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# An Efficient Model for Seismic Analysis of Building Structures with The Effect of Floor Slabs

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**Abstract**: Most of the building structures consist of structural elements such as beams, columns, braces, shear walls, foundations, and floor slabs. In general, the models used for the analysis of building structures are prepared without the floor slabs assuming that they would have negligible effects on the response of a structure. Therefore, the floor slabs are simply replaced by rigid floor diaphragms for the efficiency in the analysis. Several researchers attempted to study the effects of floor slabs using finite element models with refined plate element meshes to account for the flexural stiffness of floor slabs. Since beams and floor slabs are not located in a common plane, in general, rigid bodies shall be introduced to represent the T-beam effects. Therefore, the model used in the analysis of building structures with floor slabs would have refined finite element meshes with too many degrees of freedom to be used for the practical engineering purpose. The analytical model was proposed in this study for the efficient seismic analysis of building structures considering the flexural stiffness of the floor slabs. The proposed model employs super elements, rigid diaphragms, and the substructuring technique to minimize the number of degrees of freedom to be used in the analysis. Analyses of example structures were performed to verify the efficiency and the accuracy of the proposed model in the seismic analysis of multistory building structures. The proposed model could provide seismic response of the example structures in significantly reduced computational times while the accuracy in the analysis results such as vibration periods and response time histories were very close to those obtained from the refined model.

Keywords: flexural stiffness of the floor slab, T-section beam, substructure, super element, rigid diaphragm assumption, stick model

## 1. INTRODUCTION

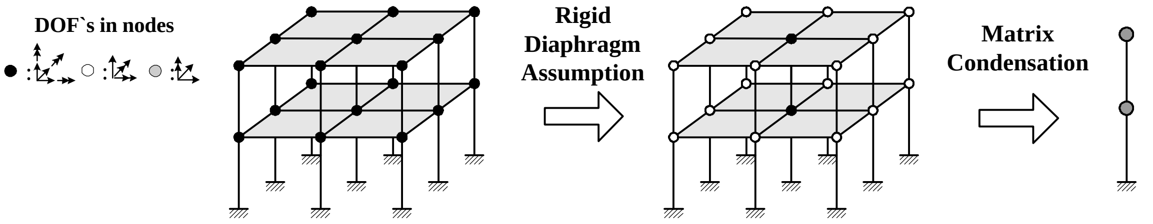
For the analysis of building structure, commercial computer programs such as ETABS have been frequently used applying the rigid diaphragm assumption for the simplicity in the analysis procedure. In this case, the flexural stiffness of the floor slabs is usually ignored in the analysis. Furthermore, even though beams are located under the floor slabs in the building structure, the analytical model was developed assuming that the axes of beams and floor slabs are located in a common plane. Therefore in dynamic analysis the analytical model, which disregard the flexural stiffness of the floor slabs and the T-beam effect would induce substantial analytical errors. The rigid links would simulate the T-beam effect most simply. Gupta and Ma [1] investigated the analytical errors caused by the rigid link elements used for considering the T-beam effect and suggested that the beam elements should be divided into several elements to minimize the errors. Between two nodal points of the beam element, Miller [2] introduced an additional beam length direction translational degree of freedom to reduce the analytical errors caused by the use of rigid link elements. Chan [3] considered T-beam effect by using the 8-node shell elements (5 degrees of freedom per node) and 3-node parabolic isoparametric beam elements. The previous researches demonstrated that the floor slabs and beams need to be divided into several elements to consider the flexural stiffness of the floor slabs and the T-beam effect. Consequentially, substantial amount of time and efforts were required to model and analyze the building structures.

In this study research on the efficient modelling techniques, which can consider the flexural stiffness of the floor slabs and the T-beam effect have been carried out. For developing the modelling techniques, the minimum number of element mesh size for beam and floor slab elements was investigated. After performing the adequate mesh size studies, an analytical model employing the substructuring technique [4,5] and a stick model with 3 degrees of freedom per story based on the rigid diaphragm assumption were proposed to reduce the required computational time of analysis. The computational time for obtaining natural frequencies, mode shapes, and dynamic response of example building structures were compared to investigate the validity of the modelling techniques proposed in this study.

## 2. ANALYTICAL MODELING OF FLOOR SLABS AND BEAMS

### 2.1 Analytical Modelling using Rigid Diaphragm Assumption and Matrix Condensation

In building structures, the flexural stiffness of the floor slabs is negligible in comparison with the in-plane stiffness of the floor slabs. When the rigid diaphragm assumption is applied for the floor slabs, two horizontal direction translation degrees of freedom and one rotation degree of freedom with respect to vertical axis are necessary to represent displacements at all nodal points at each story. Figure 1 shows the reduction of degrees of freedom when the rigid diaphragm assumption and matrix condensation are applied for floor slabs.



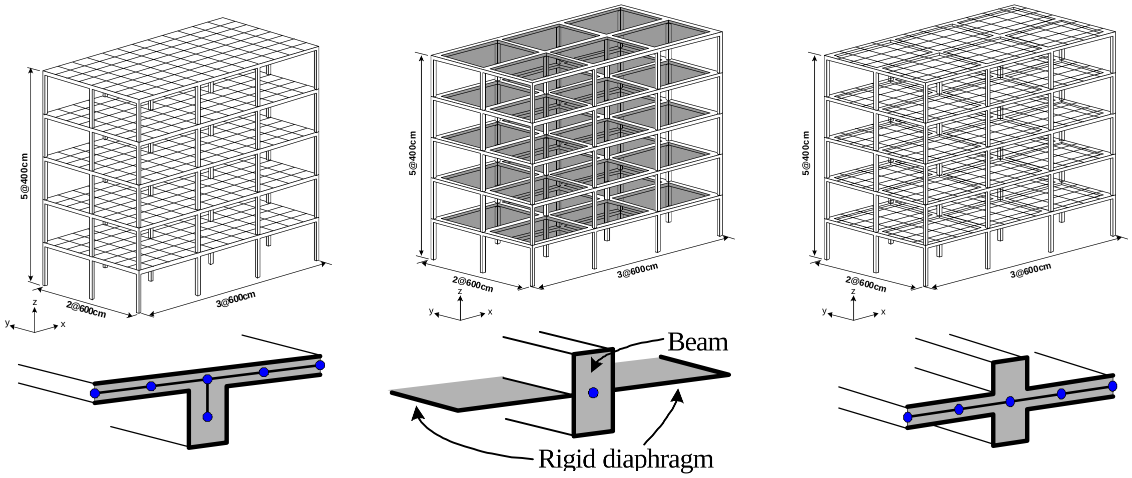
(a) General analytical model (b) Rigid diaphragm assumption model (c) Stick model

Fig. 1 Reduction of DOF`s by rigid diaphragm assumption and matrix condensation

When the rigid diaphragm assumption is applied to the general analytical model shown in Fig. 1(a), two in-plane translation degrees of freedom and one rotation degree of freedom, which is shown in Fig. 1(b), at each story can represent the structural behaviour. Furthermore, when the matrix condensation technique [6] is applied to the out of plane degrees of freedom, a stick model with 3 degrees of freedom per story as shown in Fig. 1(c), can be developed. The number of degrees of freedom per story for general analytical model, rigid diaphragm assumption model, and stick model is 54, 30, and 3 respectively. Therefore the analytical efficiency for large building structure can be improved by applying rigid diaphragm assumption and matrix condensation technique.

## 2.2 Structural Response due to Modelling Methods for Beams and Floor Slabs

To investigate the seismic behaviour of a building structure with three different modelling methods proposed in this study for beams and floor slabs, an example structure shown in Fig. 2 was selected and the eigenvalue analysis was conducted. For model A and model C, floor slabs were modelled using Lee’s plane stress element [7] and MZC rectangular element [8]. Petyt element [9] was adapted for T-beam in model A. The sizes of columns of the example structure are 60 cm x 60 cm and 45 cm x 45 cm for 1~3 stories and 4~5 stories respectively. The size of all beams is 60 cm x 40 cm and the thickness of all floor slabs is 20 cm. Model A can take the flexural stiffness of the floor slab and the T-beam effect into consideration. Model C can only take into account the flexural stiffness of the floor slab due to the assumption that the axes of beams and floor slabs are located in a common plane. To consider the effect of the floor slabs and beams more accurately for model A and model C, all floor slabs that are surrounded by beams are divided into 4 x 4 elements. Model B might be the most generally adapted model. In this model only frames are taken into account and the floor slabs are simply replaced by rigid floor diaphragms for the efficiency in the analysis. Among three analytical models, the most accurate analytical results would be obtained from model A.



(a) Model A (b) Model B (c) Model C

Fig. 2 Modeling of a beam and floor slabs

Table 1 shows the natural frequencies for each analytical model. The frequencies for model B and model C are relatively lower than those of the model A. The reason for this is Model B does not take into account the flexural stiffness of the floor slabs and T-beam effect and Model C does not take into account the T-beam effect. Therefore the lateral stiffness of model B and model C is relatively underestimated. Furthermore, from the comparison study of the natural frequencies for each analytical model, it can be observed that the T-beam effect is more influential than the flexural stiffness of the floor slab for dynamic behaviour of the structure.

Table 1 Comparison of the natural frequency (unit:Hz)

| Mode number | Model A | Model B | Model C |
|---|---|---|---|
| 1 | 1.920 | 1.404 | 1.507 |
| 2 | 1.951 | 1.461 | 1.552 |
| 3 | 2.290 | 1.794 | 1.866 |
| 4 | 4.979 | 4.035 | 4.229 |
| 5 | 5.025 | 4.142 | 4.313 |

In seismic analysis, natural frequencies of structure affect the structural response. Therefore, for accurate seismic analysis, consideration of the T-beam effect for analytical model could reduce the analytical error. However, as can be seen in table 2, the required computational time of analysis for model A and model C is longer than that of model B due to fine element meshes. These fine element divisions for large structure require longer computational time and larger computer memory capacity. Therefore an efficient analytical model, which can reduce the computational time and the computer memory capacity, need to be developed without influencing analytical results.

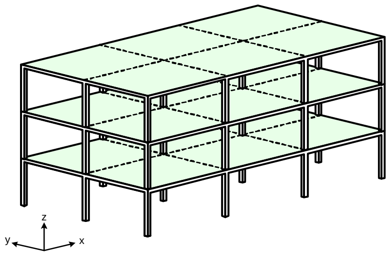
Table 2 Comparison of computation time (unit:sec)

| procedure | Model A<br>(3,510 DOF) | Model B<br>(15 DOF) | Model C<br>(3,510 DOF) |
|---|---|---|---|
| Stiffness & Mass | 62.22 | 1.63 | 61.41 |
| Eigenvalue | 20,038.69 | 0.02 | 20,037.48 |
| Time history | 77.09 | 0.12 | 77.24 |
| Total | 20,178.00 | 1.77 | 20,176.13 |

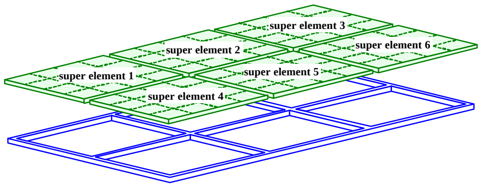
## 3. EFFICIENT SEISMIC ANALYSIS MODEL

### 3.1 Structural Modelling Employing Substructuring Technique and Super Elements

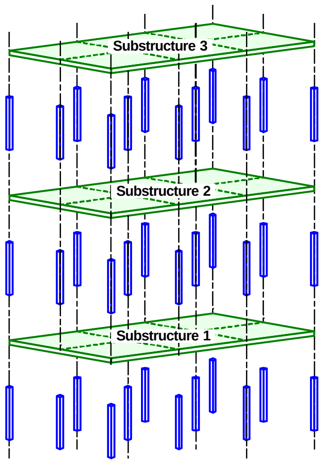
The analytical model A used in the analysis of building structures would have refined finite element meshes with too many degrees of freedom to be used for the practical engineering purpose. Therefore, analytical models were proposed in this study for the efficient seismic analysis of building structures considering the flexural stiffness of the floor slabs. The proposed models as shown in Fig. 3 employ super elements, rigid diaphragms, and the substructuring technique to minimize the number of degrees of freedom to be used in the analysis.



(a) Example building structure



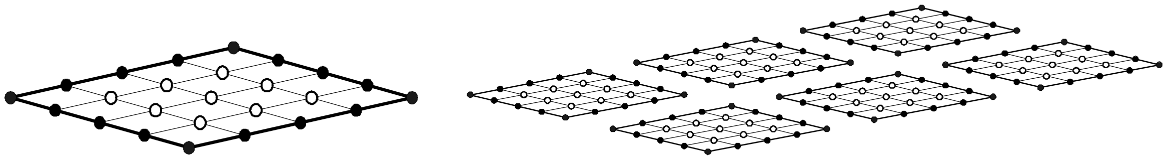
(b) Substructure modeled as a super element



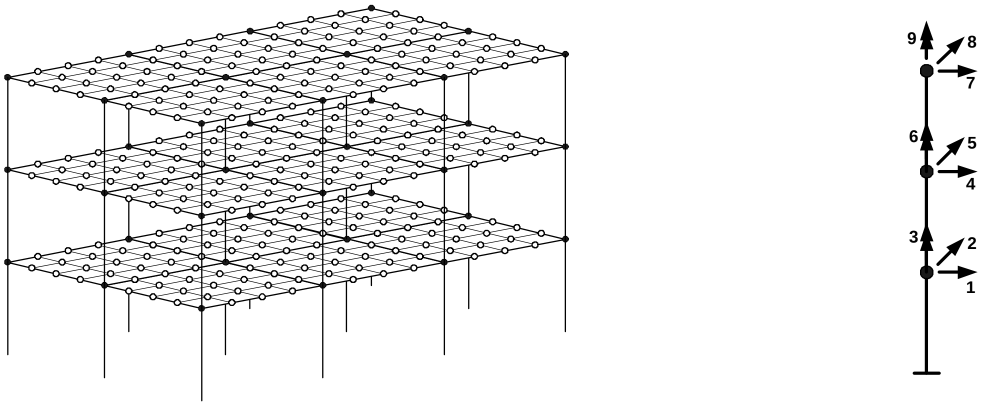
(c) Building structure divided into substructures

Fig. 3 Modeling of a beam and floor slabs

The example structure shown in Fig. 3(a) can be divided into columns, substructure 1, substructure 2, and substructure 3 as shown in Fig. 3(c) for each stories. Floor slabs that are surrounded by beams can also be divided into several super elements as shown in Fig. 3(b). For satisfying compatibility conditions at boundaries of super elements, 6 degrees of freedom per nodal point was applied as the master degree of freedom and the remaining nodal points were treated as the slave degree of freedom. Figure 4(a) shows the master degrees of freedom and the slave degrees of freedom. When the matrix condensation technique is applied to the master nodes, condensed stiffness and mass matrices can be developed. Figure 4(b) shows a substructure consisting of the identical super elements. For compatibility condition the nodal points, where substructure and columns meet, were selected as the master degree of freedoms.



(a) Master and slave DOF at super element (b) Substructure consisting of super elements



(c) Analytical model consisting of substructures (Model A-1) (d) Stick model (Model A-2)

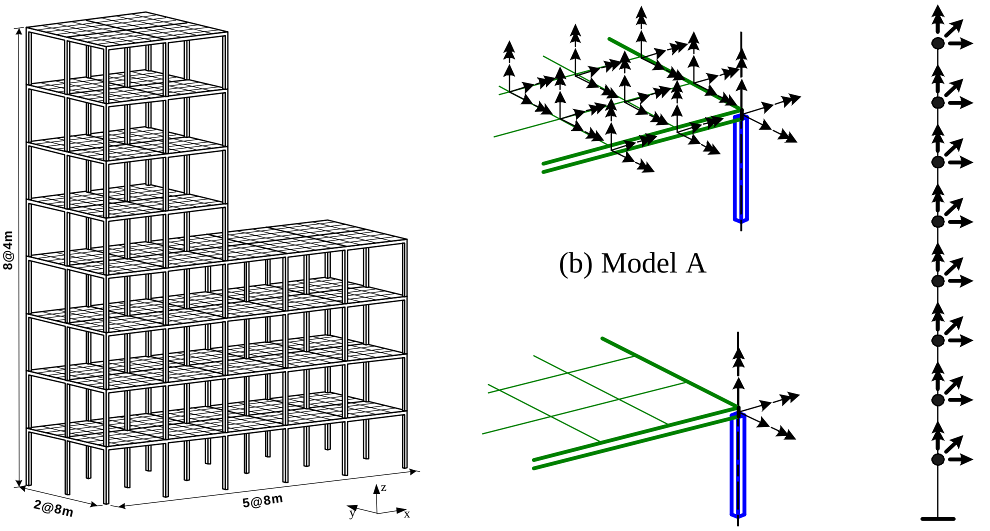
Fig. 4 Selection of degrees of freedom at each structure

The matrix condensation technique was applied to the remaining nodal points to obtain the condensed stiffness and mass matrices. Figure 4(c) shows the analytical model consisting of substructures. As can be seen in Fig. 4(c), each story consists of substructure and columns. Therefore, each substructure can be developed independently and the stiffness matrix for columns needs to be superimposed to the substructures for forming the matrices of the whole building structure. The analytical model developed so far is called model A-1 (Fig. 4(c)), that is generated from model A (Fig. 2(a)) employing the substructuring technique and super elements. When the matrices are condensed by using the super elements and substructure technique, the number of degrees of freedom can be reduced by stage; therefore the time required to condense matrices also can be reduced effectively. Particularly, the substructuring technique can usefully be applied when the building structure has a lot of identical stories. However, if there are many columns at each story, the number of degrees of freedom also increase in spite of the building structure is modelled using the substructuring technique. In this study, the rigid diaphragm assumption was applied to the master degree of freedom in Fig. 4(c). Therefore the analytical model was changed to the stick model as shown in Fig. 4(d). Because this stick model includes the flexural stiffness of the floor slabs and T-beam effect, relatively accurate structural behaviour can be obtained comparing with the stick model which adapting only the rigid diaphragm assumption shown in Fig. 1(c). The analytical model developed so far is called model A-2, which is generated from model A-1 applying the rigid diaphragm assumption.

### 3.2 Comparison of Dynamic Behaviour for Analytical Models

To examine the analytical efficiency and accuracy of selection of degrees of freedom for substructure and super elements, the seismic analysis of an example structure shown in Fig. 5(a) was conducted using El Centro 1940 NS component earthquake. All analytical models take into account the flexural stiffness of floor slabs and the T-beam effect. As shown in Fig. 5(a), floor slabs surrounded by beams are divided into 4 x 4 elements and the section dimensions for each member are summarized in table

3. The descriptions of the analytical models are shown in Figs. 5(b), 5(c), and 5(d). All degrees of freedom were considered for model A. For model A-1, 6 degrees of freedom were selected as master degree of freedom for nodes where floor slab and column meet. Model A-2 is stick model transformed from the model A-1.



(a) Example building structure (b) Model A (c) Model A-1 (d) Model A-2

Fig. 5 Analytical models

Table 3 Member size of the structure (unit:cm)

| Story | Column (bxh) | Beam (bxh) | Slab thickness |
|---|---|---|---|
| 1~2 | 75x75 | 40x60 | 20 |
| 3~5 | 60x60 | 40x60 | 20 |
| 6~8 | 45x45 | 40x60 | 20 |

Analysis was carried out in a personal computer with Pentium III 550 MHz CPU and 256 MB RAM. Table 4 shows the total number of degrees of freedom and the required computational time of analysis for each analytical model.

Table 4 Comparison of computation time (unit:sec)

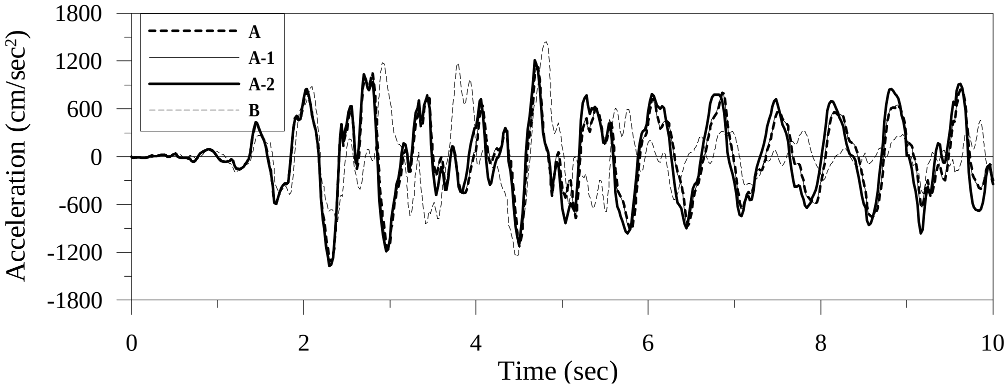
| procedure | Model A (6,588 DOF) | Model A-1 (648 DOF) | Model A-2 (24 DOF) | Model B (24 DOF) |
|---|---|---|---|---|
| Stiffness & Mass | 102.03 | 7.59 | 8.48 | 0.87 |
| Eigenvalue | 25,740.05 | 77.02 | 0.01 | 0.01 |
| Time history | 139.53 | 18.57 | 1.84 | 1.83 |
| Total | 25,981.61 | 103.18 | 10.33 | 2.71 |

Based on the analytical study, it was observed that the most computational time was consumed for carrying out the eigenvalue analysis. It was also found that not only the required computational time but also the required time for organizing the stiffness matrix and mass matrix of structure could be reduced substantially when the substructuring technique, super elements, and rigid diaphragm assumption are applied. The natural frequencies for each analytical model are summarized in table 5. In this table the frequencies for model A are assumed as the exact frequencies.

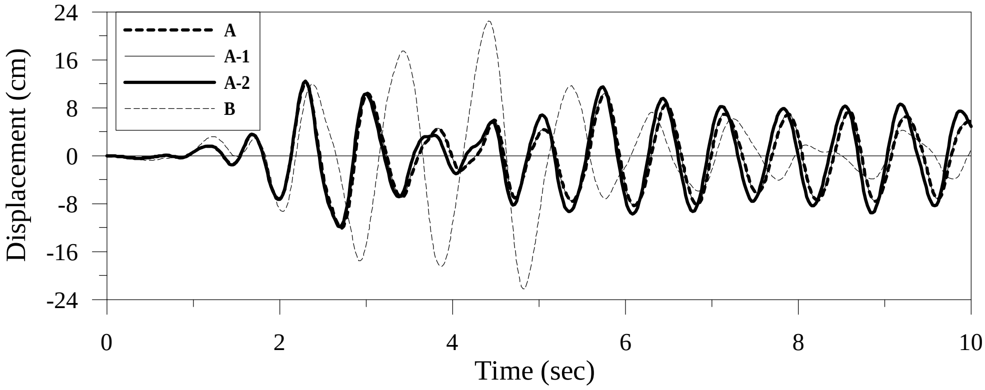
Table 5 Comparison of the natural frequency (unit:Hz)

| Mode number | Model A | Model A-1 | | Model A-2 | | Model B | |
|---|---|---|---|---|---|---|---|
| | Freq. | Freq. | Error(%) | Freq. | Error(%) | Freq. | Error(%) |
| 1 | 1.401 | 1.401 | 0.00 | 1.429 | 1.86 | 1.007 | -28.12 |
| 2 | 1.598 | 1.598 | 0.00 | 1.633 | 2.19 | 1.182 | -26.03 |
| 3 | 2.262 | 2.263 | 0.04 | 2.358 | 4.24 | 1.763 | -22.06 |
| 4 | 3.173 | 3.174 | 0.03 | 3.226 | 1.67 | 2.348 | -26.00 |
| 5 | 3.295 | 3.296 | 0.03 | 3.355 | 1.82 | 2.491 | -24.40 |
| 6 | 3.590 | 3.591 | 0.02 | 3.688 | 2.70 | 2.812 | -21.67 |
| 7 | 6.108 | 6.115 | 0.11 | 6.242 | 2.19 | 4.904 | -19.71 |
| 8 | 6.185 | 6.192 | 0.11 | 6.293 | 1.75 | 4.848 | -21.62 |
| 9 | 7.357 | 7.366 | 0.12 | 7.718 | 4.91 | 6.286 | -14.56 |
| 10 | 8.207 | 8.224 | 0.21 | 8.316 | 1.33 | 6.670 | -18.73 |

For model B, which only frames are taken into account and floor slabs are replaced by rigid floor diaphragm, relatively low natural frequencies are obtained and the errors are severe. However, the errors of the natural frequencies for model A-1 seem very minor. For model A-2, the errors of the natural frequencies is a little larger than those of the model A-1 but still very small comparing with the errors generated from the model B. These results are due to the rigid diaphragm assumption and the further reduction of errors could be expected if the structure is symmetric shape. Figure 6 shows the y-direction acceleration and displacement time histories of the structure at top story.



(a) Acceleration time history



(b) Displacement time history

Fig. 6 Comparison of time history at top floor

Comparing with model A, the response of model A-1 and A-2 is very similar because the mode shapes of low frequencies for model A, A-1, and A-2 are similar, whereas the response of model B seems different as expected. Therefore relatively accurate seismic analysis could be carried out with the proposed model A-1. On the aspect of the computational time of the analysis and the computer memory capacity, model A-2 could be an efficient analytical model for seismic analysis.

## 5. CONCLUSION

In this study efficient seismic analytical models employing substructuring technique and super elements are proposed. The effect of the flexural stiffness of floor slabs and T-beam are included in the analytical models. These effects on the dynamic behaviour for example building structure are also investigated. Through the analysis for the example structures, the efficiency and accuracy of the proposed analytical models are verified and the following conclusions can be obtained.

(1) An accurate seismic analysis can be performed with the proposed model A-1 and for the required computational time and the computer memory capacity point of view, model A-2 is efficient. Therefore model A-1 can be selected for both efficiency and accuracy. However if the computational time of analysis is more concerned, then model A-2 could be preferable.

(2) To represent the dynamic behaviour of building structure, it is desirable that the effect of the flexural stiffness of the floor slabs and the eccentric T-section beam need to be included in the analytical model.

## ACKNOWLEDGMENTS

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## REFERENCE

Gupta, A.K. and Ma, P.S., 1977, Short communications error in eccentric beam formulation, *Int. J. Numer. Meth. Engng.*, Vol 11, 1473-1483

Miller, R.E., 1980, Reduction of the error in eccentric beam modeling, *Int. J. Numer. Meth. Engng.*, Vol 15, 575-582

Chan, T.H.T. and Chan, J.H.F., 1999, The use of eccentric beam elements in the analysis of slab-on-girder bridges, *Structural Engineering and Mechanics*, Vol 8(1)85-102

Lee, D.G., Ahn, S.K., and Kim J.K., 2000, An efficient modeling technique for floor-vibration in multi-story buildings, *Structural Engineering and Mechanics*, Vol.10(6) 603-619

Ahn S.K. and Lee D.G., 2000, Efficient Seismic Analysis of Building Structures with Eccentric Beams, *Proceedings of 6th APCS, 16-18 October, 2000, Seoul, Korea*, Vol.1, 413-424

Guyan, R.J., 1965, Reduction of stiffness and mass matrices, *Am. Inst. Aeronaut. Astronaut*., Vol 11(5)380-388

Lee, D.G., 1988, An efficient element for analysis of frames with shear walls, *ICES88, Atlanta*,

Timoshenko, S. P., Weaver, W., Jr., and Young, D.H., 1990, *Vibration Problem in Engineering*, John Wiley & Sons, Fifth Edition,

Petyt, M., 1990, *Introduction to finite element vibration analysis*, Cambridge University Press, pp.294-314