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Optimal ground motion intensity measures for assessment of seismic slope displacements

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ABSTRACT: Correlating seismically induced permanent displacements to parameters characterizing the intensity of the earthquake strong ground motion allows the dynamic response computation to be decoupled from the seismic hazard evaluation in a probabilistic seismic displacement analysis. Using an earthquake database of over 1400 records, seismically induced permanent displacements were calculated using a linear and an equivalent-linear coupled stick-slip generalized single degree of freedom model. Linear and nonlinear regression analyses were performed on these results to identify Intensity Measures (IMs), which may be classified as being period-dependent or period-independent, that correlate best to the computed displacements. Optimal IMs were identified based on the efficiency and sufficiency criteria. The analyses demonstrated that the optimal IM depends on the dynamic response and strength characteristics of the earth slope. It is useful to categorize slopes as stiff or ductile and as weak or strong. The benefit resulting from the use of vectors of IMs as opposed to a scalar IM for displacement prediction depends greatly on the slope properties. An example is shown that demonstrates the benefit for using Arias Intensity as the optimal IM in the case of estimating seismic displacements of stiff slopes.

1 INTRODUCTION

Performance-based design of civil engineering structures requires the identification of critical indices of damage. In the case of seismic slope stability, performance is traditionally assessed in terms of the seismically induced permanent displacement. There exist several simplified procedures (e.g. Makdisi and Seed 1978; Bray et al. 1998) for computing permanent seismic displacements, all relating the displacements with some parameter characterizing the intensity of the strong ground motion at the site of interest. For example, Makdisi and Seed (1978), use the peak ground acceleration (PGA) at the crest of the dam, while Bray et al. (1998) use the underlying rock's PGA, mean period, and significant duration of the strong ground motion. However, it is not clear that the above parameters are optimal ones among a large set of potential candidates. This study seeks to identify optimal ground motion parameters to correlate to seismically induced permanent displacements in a rigorous way.

There are two significant benefits in correlating the seismically induced permanent displacements with a parameter characterising the intensity of the strong ground motion (herein referred to as an Intensity Measure or IM). The first benefit is that if such a correlation exists with an optimal IM, then the accuracy of the prediction of seismic displacement is improved. Hence, if ground motions are selected based on the optimal IM, as opposed to another one, to serve as input motions in a dynamic response analysis, then fewer ground motions are required to achieve the desired accuracy in the displacement prediction. Additionally, the IM versus displacement correlation can be used directly in a simplified procedure to obtain a more accurate displacement prediction. The second important benefit is the decoupling of the seismic hazard computation of the site and the dynamic response evaluation of the system, thus allowing the direct use of readily available seismic hazard maps in probabilistic seismic displacement evaluations. Such a decoupling is shown in Equation 1, which computes the probability of exceeding a certain displacement value as:

$$P(D>d)=\int_{im} P(D>d\,|\,IM)\cdot\left[\sum_{i} N_i(M_{min})\cdot\int_{m}\int_{r} f(im\,|\,M,R)\cdot f(m)\cdot f(r)\cdot dmdr\right]dim \qquad (1)$$

where P(D>d) = probability of the displacement D exceeding a test value d; $N_i(M_{min})$ = the rate of earthquakes above magnitude M_{min} for the ith source, im and IM = Intensity Measure, m and M = magnitude, r and R = distance, f(x) = probability density function for the random variable X.

2 IDENTIFICATION OF OPTIMAL INTENSITY MEASURES

2.1 General

Optimal Intensity Measures (IMs) were identified through linear and non-linear regression analyses correlating the calculated seismically induced permanent displacements with various IMs. Intensity Measures were rated based on two criteria: their efficiency and sufficiency (Cornell and Luco 2001). Due to the lack of well-documented relevant case histories, the seismically induced permanent displacements were simulated by means of two idealised slope models. The first, proposed by Rathje and Bray (1999), is a coupled, stick-slip, linear generalized single degree of freedom model (at a specified level of equivalent viscous material damping = 10%). The model is characterized by its stiffness and geometry, as represented by the initial, elastic fundamental period (T_s), and by its strength, as represented by the yield acceleration (k_y). Both were parametrically varied in the simulations obtaining the discrete values of T_s = 0, 0.3, 0.5, 0.7, 1.0, and 2.0 seconds, and k_y = 0.05, 0.1, and 0.2. The second model is a coupled, stick-slip, equivalent-linear generalized single degree of freedom model (Rathje and Bray 2000). It has the added feature of strain compatible shear modulus and material damping values that were calculated in an iterative procedure using the Vucetic and Dobry (1991) PI=30 shear modulus reduction and damping versus strain relationships.

A ground motion database comprising 1447 horizontal components recorded during 45 worldwide earthquakes in active plate margins was used as excitation to the idealized slope models. Only records from earthquakes with moment magnitudes $5.5 \le M \le 7.6$ recorded at rupture distances $R \le 100$ km on sites classified as B (rock), C (weathered, soft rock / shallow stiff soil), or D (deep stiff Holocene or Pleistocene soil) following the Bray and Rodriguez-Marek (1997) classification scheme were used. Moreover, the records had a high pass filter hp ≥ 0.25 Hz, and a low pass filter lp ≤ 10 Hz. Records on soft clay or liquefied sites were excluded from the analyses.

Linear and nonlinear regression analyses (Equations 2 and 3) were performed relating the seismically induced permanent displacements individually with a comprehensive list of IMs, the most important of which are shown in Table 1, with these relationships:

$$ln(D)=a+b\cdot ln(IM)+c\cdot\left(ln(IM)\right)^2+\varepsilon \qquad (2)$$

$$ln(D)=a+b\cdot ln(ln(IM)+c)+\varepsilon \qquad (3)$$

where D = seismically induced permanent displacements; IM = Intensity Measure; a, b, c = coefficients determined by the regression; and ε = normally distributed random error with zero mean and standard deviation σ.

The two equations result in very similar curves, with the exception of near the limits of the data. Hence, Equation 2 was chosen as the basis of comparison of the different IMs, because of its computational simplicity. However, it should not be used as a displacement prediction tool, because it cannot be reliably extrapolated to high levels of intensity of ground motion.

Table 1. Definition of a subset of the principal Intensity Measures used in this study

IM	Name	Definition	Units
PGA	Peak Ground Acceleration	$\max_t(\lvert a(t)\rvert)$	g
SA	Spectral Acceleration	$SA(T_s)$	g
a_{rms}	Root Mean Square Acceleration	$\sqrt{\frac{1}{D_{595}}\int_{T_1}^{T_2}[a(t)]^2 dt}$	g
PGV	Peak Ground Velocity	$\max_t(\lvert v(t)\rvert)$	cm/s
EPV	Effective Peak Velocity (ATC 1978)	$SV(T_s \approx 1)/2.5$	cm/s
I_a	Arias Intensity, (Arias 1970)	$\frac{\pi}{2g}\int_0^{\infty}[a(t)]^2 dt$	cm/s
CAV	Cumulative Absolute Velocity	$\int_0^{\infty}\lvert a(t)\rvert dt$	cm/s
PGD	Peak Ground Displacement	$\max_t(\lvert d(t)\rvert)$	cm
SI	Response Spectrum Intensity, (Housner 1959)	$\int_{0.1}^{2.5} PSV(\xi = 0.05, T_s)dT$	cm
I_c	Characteristic Intensity (Ang 1990)	$a_{rms}^{1.5}(D_{595})^{0.5}$	$cm^{1.5}s^{-2.5}$
T_p	Predominant Period	$T(\max(SA))$	s
D_{595}	Significant Duration, (Trifunac & Brady 1975)	$t(0.95I_a) - t(0.05I_a)$	s

2.2 **Efficiency and sufficiency evaluation**

An efficient Intensity Measure is one that results in a relatively small variability of displacements given the IM (Cornell and Luco 2001). This property can be quantified by the standard deviation of the random error of the regression of seismic displacement as a function of the IM. Figure 1 compares the different IMs shown in Table 1 with respect to their efficiency by plotting the standard deviation of the random error of the regression described in Equation 2 for four slopes with different fundamental periods and yield accelerations. The highest efficiency IM depends primarily on the slope stiffness and strength, and secondarily on the type of model used. The latter primarily affects the optimal IM in the case of ductile rather than stiff slopes.

For a stiff, weak slope, Arias Intensity, Characteristic Intensity (I_c), and RMS acceleration (a_{rms}) are the most efficient period-independent IMs (i.e. they are not functions of the fundamental period of the slope), and this finding is applicable to both the linear and equivalent-linear models. Spectral acceleration at the initial elastic period and at a degraded period selected to be twice the initial elastic period are the best period-dependent IMs to be used for the linear and equivalent-linear model, respectively. The value of the degraded period ($2T_s$) was selected based on an average earthquake-induced peak shear strain in the slope of 0.1% to 2%. For slopes having higher fundamental periods, effective peak velocity (EPV), and response spectrum intensity (SI) are the most efficient period-independent IMs for the linear model, while the same measures together with peak ground velocity

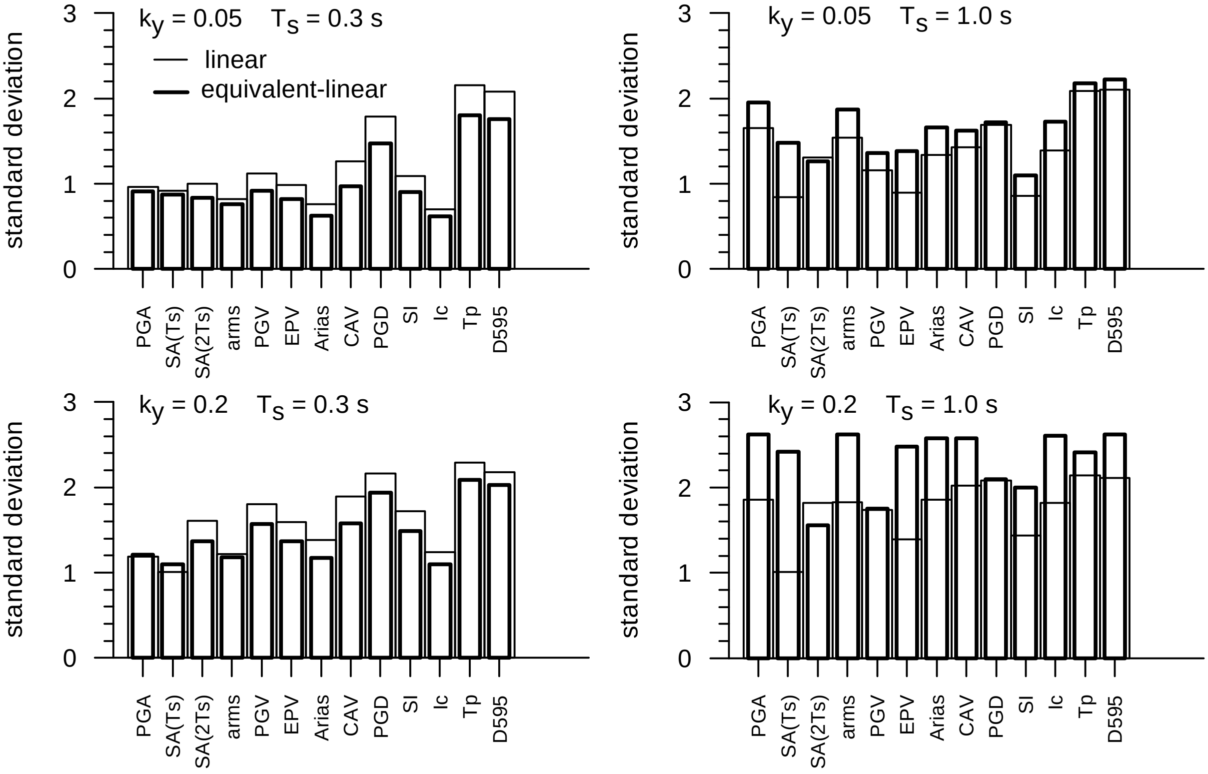

Figure 1. Standard deviation of the random error in Equation 2 using different IMs. Four different combinations of the stiffness and strength of the slope are shown.

(PGV) are the best for the equivalent-linear model. Among the period-dependent IMs, spectral acceleration at the initial fundamental period of the slope ($SA(T_s)$) is best for the linear model, while a degraded period ($SA(2T_s)$) is best for the equivalent-linear model.

In general, the standard deviation increases with increasing slope ductility and strength. Moreover, for stiff systems, the dispersion in the predicted displacements is smaller for an equivalent-linear model than a linear model. For stiff systems, the period degradation is not that significant, and therefore, it can act as an averaging effect on the displacements. Conversely, for more ductile slopes, the period degradation due to the different ground motions may result in systems with very different behaviours. Hence, the dispersion in the displacement prediction for the linear case is smaller compared to the equivalent-linear. IMs with units of time (i.e. T_p, D_{595}), are not good predictors of seismic displacements when used alone. Lastly, it was found that the random error in most of the above correlation is heteroskedastic, and in fact, decreases with increasing level of intensity.

A sufficient IM is one that renders the seismically induced permanent displacements independent of the earthquake magnitude and distance to the site given the IM (Cornell and Luco 2001). This notion of sufficiency can be extended to other variables as well, such as the fault mechanism, site category, and near-fault forward-directivity effects. In this study, the sufficiency of the most efficient IMs was examined with respect to the five variables already mentioned. To check sufficiency with respect to magnitude and distance these two variables were added to the regression equation by means of linear terms with respect to magnitude and to the logarithm of distance, as shown in Equation 4.

$$ln(D) = a + b \cdot ln(IM) + c \cdot (ln(IM))^2 + d \cdot M + e \cdot ln(R) + \varepsilon \qquad (4)$$

where D = seismically-induced permanent displacements; IM = Intensity Measure; M = moment magnitude; R = rupture distance; ε = normally distributed random error with zero mean and standard deviation σ; and a, b, c, d, and e = coefficients determined by the regression. A zero value of the coefficient d and e (at a certain statistical significance) would indicate sufficiency of the IM with respect to magnitude and distance respectively. A different approach was taken to check for sufficiency with respect to the fault mechanism, soil category and near-fault effects. The means of subsets of residuals from the regression described in Equation 2 corresponding to different categories were compared. Equal means at a certain statistical level (here selected to be 0.01) signify sufficiency of the IM, while unequal means would indicate that the IM is not sufficient.

The most important finding from the investigation of sufficiency is that it is a property significantly affected by the dynamic response and strength characteristics of the slope and by the type of model used. As a consequence, none of the IMs satisfies the sufficiency criterion with respect to the above parameters in a rigorous way for all possible values of the slope properties. It was shown by Shome (1999) that the error in the prediction of the damage measure using a hazard decoupling assumption with an insufficient IM can be as small as ± 10%. Moreover, in environments where the seismic hazard is often controlled by a single magnitude event, such as in the San Francisco Bay Area, sufficiency of the selected IM becomes a secondary issue.

2.3 **Vectors of intensity measures**

There are two compelling reasons for wanting to correlate the seismically induced permanent displacements to a vector as opposed to a scalar IM. The first reason is when the efficiency in the displacement versus IM correlation substantially increases due to the use of an additional IM. The second reason is when the random variable that we seek to regress on is strongly jointly dependent on two predictor variables, so it would be a mistake to consider only one (Bazzuro, 1998). In the case of the linear model, the spectral acceleration at the initial elastic fundamental period of the slope ($SA(T_s)$) was considered as the first component of the vector; whereas for the equivalent-linear case the spectral acceleration at a degraded period equal to twice the initial elastic fundamental period was considered as the first component of the vector. These IMs were selected based on their overall efficiency. The adequacy of the remaining IMs to constitute the second component of the vector was quantified based on the increase of the R-square statistic when a vector as opposed to a scalar was used as a regressor. The functional form used for the regression on a vector of IMs {SA, IM_2) is described as:

$$ln(D) = a + b \cdot ln(SA) + c \cdot \left(ln(SA)\right)^2 + d \cdot ln(IM_2) + \varepsilon \tag{5}$$

where D = seismically-induced permanent displacements; SA = spectral acceleration at the initial or twice the initial elastic fundamental period for the linear and equivalent-linear case respectively; IM_2 second Intensity Measure; ε = normally distributed random error with zero mean and standard deviation σ; and a, b, c, and d = coefficients determined by the regression. Equation (5) was selected after observing that the residuals of the regression on the spectral acceleration only had a linear dependence on the logarithm of the remaining IMs when plotted against them.

The relative increase in the R-square statistic when a vector of IMs as opposed to a scalar IM is used, is plotted in Figure 2 for the different IMs shown in Table 1 and for four different combinations of the stiffness and strength characteristics of the slope. There is a significant improvement in the efficiency resulting by the addition of a second IM for weaker and stiffer slopes. This is no longer true for more ductile and stronger slopes where a vector of IMs does not significantly improve efficiency. It also appears that the linear model benefits more by the use of a vector, especially in the case of a stiff and weak system. Overall, it is not obvious whether there is an optimal IM that should be used as a second component to a vector of IMs. This decision rather depends on the model type and slope properties.

To perform a probabilistic seismic displacement assessment using a vector of IMs, it is essential that the seismic hazard be given for this vector. This is not a trivial task, and this information is not readily available. Hence, the engineer should decide on a project basis whether the improvement in efficiency by the use of a vector IM offsets the complications in the vector seismic hazard evaluation.

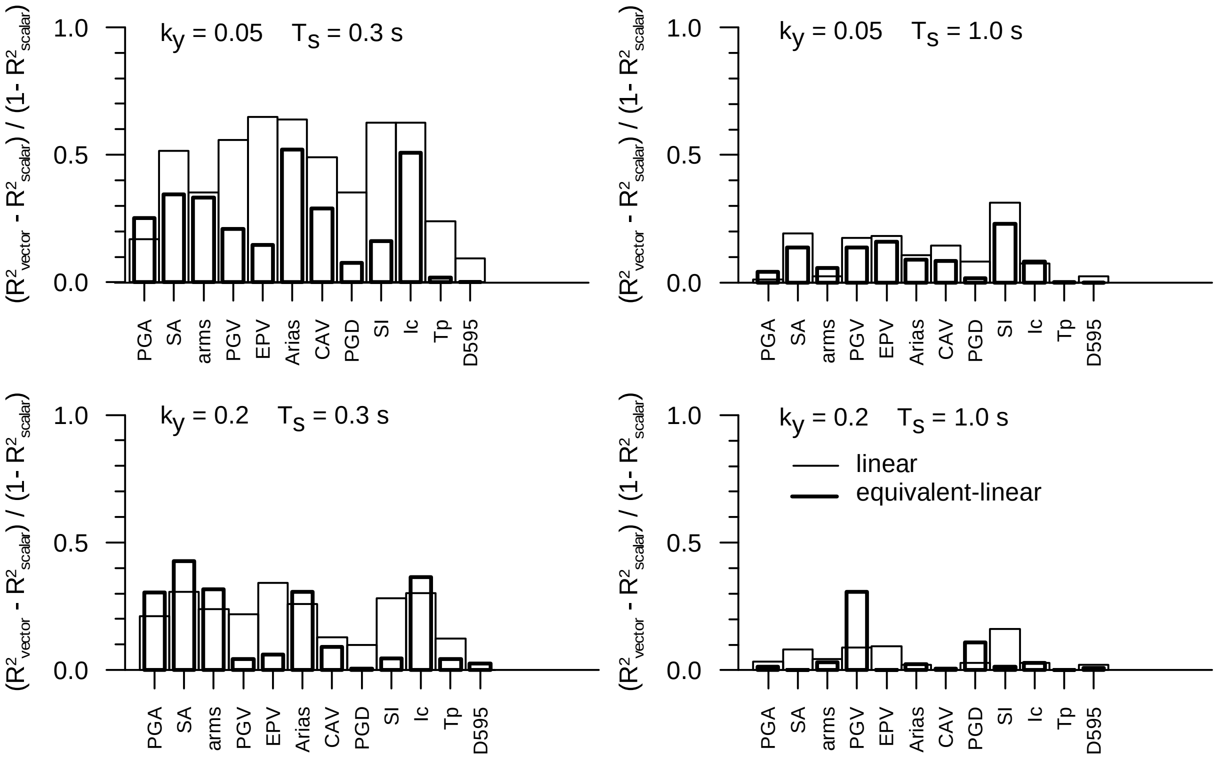

Figure 2. Relative improvement of the R-square statistic through the use of vector of IMs. The vector is $\{SA(T_s), IM_2\}$ and $\{ SA(2T_s), IM_2\}$ for the linear and the equivalent-linear model respectively. In this figure, SA represents $SA(2T_s)$ and $SA(T_s)$ for the linear and equivalent-linear model, respectively.

3 PROBABILISTIC DISPLACEMENT COMPUTATION

As an illustrative example, the probability distribution function for the seismically induced permanent displacements is computed through the use of three different IMs for the linear and equivalent-linear models. For this example, a deterministic earthquake scenario represented by a strike-slip magnitude 7.5 earthquake at 10 km from the site on a site class D was selected. The structure of interest is an earth slope having an initial elastic fundamental period of T_s = 0.3 seconds and a yield acceleration k_y = 0.05. The probability density function for the displacements is given by Equation 6.

$$f(d\,|\,event) = \int_{\varepsilon} f(d\,|\,IM_{median},\varepsilon)\cdot f(\varepsilon)d\varepsilon \qquad (6)$$

where f(x) = probability density function for the random variable X; d = seismically induced permanent displacements; IM_{median} = median value of the intensity measure given an earthquake scenario; ε = number of standard deviations in the prediction of the IM.

It is assumed that f(ε) follows the standard normal distribution and that the probability density function of the displacement given the value of the IM follows the lognormal distribution given by Equation 7.

$$f(d\,|\,IM_{median},\varepsilon) = \frac{1}{\sqrt{2\pi}\cdot\sigma_d}\cdot\exp\left(-0.5\cdot\left(\frac{\ln(d)-\ln(\hat{d})}{\sigma_d}\right)^2\right) \qquad (7)$$

where σ_d = standard deviation in the correlation of displacements versus an IM; $\hat{d}$ = the median displacement computed by the displacement versus IM correlation.

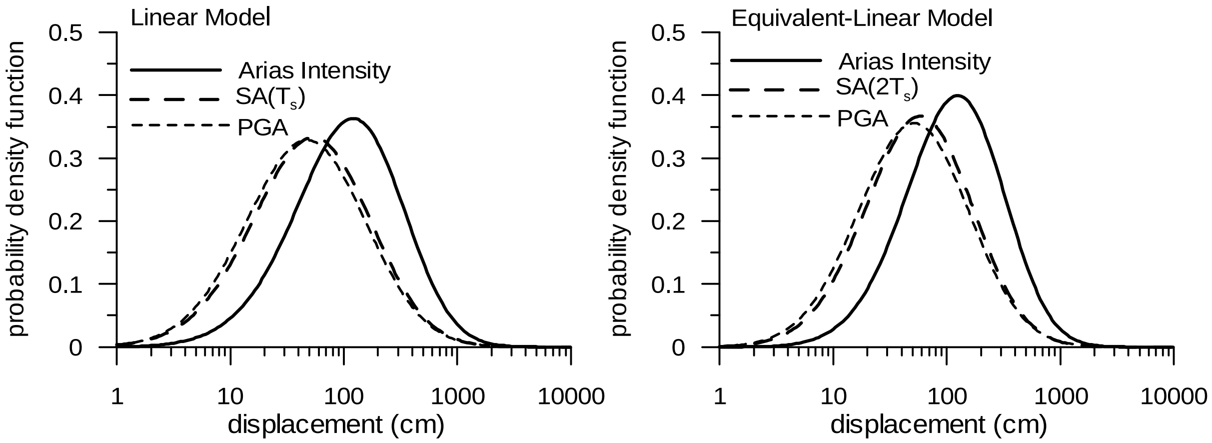

Figure 3. Displacement probability density function as computed using different IMs for a magnitude 7.5 strike-slip earthquake at 10 km for a class D site. The earth structures has $T_s = 0.3$ s and $k_y = 0.05$.

Figure 3 shows the displacement probability density function as computed using three different intermediate IMs for the linear and equivalent-linear models. For the linear model, the three IMs used are: Arias Intensity, spectral acceleration at the initial elastic period, and peak ground acceleration; whereas, for the equivalent-linear model, the IMs used are: Arias Intensity, spectral acceleration at a degraded period of twice the initial elastic period, and peak ground acceleration. The high efficiency of Arias Intensity for stiff slopes offsets the large dispersion in its estimation given a specified earthquake magnitude, site-to-source distance, fault type, and soil class (Travasarou et al. 2003), resulting in a smaller dispersion in the displacement distribution compared to that predicted by the other two IMs. However, the median displacement values for this earthquake scenario predicted using different IMs are appreciably different. This is attributed to the different databases used to develop the relevant ground motion attenuation relationships and constitutes a limitation of these comparisons.

4 CONCLUSIONS

Linear and nonlinear regression analyses were performed to correlate the seismically induced permanent displacements computed from the response of a linear and an equivalent-linear coupled generalized single degree of freedom system to several IMs characterizing the intensity of the strong ground motion at a site due to an earthquake event. The analyses were carried out with the intention to identify optimal IMs to correlate to the seismically induced permanent displacements. The identification of such measures has two benefits: (1) selection of earthquake records based on them allows the engineer to reduce the number of structural response analyses to achieve the same accuracy in the displacement prediction, and (2) it allows the probabilistic seismic displacement evaluation to be performed in two steps, the first being the probabilistic seismic hazard analysis, and the second, the dynamic response analysis of the structure.

The parameters examined were rated based on their ability to satisfy the criteria for efficiency and sufficiency. It was found that the Spectral Acceleration at the slope elastic fundamental period (SA) is the best overall IM when the slope is modelled as linear. The spectral acceleration at a degraded period of twice the initial fundamental period is the best overall IM when an equivalent- linear model is used. For applications where the period of the slope cannot be accurately estimated, or for more generic applications, such as regional studies of slope stability, the use of period-independent measures, such as Arias Intensity (for stiff systems with $T_s \leq 0.7$ s) or Response Spectrum Intensity (SI) for more ductile cases ($T_s > 0.7$ s), is recommended for the linear model. For the equivalent-linear model, Arias Intensity remains a good IM in the case of stiff slopes, but for more ductile slopes, where the standard deviation in the correlation significantly increases, there does not appear to be an optimal period-independent IM. Judging from the dispersion in the seismic displacements computed for the two models, the use of a relatively simple linear model to represent an earth slope is less important for stiff slopes, but for slopes with larger fundamental periods, a more sophisticated equivalent-linear or nonlinear slope model is more appropriate.

Although none of the IMs examined is completely sufficient for all cases of slope properties, the accuracy in the prediction of slope displacements when a probabilistic approach is used is not undermined significantly. This is particularly true for cases where the seismic hazard is dominated by a narrow range of earthquake magnitudes, which is often the case in highly seismic regions.

The use of a vector of IMs, which has as its first component the spectral acceleration at the elastic fundamental period for a linear model, or the spectral acceleration at twice the elastic fundamental period for an equivalent-linear model and as its second component an IM that better characterizes the longer period range (i.e. EPV, SI, PGV, SA($2T_s$)) for the linear model and the short period range for the equivalent-linear model, largely improves the efficiency in the displacement versus IM correlation. However, the engineer should decide whether this improvement in efficiency outweighs the complications in the vector seismic hazard calculation.

Finally, prediction of the seismic displacement through different IMs can lead to more accurate predictions when the optimal IM is used, such as Arias Intensity for the case of stiff slopes. However, there exists the problem of non-centering of the predicted median values due to the different ground motion databases used by researchers to develop attenuation relationships for the different IMs.

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