



## A simple method for stochastic dispersion of earthquake waves

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**ABSTRACT:** The analyses of the dynamic response of extended structures to asynchronous earthquake input motions need more than one input motion. The recently developed theoretical framework of conditional random field simulation of ground motion is too complex to be used by engineers. In this paper, a simple method to generate ground motions with dispersion from original seismic wave is proposed. The proposed method is based on the two assumptions that in time domain the components of discretized space-time random field  $\{f_{t_1}(x), f_{t_2}(x), \dots, f_{t_n}(x)\}$ , are mutually uncorrelated, and only the correlation of the predominant frequency of the earthquake is considered for the frequency dependent spatial correlation function of the ground motion field. With the aid of these assumptions, the modified Kriging method for multi-variate Gaussian random fields can be easily performed in the time domain. A description of an algorithm for the simulation of ground motions at  $M$  locations conditioned by the recorded time-histories from  $n$  locations is presented. A numerical example to generate the input ground motions for several bridge support points from one specified time-history is given and some results are presented.

### 1 INTRODUCTION

Monte Carlo simulation is a method widely used in engineering to solve complicated problems that cannot be treated effectively by purely analytical tools (Shinozuka and Deodatis 1991). In earthquake engineering, stochastic approaches are often used to simulate seismic ground motions. Recently, conditional simulation of random processes and fields has been studied in connection with its application to urban earthquake monitoring. The conditional nature of the simulation stems from the fact that the realizations of the random processes or fields have been recorded at only some locations. One needs to simulate the full random field from the recorded information given. So far, the conditional simulation can be carried out by using either the Kriging method (Kriging 1966, Journel and Huijbregts 1978) or the conditional probability density function (CPDF) method (Kameda and Morikawa 1994).

Kriging methodology has been developed by many researchers mainly in geostatistics, which provides the best linear unbiased estimate built on data of a stochastic field. Vanmarcke and his co-workers directly applied the Kriging method to conditional simulation problems in earthquake engineering (Vanmarcke and Fenton 1991, Vanmarcke et al. 1993). Hoshiya and his co-worker modified the Kriging method and used the modified version for conditional simulation in relation to earthquake engineering applications (Hoshiya 1994, Hoshiya and Marugama 1994, Hoshiya 1995). The modification by Hoshiya and his co-worker represents a significant improvement that has made the Kriging method theoretically much cleaner and computationally more efficient (Shinozuka and Zhang 1996).

The conditional probability density function (CPDF) method was developed and applied in earthquake engineering by Kameda and Morikawa. This method takes advantage of the ease with which the conditional probability density function can be analytically derived for Gaussian variables, and presents (mathematically) a more straightforward method.

Although the theoretical framework of conditional random fields has been established, its use by the earthquake engineering community is viewed as impractical due to its complexity and the difficulty of

estimating the spatial correlation of fields. In this paper, a simple method for conditional simulation of random field is proposed, which is intended to be used for engineering purposes to study the effect of spatial variation of seismic input motions on the responses of extended structures. In this method two assumptions are adopted, which in time domain the components of discretized space-time random field  $\{f_{t_1}(x), f_{t_2}(x), \dots, f_{t_n}(x)\}$ , are mutually uncorrelated, and only the correlation of the predominant frequency of the earthquake is considered for the frequency dependent spatial correlation function of the ground motion field. With the aid of these assumptions, the modified Kriging method could be easily used to conditionally simulate ground motions in the time domain. Finally, the proposed method is applied to generate the input ground motions for several bridge support points when the time-history is specified for one support point

## 2 CONDITIONAL SIMULATION OF MULTI-VARIATE GAUSSIAN FIELDS (Ren et al.1995)

Assume that  $\mathbf{F}(x) = [F_1(x), F_2(x), \dots, F_n(x)]^T$  is a homogeneous n-variate Gaussian random vector field with zero-mean and cross-covariances  $R[F_k(x_i), F_l(x_j)] = E[F_k(x_i)F_l(x_j)]$  ( $k, l = 1, 2, \dots, n$ );  $\mathbf{f}(x_i)$  ( $i = 1, 2, \dots, N$ ) is a set of realizations of the vector field  $\mathbf{F}(x)$  at locations  $x_i$ . Following Hoshiya's technique, the actual field  $\mathbf{F}(x)$  could be represented by its simulated counterpart  $\mathbf{F}^S(x)$ , i.e.  $\mathbf{F}(x) = \mathbf{F}^S(x)$ . In component form, they are

$$\begin{aligned} F_l^s(x) &= F_l(x) \\ &= F_l^e(x) + [F_l(x) - F_l^e(x)] \\ &= F_l^e(x) + \mathbf{e}_l(x) \end{aligned} \quad (1)$$

where  $F_l^e(x)$  is the Kriging estimate of  $l$ th component  $F_l(x)$  of the multi-variate random field  $\mathbf{F}(x)$ , and has the following expression

$$F_l^e(x) = \sum_{i=1}^N \sum_{k=1}^n \mathbf{I}_{ikl}(x) F_k(x_i) \quad (l = 1, 2, \dots, n) \quad (2)$$

where  $\mathbf{I}_{ikl}$  are Kriging weights.  $\mathbf{e}_l(x)$  in equation (1) is the error between the exact field  $F_l(x)$  and its Kriging estimate  $F_l^e(x)$

$$\mathbf{e}_l(x) = F_l(x) - F_l^e(x) \quad (3)$$

In order for equation (2) to be the best linear unbiased estimator, it is required that the variance of the error  $\mathbf{e}_l(x)$  attains a minimum. At any unrecorded location  $x_r$  the variance of the error is of the form

$$\begin{aligned} \text{Var}[\mathbf{e}_l(x)] &= E \left\{ \left[ F_l(x_r) - F_l^e(x_r) \right]^2 \right\} \\ &= \text{Var}[F_l(x_r)] - 2 \sum_{i=1}^N \sum_{k=1}^n \mathbf{I}_{ikl}(x_r) R[F_k(x_i), F_l(x_r)] \\ &\quad + \sum_{i=1}^N \sum_{j=1}^N \sum_{k=1}^n \sum_{m=1}^n \mathbf{I}_{ikl}(x_r) \mathbf{I}_{jml}(x_r) R[F_k(x_i), F_m(x_j)] \end{aligned} \quad (4)$$

Minimizing equation (4) with respect to Kriging weights  $\mathbf{I}_{ikl}(x_r)$ , the following set of equations are obtained:

$$\sum_{j=1}^N \sum_{m=1}^n \mathbf{I}_{jml}(x_r) R[F_k(x_i), F_m(x_j)] = R[F_l(x_r), F_k(x_i)] \quad (i=1, 2, \dots, N; k=1, 2, \dots, n) \quad (5)$$

Equation (5) consists of  $n \times N$  equations, from which  $n \times N$  unknown Kriging weights  $\mathbf{I}_{ikl}(x_r)$  for the component  $f_l(x)$  at location  $x_r$  can be determined.

Using equation (5) it could be found that the error vector  $\mathbf{e}(x_r) = [\mathbf{e}_1(x_r), \mathbf{e}_2(x_r), \dots, \mathbf{e}_n(x_r)]^T$  of  $\mathbf{F}(x)$  at an unrecorded location  $x_r$  is independent of the random vector  $F(x_i)$  as well as the error vector  $\mathbf{e}(x_i)$  at recorded location  $x_i$  ( $i=1, 2, \dots, N$ ). In addition, it is uncorrelated with the Kriging estimate  $F_l^e(x_r)$  ( $l=1, 2, \dots, n$ ) at unrecorded location  $x_r$ . However, different error components are correlated with each other. The correlation function is given by

$$\begin{aligned} E[\mathbf{e}_l(x_r) \mathbf{e}_k(x_r)] &= E\left\{ \mathbf{e}_l(x_r) [F_k(x_r) - F_k^e(x_r)] \right\} \\ &= E[\mathbf{e}_l(x_r) F_k(x_r)] \\ &= R[F_k(x_r), F_l(x_r)] - \sum_{j=1}^N \sum_{m=1}^n \mathbf{I}_{jml}(x_r) R[F_k(x_r), F_m(x_j)] \end{aligned} \quad (6)$$

The above important properties of  $\mathbf{e}(x_r)$  guarantee that the error vector  $\mathbf{e}(x_r)$  could be simulated separately from the Kriging estimate. Hence, to simulate the Gaussian random vector field  $\mathbf{F}(x)$  at the desired location  $x_r$ , under the condition of known realizations  $f(x_i)$  ( $i=1, 2, \dots, N$ ) at the recorded locations, one can calculate the Kriging estimate of each component  $f_l^e(x_r)$  ( $l=1, 2, \dots, n$ ) and simulate the error vector  $\mathbf{e}(x_r)$  separately, and formulate their sum

$$F_l^e(x_r) = \sum_{i=1}^N \sum_{k=1}^n \mathbf{I}_{ikl}(x_r) f_k(x_i) + \mathbf{e}_l(x_r) \quad (7)$$

In the particular case that components of  $F(x)$  are mutually uncorrelated, namely  $R[F_k(x_i), F_m(x_j)] = 0$  for  $k \neq m$ , equation (5) reduces to, by letting  $\mathbf{I}_{jl}(x_r) = \mathbf{I}_{jll}(x_r)$  for simplicity

$$\sum_{j=1}^N \mathbf{I}_{jl}(x_r) R[F_l(x_i), F_l(x_j)] = R[F_l(x_r), F_l(x_i)] \quad (i=1, 2, \dots, N) \quad (8)$$

Equation (8) has a unique set of solution  $\mathbf{I}_{jl}(x_r)$  unless the covariance matrix  $R[F_l(x_i), F_l(x_j)]$  of the  $l$ th component  $F_l(x)$  is singular. Equation (2) becomes,

$$F_l^e(x_r) = \sum_{i=1}^N \mathbf{I}_{il}(x_r) F_l(x_i) \quad (9)$$

This implies that for the case of uncorrelated component random field each component  $F_l(x)$  of  $F(x)$  can be simulated separately, as in the case of uni-variate random field.

### 3 SIMULATIONS OF GROUND MOTIONS

The two effects that give rise to the spatial variability of seismic ground motions, i.e. the ‘incoherence effect’ and the ‘wave passage effect’, were considered separately in the proposed method to generate seismic ground motions with dispersion from the original motions. First, only the ‘incoherence effect’

was concerned, the ground motions were assumed to be a space-time random field  $f(t, x)$  that might be approximated by its discretized version  $\{f_{t_1}(x), f_{t_2}(x), \dots, f_{t_n}(x)\}$  with  $f_{t_i}(x)$  denoting  $f(t_i, x)$ . Further more, it was assumed that  $\{f_{t_1}(x), f_{t_2}(x), \dots, f_{t_n}(x)\}$  was a homogeneous n-variate Gaussian random vector field with zero-mean and cross-covariances  $R[f_{t_k}(x_i), f_{t_l}(x_j)] = E[f_{t_k}(x_i)f_{t_l}(x_j)]$  ( $k, l = 1, 2, \dots, n$ ); Thus the problem now became one to simulate a homogeneous n-variate Gaussian vector field. For reasons of simplicity, assume that in the time domain the components of the discretized space-time random field were mutually uncorrelated, i.e.,  $R[f_{t_k}(x_i), f_{t_m}(x_j)] = 0$  for  $k \neq m$ . As mentioned before, for a multi-variate random vector field with uncorrelated components, each component  $f_{t_i}(x)$  could be simulated separately, as in the case of a uni-variate random field. The wave passage effect on the simulated motions was then included by introducing a time lag between the different points. The time lag  $\tau$  between any two supports was given by  $\tau = r_{ij}/v_a$ , where  $v_a$  was the apparent wave propagation velocity and  $r_{ij}$  was the separation between the two supports projected parallel to dominant wave propagation direction. A description of an algorithm for the simulation of ground motions at  $M$  locations conditioned by the recorded time histories from  $n$  points was presented below:

(1) At the first time step  $t_1 = \Delta t$ :

The acceleration values  $f_{t_l}$  ( $l = 1, 2, \dots, n$ ) for  $n$  supports were known, thus the conditional simulation values  $f_{t_m}^c$  ( $m = n+1, n+2, \dots, M$ ) at other sites could be obtained by means of the following steps:

(i) Found the estimate of the Kriging values  $f_{t_m}^e$  ( $m = n+1, n+2, \dots, M$ ) on the basis of the known values  $f_{t_l}$  ( $l = 1, 2, \dots, n$ ) by using equation :

$$f_{t_m}^e = \sum_{l=1}^n I_{lm} f_{t_l}$$

where  $l$  was a number of supports at which the ground motions are known, and  $I_{lm}$  were Kriging weights, which could be obtained from equation (8).

(ii) Simulated the error  $e_{t_m}^s$  ( $m = n+1, n+2, \dots, M$ ), which was a multi-variate normal distribution random variety with zero means. The variance matrix for stochastic variates  $E_m$  was:

$$\mathbf{V} = \begin{bmatrix} E[E_{(n+1)(n+1)}^2] & E[E_{n+1}E_{n+2}] & \cdots & E[E_{n+1}E_M] \\ E[E_{n+2}E_{n+1}] & E[E_{(n+2)(n+2)}^2] & \cdots & E[E_{n+2}E_M] \\ \vdots & \vdots & \ddots & \vdots \\ E[E_M E_{n+1}] & E[E_M E_{n+2}] & \cdots & E[E_{MM}^2] \end{bmatrix}$$

where  $E[E_n E_m]$  could be obtained from equation (6).

Unconditionally simulated the errors  $e_{t_m}^s$  using the Cholesky decomposition method on the basis of the above variance matrix (Dagpunar 1988).

(iii) The conditionally simulated values were obtained as

$$f_{t_m}^c = f_{t_m}^e + e_{t_m}^s \quad (m = n+1, n+2, \dots, M)$$

- (2) Repeated the steps (i), (ii), (iii) for time steps  $t_2 = 2\Delta t$ ,  $t_3 = 3\Delta t$ ,  $\dots$ ,  $t_N = N\Delta t$  respectively.
- (3) Put  $f_{t_m}$  ( $i = 1, 2, \dots, N$ ) together to form the total conditionally simulated accelerogram for support  $m$ , ( $m = n+1, n+2, \dots, M$ ).

Then the acceleration values were shifted by time-delay parameters  $\Delta T_{nm}$  at support  $m$ , ( $m = n+1, n+2, \dots, M$ ).

#### 4 AUTOCORRELATION FUNCTION OF A RANDOM FIELD

The autocorrelation function of an isotropic, zero-mean uni-variate random field is

$$R[F(x_i), F(x_j)] = E[F(x_i)F(x_j)] = E[F(x_i)F(x_j + r_{ij})] = R_{ij}(r_{ij}) \quad (10)$$

where  $r_{ij}$  is the distance between the two points  $i, j$ . The spatial variability of the seismic motions is, generally, obtained from the time domain analyses of the recorded data, which are assumed to be stationary, and is usually described by a function that decays exponentially with separation distance and frequency. Hence the autocorrelation function  $R_{ij}(r_{ij})$  adopted here is the negative exponential form [Zerva and Shinozuka 1991]

$$R_{ij}(r_{ij}) = \mathbf{s}^2 \exp\left(-\frac{|r_{ij}|}{b}\right) \quad (11)$$

where  $\mathbf{s}$  is a standard deviation of the field and  $b$  indicates a correlation length,  $b > 0$ . Assuming the ergodicity of the ground motion, the value of  $\mathbf{s}$  can be calculated from the formula for standard deviation of a history record with zero mean at a given point:

$$\mathbf{s} = \left[ \frac{1}{N-1} \sum_{i=1}^N f_i^2 \right]^{1/2} \quad (12)$$

where  $N$  is the number of values in the record. The spatial correlation function is frequency-dependent and makes simulation of the conditional ground motion relatively troublesome when the simulation is performed in time domain. Since the band of frequencies which dominates the response of engineering structures like highway elevated bridges or multi-supported pipelines is narrow, one spatial correlation function is assumed to represent the correlation for the band of frequencies of interest (Jankowski and Wilde 2000). Moreover, the functional dependence of the coherency function on distance and frequency has not been fully established. The coefficient  $b$  may be described in terms of the predominant frequency of earthquake,  $\mathbf{w}_d$ , and the mean apparent seismic wave velocity,  $v$ , (Jankowski and Wilde 2000, Vanmarcke and Fenton 1991)

$$b = \frac{2pvd}{\mathbf{w}_d} \quad (13)$$

where  $d$  is a dispersion factor,  $d > 0$ , which depends on local geological and topographical conditions. The bigger the value of  $d$  the higher is the correlation expected between points of the random field. The predominant frequency of the earthquake,  $\mathbf{w}_d$ , can be determined from the acceleration spectrum of the earthquake record. Substituting equation (13) into equation (11) yields

$$R_{ij}(r_{ij}) = \mathbf{s}^2 \exp\left(-\frac{\mathbf{w}_d |r_{ij}|}{2pvd}\right) \quad (14)$$

## 5 EXAMPLE OF SIMULATION OF GROUND MOTION FIELD

The proposed method was applied to generate the input ground motions for several bridge support points with the specified time-history for one support point. The bridge under consideration is shown in Figure 1. The North-South component of El Centro (May 18, 1940) earthquake record was used as the specified time-history at abutment 1. The predominant frequency used for this earthquake wave was 11 rad/sec. The seismic wave was assumed to travel along the bridge in the longitudinal direction. The generated time-histories with a propagation velocity  $v = 200$  m/s and dispersion factor  $d=10$  for the support points of piers 2, 4, 6, 8, and abutment 10, together with the difference between the generated time-histories and the specified one are presented in Figure 2. The spectral accelerations of the generated time-histories and the original one are shown in Figure 3. As expected the generated time-histories look quite similar to the specified one, and their spectra are also very close each other.

Several other cases were run with different travelling wave velocities of  $v = 100, 400, 1000, 2000$  m/s, and different dispersion factors  $d = 1, 100$ . The differences between the simulated time-histories and the specified earthquake record varied with the separation, the dispersion factor and the traveling wave velocity as shown in Figures 4, 5 and 6. In these figures, the variations of the acceleration differences with separation distance, traveling velocity and dispersion factor  $d$  show the expected exponential decay that arises from the rule adopted for the simulation described previously.

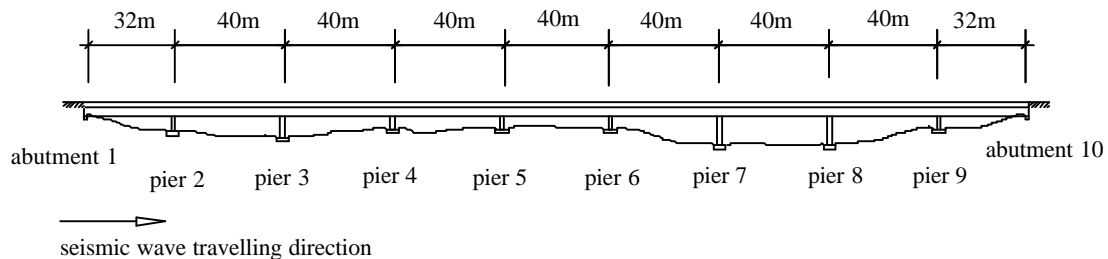


Figure 1 Bridge Elevation

## 6 CONCLUSIONS

A simple method for stochastic dispersion of earthquake wave was proposed in this paper, by using the modified Kriging method for multi-variate Gaussian fields. The proposed method was based on the two assumptions that the components of discretized space-time random field  $\{f_{t_1}(x), f_{t_2}(x), \dots, f_{t_n}(x)\}$  were mutually uncorrelated, and only the correlation of the predominant frequency of the earthquake was considered for the frequency dependent spatial correlation function of the ground motion field. The numerical example showed that the method was simple and effective. The results were reasonable because the spectra of simulated time-histories and the specified earthquake record were very close to each other. The variation of the accelerations with the separation and the propagation velocity followed the rule adopted. This method could be easily used for engineering purposes.

## ACKNOWLEDGEMENT

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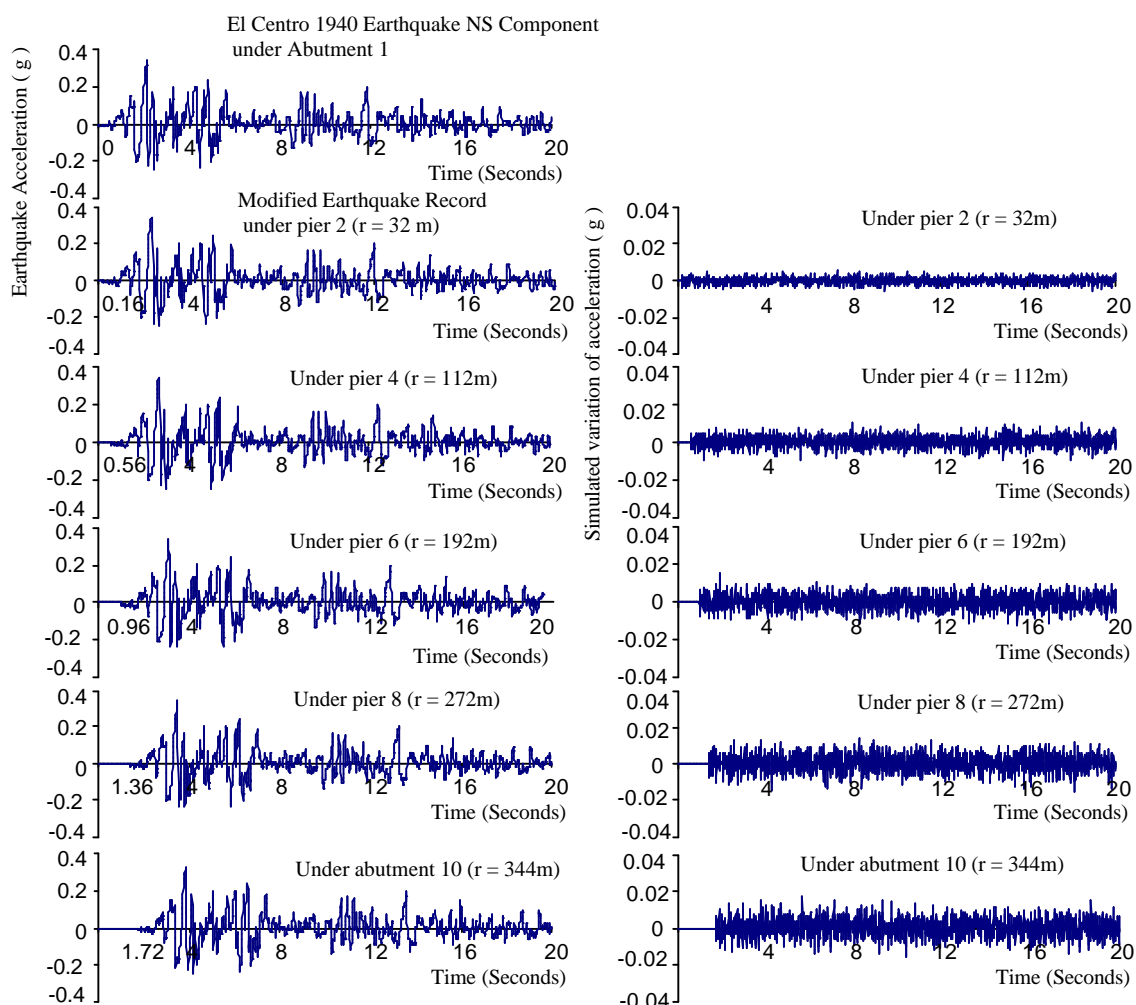


Figure 2 The specified time-history (top left), the simulated time-histories (left), and the differences from the specified input at abutment 1 (right)

Shinozuka, M., and G. Deodatio. 1991. "Simulation of Stochastic Processes by Spectral Representation." *Applied Mechanics Reviews*, Vol. 44(No. 4), 191-204.

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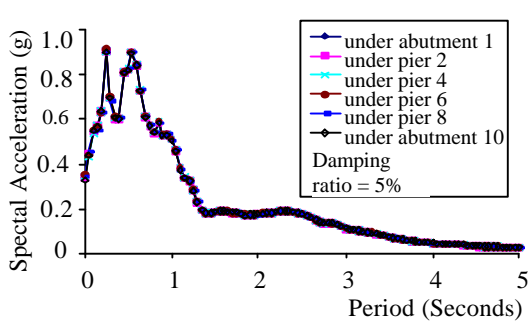


Figure 3 The Spectra of Time-histories

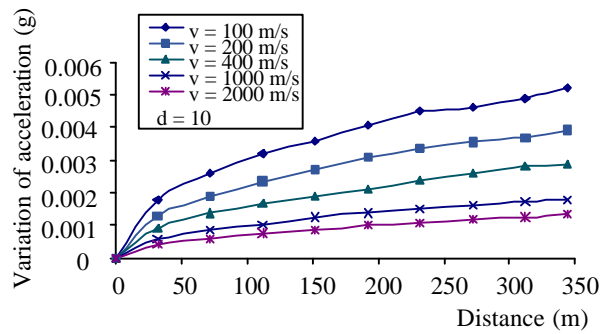


Figure 4 The variation of accelerations with the distance

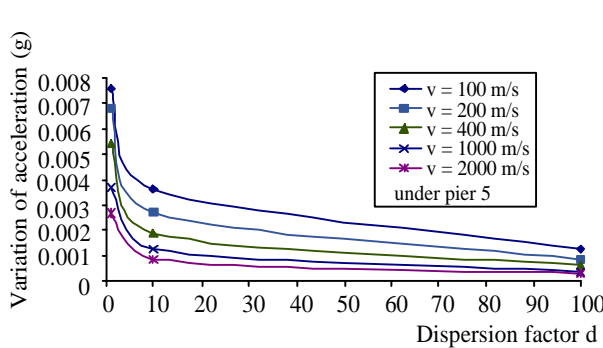


Figure 5 The variation of accelerations with the dispersion factor

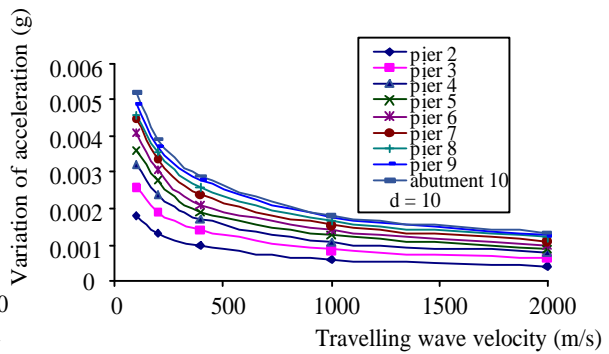


Figure 6 The variation of accelerations with propagation velocity