2003 Pacific Conference on
Earthquake Engineering

Factors bounding prograde Rayleigh-wave particle motion in a soft-soil layer

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ABSTRACT: The ellipticity of particle orbits is calculated for Rayleigh waves travelling in a soil layer lying upon a rock half-space, for a selection of different combinations of Poisson ratio in the soil, and shear wave impedance contrast between the soil and the rock. If either or both the Poisson ratio in the soil, and shear wave impedance contrast between the soil and the rock, are high, there is a range of frequencies for which the particle orbit becomes prograde. In transitions between prograde and retrograde motion the Horizontal-to-Vertical Spectral Ratio (HVSR) of the waves becomes in turn very large then very small with increasing frequency. Assuming that microtremors are a result of passing Rayleigh waves, it follows that any site with an HVSR that has the typical structure of a high peak followed by a low trough, must be soft, must trap energy by reflection, and will be particularly prone to amplify earthquake shaking.

1 INTRODUCTION

When recordings of microtremors are analysed using Nakamura’s method (Nakamura 1989), the Horizontal-to-Vertical Spectral Ratio (HVSR) plots for certain soft sites have the characteristic appearance of a peak, with a height of more than five, followed by a trough with a height of less than one, at a higher frequency, as exemplified by the solid line of figure 1. This appearance is seldom commented on, but can be related to the theory of Rayleigh waves propagating in a layer. According to Haskell (1953), in the course of changing from retrograde to prograde particle orbits as frequency increases, the particle motion theoretically becomes a horizontal line, and in changing from prograde to retrograde particle orbits at a higher frequency, the motion theoretically becomes a vertical line. These transitions would result in an infinite, and a zero HVSR respectively. It has therefore been suggested (Lachet & Bard 1994, Lermo & Chavez-Garcia 1994, Konno & Ohmachi, 1998) that the mechanism which governs Nakamura’s method is the propagation of Rayleigh waves, contrary to the original view (Nakamura 1989) that any Rayleigh wave contribution detracts from the method.

Accepting this interpretation of the origin of the peak/trough structure of many HVSR plots, it becomes important to investigate the geotechnical circumstances under which prograde Rayleigh wave orbits (and therefore an HVSR peak and trough structure) may arise.

2 THE GENERIC MODEL

A Rayleigh wave travelling along the interface between a homogeneous half space and the empty space above it has a particle motion which is elliptical in a plane containing vertical and the propagation direction (Bullen & Bolt 1985). In contrast to water waves, the particle motion is always retrograde, that is, if a Rayleigh wave is travelling from left to right the particle motion describes an anticlockwise ellipse. However when the Rayleigh wave travels in a more flexible layer on top of the half space, there may be a range of frequencies for which the particle motion is prograde, that is, if a wave is travelling from left to right the particle motion describes a clockwise ellipse. In changing between prograde and retrograde the motion must at some frequency become a straight line (which is neither retrograde nor prograde). It sometimes happens (Haskell 1953) that for one transition the straight line is vertical, and for the other it is horizontal. If the motion is completely horizontal at some frequency, the HVSR will be infinite at that frequency, and if the motion is completely vertical at some other frequency, the HVSR will be zero at that frequency. It was shown (Lachet & Bard 1994) that the peak of a Nakamura's ratio plot corresponds to the resonant frequency of the layer. Further modelling (Lermo & Chavez-Garcia 1994) showed the theoretical existence of a trough as well, but the significance of the trough was not remarked upon. Subsequently the peak/trough structure was noted (Konno & Ohmachi 1998), with the comment that the frequency ratio between trough and peak is approximately two, and that the frequency of the trough could be used as an additional control on the natural frequency of the layer. It was also asserted (Konno & Ohmachi 1998) that some layer/half space stratigraphies can result in a trough/trough structure. Such a result has also been encountered in the investigations described here, but it is a relatively rare occurrence.

The dashed line of figure 1 shows how the HVSR predicted by a simple one-layer model has a close correspondence to the actual frequencies at which the HVSR peak and trough occur.

The very existence of sharp resonant peaks in some HVSR plots is suggestive of an abrupt change in shear wave velocity at some depth, and therefore a suitable tactic to start investigations must lie in exploring the properties of a simple homogeneous layer lying on top of a stiffer half space, even though such simple circumstances must seldom occur in nature. Accordingly, a uniform layer upon a half space, as shown in figure 2, is the model adopted for this work. In addition it is useful to adopt likely values for the layer and half space properties. A typical soft site has a superficial layer of saturated soil, which has a high p-wave velocity and a low s-wave velocity (Poisson ratio ν approaching 0.5). By contrast rock is closer to being a Poisson solid, (which has Lamé constants λ and μ equal, and therefore with Poisson's ratio $\nu = 0.25$ and p/s velocity ratio = v3).

In studies of waves propagating in layers (Ewing 1957) it is usual to adopt a nomenclature similar to that in figure 2, where p-wave velocities are denoted by α, and s-wave velocities by β. Layer properties have the subscript 1 and half space properties have the subscript 2.

This study seeks combinations of α_1/β_1 and β_2/β_1 which allow prograde motion over some frequency range. The ratio α_1/β_1 is related to the Poisson ratio and the ratio β_2/β_1 is related to the s-wave impedance ratio. It is convenient to work in terms of α_1/β_1 and β_2/β_1 because velocities are parameters which are often measured directly at a site, rather than being derived quantities.

3 METHOD AND RESULTS

The process of evaluating Rayleigh wave propagation in layered media is a routine computational matter these days. One useful set of codes (Herrmann 1984) is driver-file oriented and runs on UNIX platforms, so it is relatively easy to call the routines from within some other program. In the present case that other program is a simplex search program (Nelder 1965). In this exercise a range of the ratio β_2/β_1 was specified, and for each ratio the simplex algorithm was used to locate the corresponding value of α_1/β_1 for the onset of prograde motion. In each instance of the evaluation of a model the layer thickness was taken to be 30m, its shear wave velocity 50m/s, and its density 1.2 tonne/m^3. The half space was taken to be a Poisson solid, with density 1.8 tonne/m^3. Repeated use of the simplex algorithm defined the black line shown in figure 3 as the boundary between situations where prograde motion can, and cannot, occur. Between the black line and the blue line, the transitions between prograde and retrograde motion both involve an occurrence of vertical motion (Konno & Ohmachi 1998) and therefore a double trough structure in the HVSR plot. The small size of the region between the heavy line and the blue line of figure 3 implies that a double trough structure should only be expected on rare occasions. Above, and to the right of the blue line, a peak/trough structure is expected.

In a single-layer situation, the black line shown in figure 3 is crucial. It shows that if the value of α_1/β_1 is less than 1.68 ($\nu < 0.23$) or if the value of β_2/β_1 is less than 2.09 there can be no prograde motion.

However at sites where the peak/trough structure is seen, it is usual for the ratio of trough frequency to peak frequency to be approximately 2 (Konno & Ohmachi 1998). It is therefore of interest to establish the conditions under which a trough/peak frequency ratio of two can occur, using an extension of the simplex method previously used to establish the conditions for which prograde motion can occur. The results are incorporated in figure 3 as a red line. It is evident that if the value of α_1/β_1 is less than 2.65 ($\nu < 0.42$), or if the velocity ratio β_2/β_1 is less than 3.9, a trough/peak frequency ratio of two or greater is impossible, and that field measurements of microtremors are likely either not to have a peak/trough structure, or to have a trough/peak frequency ratio of about 2. This is in agreement with observations (Konno & Ohmachi 1998). In addition the present study indicated that trough/peak frequency ratios of more than about 2.3 required unrealistic ratios of α_1/β_1 and/or β_2/β_1.

4 DISCUSSION

If the Rayleigh-wave-orbit basis of explaining microtremor HVSR is correct, the curve for a trough/peak frequency ratio of two becomes extremely relevant to the interpretation of HVSR values for microtremors. The previous analysis shows that sites with trough/peak frequency ratios of around two are guaranteed to have soils with high Poisson ratio and high impedance contrast to the substrate. They must therefore support Rayleigh wave propagation, and must be effective at trapping energy carried by elastic waves. Such sites will amplify ground motion, and will be associated with greater shaking damage than other sites. In part any excess damage will be a result of extended shaking duration, in turn caused as surface waves reverberate within the layer, travelling at relatively low speeds.

Even though the theoretical Rayleigh wave particle orbits become first horizontal, then vertical as frequency increases, a typical microtremor recording will contain energy other than that carried by Rayleigh waves. There is a possibility of system noise, a possibility of Love waves, and a high likelihood of body waves at any site, so that the HVSR maximum and minimum will reflect the relative amount of this extraneous noise, together with the extent of any spectral smoothing which has been employed. Accordingly, the peak height and trough depth will have no meaning other than quantifying the proportion of signal due to Rayleigh waves at the site in question. It would not be expected to correlate with the amount of site amplification.

Experience (to date the Institute of Geological and Nuclear Sciences has accumulated microtremor recordings at 418 soft sites) shows that the peak/trough structure may be relatively uncommon. Some areas of soft soil, such as south Auckland, may contain 20% of sites with the peak/trough structure, while others such as Wanganui and Suva have as few as 2% of sites with it. One obvious possible reason why few sites have the peak/trough characteristic is that few sites meet both the high α_1/β_1 and high β_2/β_1 criteria. However it is also possible that although many sites actually meet both the high α_1/β_1 criterion and the high β_2/β_1 criterion, Rayleigh waves for some reason are not present, so the microtremor HVSR does not have the peak/trough characteristic.

Temporarily accepting the scenario that a peak/trough structure is only present at sites with high α_1/β_1 and high β_2/β_1, we see that a new view of microzoning appears, which holds that extreme site-related amplification is relatively rare, but significant in its impact. According to this view, any HVSR with a peak/trough structure identifies a site which will experience extreme resonant amplification. Such a view is backed by the presentation (Seekins *et al.* 1996) of a peak/trough structure for a site near the Embarcadero Freeway which collapsed during the 1989 Loma Prieta earthquake, but not at other, less damaged, soft soil sites. Another example is the reported overthrow of trees in Wainuiomata, New Zealand, during the June 1942 earthquake (Perrin 2001), at a site where a peak/trough HVSR has been noted (Singh *et al.* 1998).

HVSR plots with a peak/trough structure are not an exclusively New Zealand phenomenon, although their incidence in other countries could be influenced by details of instrumentation or data processing. For instance, too much spectral smoothing could disguise the phenomenon. Good examples of peak/trough structure are provided at site F of the JSKA array in Kushiro City, Hokkaido, Japan (Zhao *et al.* 1997), at site ACAZ in Acapulco, Mexico (Lermo & Chavez-Garcia 1994), in the Po Valley of Italy (Mucciarelli 1998), at Baguio site BG2 in the Phillipines (Ohmachi & Nakamura 1992), and at the Embarcadero Freeway in San Francisco (Seekins *et al.* 1996).

Although it is proposed that the peak/trough structure should be viewed as diagnostic of a site prone to great increases in shaking-related damage, the converse does not necessarily hold true. If microtremors had a significant component of Love waves or body waves, the trough depth would be reduced or the trough made entirely absent. Again, if higher mode Rayleigh waves were present to any extent, they could similarly modify the trough. This type of difficulty applies in the case of Parkway valley, Wainuiomata, New Zealand, where the bimodal HVSR curves shown in figure 4 presumably allow any trough structure to be nullified.

Given that a peak/trough structure is characteristic of a small proportion of soft sites, and that those sites are particularly likely to be associated with great damage, there emerges the idea that current site classification schemes may be open for revision on a basis that there is a small class of soft sites which tend to amplify more than other soft sites. Adopting such an approach would not merely add another ground class, but would reduce design loadings for some existing classes on the basis that they have previously been over-assessed by incorporating the new class. In a nutshell, it is proposed that most sites classified as class D are not as bad as believed, but a small proportion of them are worse than believed.

This paper has emphasised the role of transitions of particle motion between prograde and retrograde, and the way that such transitions are reflected in HVSR plots. However it is also possible for sites with no such transitions to adopt very flat particle orbits and hence to have substantial HVSR peaks. Not surprisingly, the modelling for this study showed that high HVSR peaks arising in this way appear only to occur for sites having a high s-wave velocity contrast between the layer and the substrate. However the height of the peak in a “peak only” situation appears to be determined by both the impedance contrast and the Poisson’s ratio. Illustrating this point, figure 5 shows the way in which the peak height in a “peak only” situation varies with impedance contrast for two values of Poisson’s ratio. It follows that in a “peak only” situation it would be surprising if the HVSR peak height, however filtered, could reflect site amplification, unless all sites had nearly identical Poisson’s ratios.

A possible example of a “peak only” HVSR associated with large amplification is seen at site SCT1 in Mexico City (Lermo & Chavez-Garcia 1994). SCT1 is clearly resonant on the basis of standard spectral ratios, but has an HVSR plot with no distinct minimum even though the values for frequencies above the peak are all much less than unity.

5 CONCLUSIONS

- The presence of a peak/trough structure in a microtremor-derived HVSR is an indicator of extreme site amplification.
- The height of a peak in a microtremor-derived HVSR does not necessarily correlate with the amount of site amplification.
- The absence of a peak in a microtremor-derived HVSR does not necessarily denote a lack of site amplification

6 ACKNOWLEGEMENTS

The work described here was funded by the New Zealand Foundation for Research, Science and Technology. The results owe a lot to the views and tools of Paco Chávez-García of UNAM’s Institute of Engineering, and discussions with Cinna Lomnitz of UNAM’s Institute of Geophysics.

REFERENCES:

Bullen, K.E.; Bolt, B.A. 1985. An introduction to the theory of seismology. Cambridge University Press.

Ewing, M., W. Jardetzky and F. Press 1957: Elastic Waves in Layered Media. New York, McGraw-Hill Inc.

Haskell, N.A. 1953. The dispersion of surface waves on multilayered media. *Bulletin of the Seismological Society of America 43*: 17-34.

Herrmann, R.B. 1984. Computer programs in earthquake seismology; Vol I-Vol VIII. *56th annual meeting, Eastern Section of the Seismological Society of America 55*: 13.

Konno, K.; Ohmachi, T. 1998. Ground-motion characteristics estimated from spectral ratio between horizontal and vertical components of microtremor. *Bulletin of the Seismological Society of America 87*: 228-241.

Lachet, C.; Bard, P.-Y. 1994. Numerical and theoretical investigations on the possibilities and limitations of Nakamura's technique. *Journal of Physics of the Earth 42*: 377-397.

Lermo, J.; Chavez-Garcia, F.J. 1994. Are microtremors useful in site response evaluation? *Bulletin of the Seismological Society of America 84*: 1350-1364.

Mucciarelli, M. 1998. Reliability and applicability of Nakamura's technique using microtremors: an experimental approach. *Journal of Earthquake Engineering 2*: 625-638.

Nakamura, Y. 1989. A method for dynamic characteristics estimation of subsurface using microtremor on the ground surface. *QR of RTR 30*: 25-33.

Nelder, J.A., and Mead, R. 1965. A Simplex Method for Function Minimization. *Comput. J. 7*: 308-313.

Ohmachi, T.; Nakamura, Y. 1992. Local site effects detected by microtremor measurements on the damage due to the 1990 Philippine earthquake *In* Tenth World Conference on Earthquake Engineering, Madrid, Spain, 1992. *Vol. 2*, Pp. 997-1002.

Perrin, N.D. 2001. Pers. Comm.

Seekins, L.C.; Wennerberg, L.G.; Margheriti, L.; Liu, H.-P. 1996. Site amplification at five locations in San Francisco, California; a comparison of S waves, codas, and microtremors. *Bulletin of the Seismological Society of America 86*: 627-635.

Singh, A., Stephenson, W.R., and Hull, A. 1998. Assessment for amplification of earthquake shaking by soft soils in Suva. 0250-7234, Mineral Resources Department, Suva, Fiji.

Zhao, B.; Horike, M.; Takeuchi, Y.; Kawase, H. 1997. Comparison of site-specific response characteristics inferred from seismic motions and microtremors. *Zisin = Jishin 50*: 67-87.

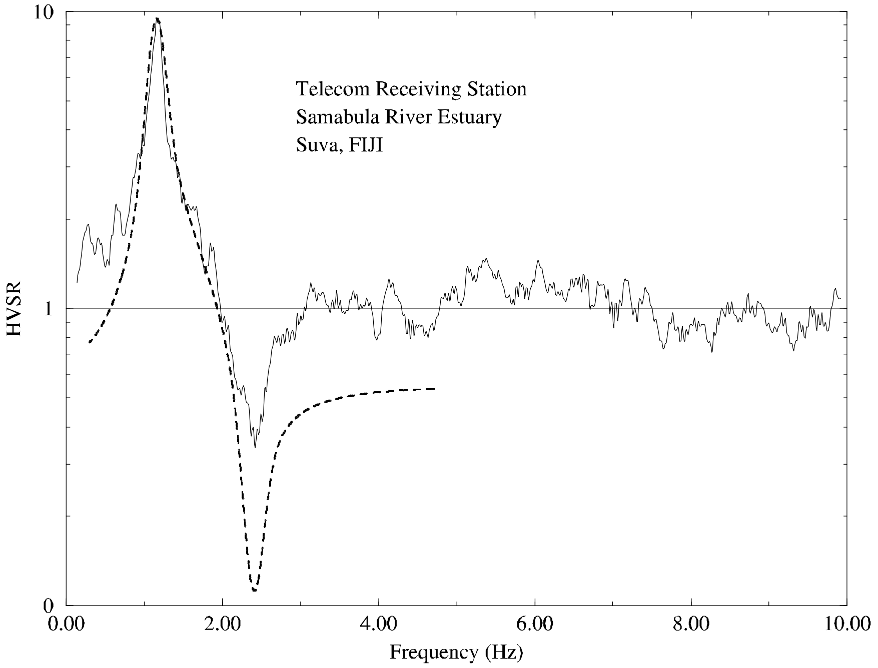

Figure 1. Solid line – HVSR derived from microtremors recorded at a soft soil site. Dashed line – HVSR derived from the theoretical particle orbit of a fundamental mode Rayleigh wave travelling through a site which has 21.3m of soil with a shear wave velocity of 100m/s, over a half space of rock with a shear wave velocity of 2.7km/s.

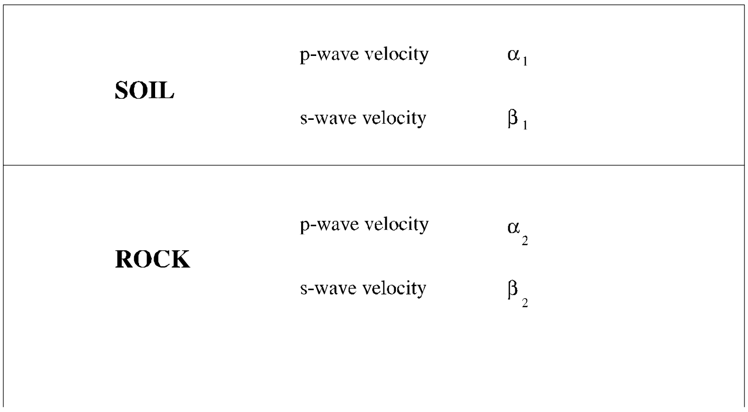

Figure 2. Nomenclature used to describe a soil layer upon a rock half space.

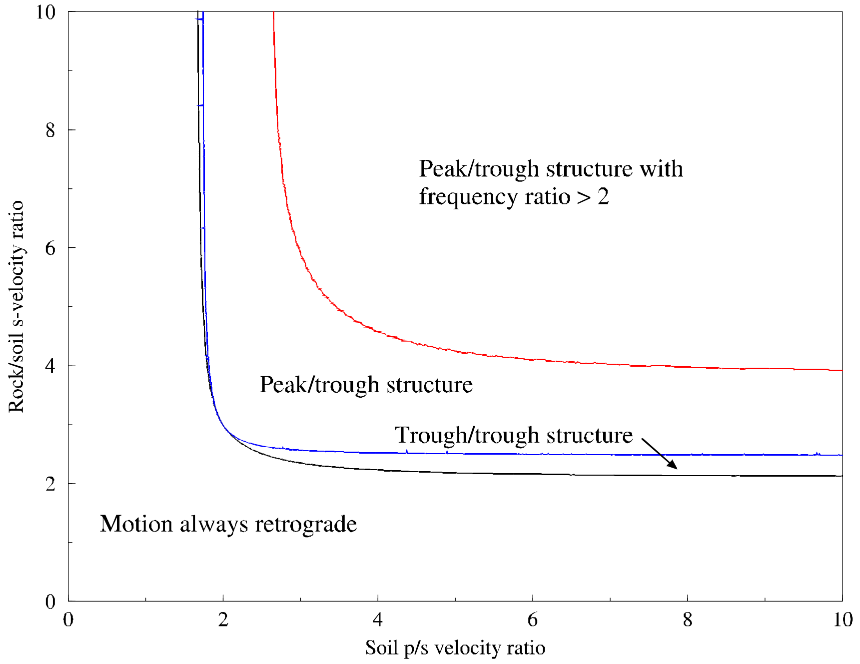

Figure 3. Transitions between prograde and retrograde particle motion for a fundamental mode Rayleigh wave travelling in a soil layer upon a rock half space. Left of, and below the black line there cannot be prograde motion. Right of, and above the red line, the HVSR should have a peak/trough structure, with the ratio of trough frequency to peak frequency being greater than two. There is a small region (between the black line and the blue line) for which a trough/trough structure may be expected.

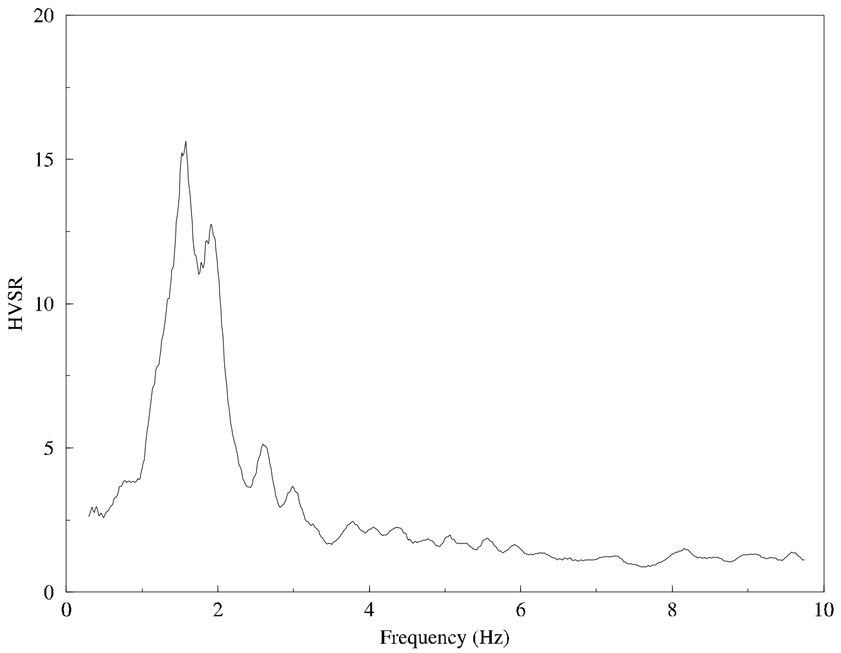

Figure 4. HVSR plot for microtremors recorded in Parkway valley, New Zealand. The two peaks (of height 15.6 and 12.7) reduce the chance that a peak/trough structure may occur, because the skirt of one peak is likely to fill in the trough associated with the other peak. Before such concealment these troughs would have heights of less than one.

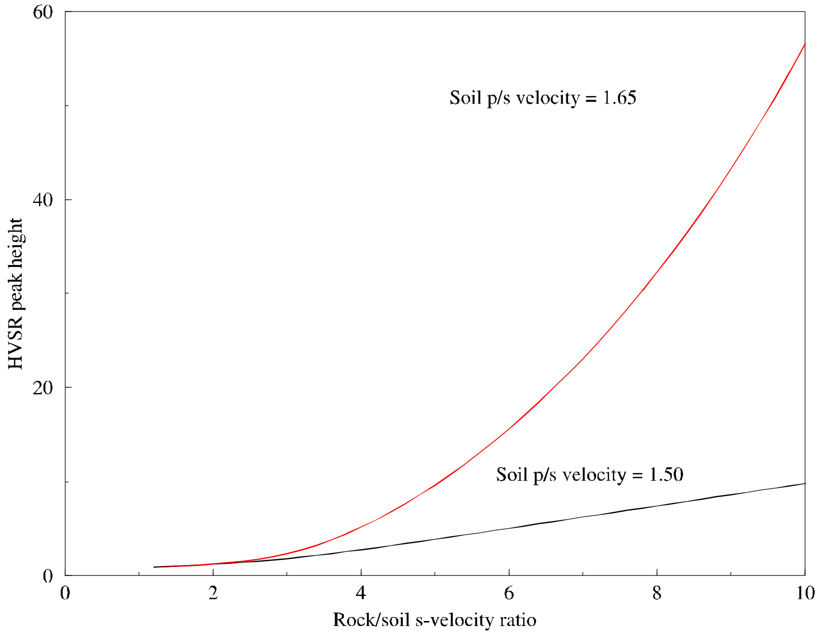

Figure 5. The expected height of an HVSR peak due to fundamental mode Rayleigh wave propagation, in two cases where no peak/trough structure is expected. The two cases have different ratios of p-wave velocity to s-wave velocity in the soil layer; red 1.65; black 1.50. It is clear that an HVSR peak derived from microtremors (presumed to be Rayleigh waves) will have a height that is sensitive to factors other than site amplification. In this case the peak height is sensitive to Poisson ratio.