principles. *ASCE, J. Geotech. Eng.* 120 (9): 1554-1569.

Nemat-Nasser, S. & Shokooh, A. 1979. A unified approach for densification and liquefaction of cohesionless sands in cyclic loading. *Can. Geot. J,* 16: 659-678.

Okada, N. & Nemat-Nasser, S. 1994. Energy dissipation in inelastic flow of saturated cohesionless granular media. *Geotechnique.* 44 (1): 1-19.

Thevanayagam, S. 1998. Relative roles of coarse and fine grains on the mechanical response of granular mixes. *ASCE, J. Geotech. & Geoenv. Eng.* in review.

Thevanayagam, S., Shenthan, T., Mohan, S. and Liang, J. 2002. Undrained fragility of sands, silty sands and silt. *ASCE, J. Geotech. & Geoenv. Eng.* 128 (10): 849-859.

Thevanayagam, S. 2000. Liquefaction potential and undrained fragility of silty sands. In R. Park (ed.). *Proc. 12th International Conf. Earthq Eng.* Auckland, New Zealand, paper # 2383.

Thevanayagam, S., Liang, J., and Shenthan, T. 2000. A contact index for liquefaction potential analysis of silty/gravely soils. In Tassoulas (ed.). *EM2000, Proc. 14th ASCE EMD Spec. Conf.* Austin, Texas.

Thevanayagam, S. 2001. Role of intergranular contacts on mechanisms causing liquefaction and slope failures in silty sands. *http://erp-web.er.usgs.gov/reports/annsum/vol43/pt/pt_vol43.htm*. USGS web.

Thevanayagam, S., Kanagalingam, T, Martin, G.R., and Shenthan, T. 2001. Cyclic resistance of sand, silty sand, and silts. *Can. Geo.J.* in review.

APPENDIX I. Contact density Indices

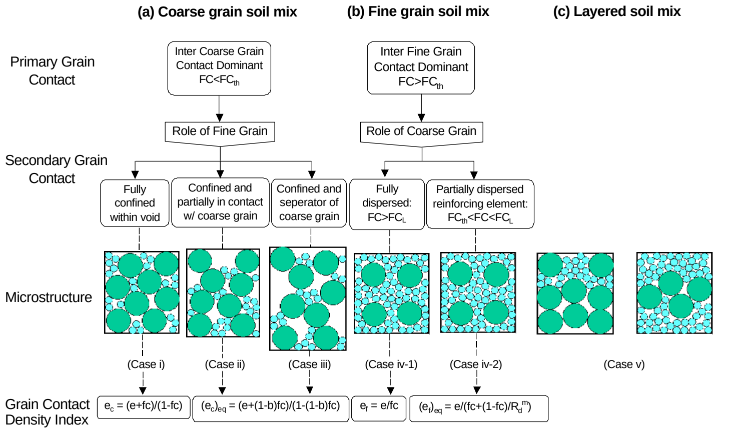
How and when coarse and fine grains contribute to the mechanical response of a granular mix is a complex problem. It depends on a number of factors, including fines content, void ratio, size disparity, particle shape, and gradation of the host coarse and fine grain soils.

Consider a granular mix containing coarse and fine particles of size D and d, respectively (Fig.I, Thevanayagam 2001). The microstructure of this granular mix can be formed in many different ways. Each one of them leads to a different internal force chain network among particles and hence each exhibits a different stress-strain response. Among many variations, the first category in Figure I-a occurs when the fine grains are fully confined within the void spaces between the coarse-grains with no contribution whatsoever in supporting the coarser grain skeleton. This requires that d is much smaller than the pore size between the coarse grains (R_d=D/d>>6.5). It is also essential that the fine grain content (FC) is less than a certain threshold value (FC_{th}) such that the inter coarse-granular voids are not completely full of fine grains so that the fine grains do not constitute a strong contact force chain among themselves nor contribute to the coarse grain force chain. When FC is sufficiently high (>FC_{th}), direct fine-grain-to-fine-grain contacts begin to constitute a strong force chain. Soil microstructure that satisfies the two constraints (FC<<FC_{th} and R_d>>6.5) is categorized as case i. FC_{th} is given by $100e/e_{max,HF}$ where $e_{max,HF}$ = void ratio of the host fine grain soil above which it has no appreciable strength.

At FC<FC_{th}, however, other microstructures are possible. Consider changing the position of some of the fine grains, while maintaining the fine grain content. Microstructure corresponding to case ii or iii in Figure I-a forms. Essentially, the microstructure for these cases is made of a coarse grain skeleton where some of the coarse grain contacts form via fine grains while some fine grains remain confined within the voids between the coarse grains. The 'separating fines' actively participating in the internal contact force chain needs to be accounted for in order to explain the mechanical response of the mix.

If FC exceeds sufficiently beyond FC_{th}, a fourth category (Fig.I-b) occurs naturally. The fine grains make active contacts among themselves while the coarse grains disperse and act as embedded reinforcement elements within the fine grain matrix until they disperse sufficiently far from each other. This imposes a limiting fines content FC_L. Beyond FC_L, fine-grain-to-fine-grain contact governs the behavior of the soil mix. The transition zone between FC_{th} and FC_L is case iv-2 and the zone corresponding to $FC>FC_L$ is case iv-1. The case v (Fig.I-c) occurs when coarse and fine grains constitute a fully layered system. A composite system consisting of these cases is also possible.

Based on contact density (per grain) considerations, equivalent intergranular void ratios $(e_c)_{eq}$ and $(e_f)_{eq}$ have been developed as indices of active grain contact density. Neglecting the differences in specific gravities between the coarse and fine grains, for case i, e_c $(=(e+fc)/(1-fc))$; $fc=FC/100$; e=global void ratio) may be used as the contact density index. For reasons discussed before, however, fine grains do contribute to the contact force chain. In such cases, an equivalent intergranular void ratio $(e_c)_{eq}$ $(=e+(1-b)fc)/((1-(1-b)fc))$ has been defined as the contact density index. The term b in $(e_c)_{eq}$ refers to the contribution by fine grains to the force chain. For case iv-1, an equivalent interfine void ratio $(e_f)_{eq}$ $(=e/(fc+(1-fc)/R_d^m)$ has been defined as the contact density index. The term m refers to the contribution by dispersed coarse grains to the force chain. At $FC>FC_L$, $(e_f)_{eq}$ becomes the interfine void ratio e_f $(=e/fc)$, the void ratio of the fine-grained soil, ignoring the presence of coarse grains. The corresponding equivalent intergranular relative densities are: $(D_{rc})_{eq}=(e_{max.HC}-(e_c)_{eq})/(e_{max.HC}-e_{min,HC})$ for $FC<FC_{th}$ and $(D_{rf})_{eq}=(e_{max.HF}-(e_f)_{eq})/(e_{max.HF}-e_{min,HF})$ for $FC>FC_{th}$, where $e_{max.HC}$=maximum void ratio of the host coarse grain soil; $e_{min.HC}$ and $e_{min.HF}$ = minimum void ratios of the host coarse- grained and host fine-grained soils, respectively. Using the respective equivalent contact density indices at $FC<FC_{th}$ and $FC>FC_{th}$, the mechanical response of a granular mix may be compared with the response of the host coarse-grained or host fine-grained soil, respectively. For granular mixes containing graded coarse and fine-grained soils, the same relationships may be applicable, by replacing R_d by D_{50}/d_{50}. Figure II (Thevanayagam 2001) presents the experimental relationship for b and m as a function of the coefficients of uniformity C_{uc} and C_{uf} of the coarse and fine grained soils, respectively, and D_{50}/d_{50} $(=R_{d50})$. Further details are found in Thevanayagam (1998) and Thevanayagam et al. (2001, 2002).



b=contact contribution factor (by fine grains); e=global void ratio; FC=fine grains content % by dry weight; FC_{th}=threshhold fine grains content, $FC_{th}<(100e/e_{max,HF})\%$; FC_L=limit fines content, $FC_L>100(1-\pi(1+e)/(6s^3))\%>FC_{th}$; m: reinforcement factor; R_d=D/d=particle size disparity ratio; $s=1+a/R_d$, a~10; $e_{max,HF}$: the maximum void ratio of host fine grain soil

Figure I Intergranular soil mix classification

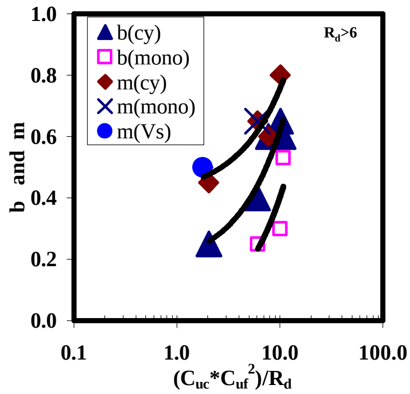


Figure II b and m versus $C_{uc}C_{uf}^2/R_{d50}$

(cy=cyclic shear; mono=monotonic shear, Vs= shear wave velocity)