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Stationarity of seismic noise and SPAC. Results of a new approach

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ABSTRACT: The SPAC method to analyse ambient noise vibration was introduced quite a few years ago by Aki (1957). This method makes recourse to a spatial averaging (through cross-correlation functions) of microtremor measurements using an array of stations. This idea has popped up repeatedly, but no significant modifications have been contributed to the original technique. In this paper we propose a fresh look at the SPAC method, introducing the idea of expoiting temporal stationarity as a subtitute for spatial averaging. This idea has several advantages from which we cite the two most important: there is no need for simultaneous recordings using an array of stations, whose locations must obey a very rigid scheme; we can obtain results for a large number of closely spaced distance intervals. Our proposal is tested using data from the Parkway, Wainuiomata, temporary array, that operated for almost two months in 1994. The results are excellent.

1 INTRODUCTION

Site effects play a major role in destructive ground motion. The effects of local geology may modify the ground motion observed on soft soils in a very significant way, the case of Mexico City earthquakes during 1985 being a foremost example. Ground motion was amplified by a factor of about 40 at the resonant frequency of the very soft soil layer covering the ancient lake bed zone. Site effects may be taken into account easily when abundant records of ground motion exist for nearby stations on different soil conditions using for example spectral ratios (e.g. Chávez-García et al., 1990). Usually, however, we must have recourse to exploration techniques to determine the subsoil structure and thence deduce the expected amplification.

More than 40 years ago, Aki (1957) proposed an innovative technique to use noise measurements from an array of stations to determine the underlying subsoil structure. The method is based on the fact that the spatial autocorrelation function of noise measurements between equidistant stations takes a special form (the form of a zero order, first kind Bessel function) when they are spatially averaged. The SPAC method allows the processing of microtremor records to obtain phase velocity dispersion curves for surface waves. In the original version of the method, vertical component records were used, and thus, phase dispersion curves corresponded to Rayleigh waves. More recent applications of the method have tried to use the horizontal components. A restriction already established by Aki (1957) was that the crosscorrelation of noise records had to sample different azimuths for a given inter-station distance.

The original paper by Aki was based on data from **three** analog seismographs. Today, thanks to the revolution in technology, high quality digital recorders at affordable prices have drastically modified the use of arrays in seismology, the most extreme example being the Kyoshin-net in Japan consisting of 1000 uniform instruments, installed after the 1995, Kobe, earthquake. More modest examples are temporary networks installed to investigate seismic emission in volcanoes (Metaxian 1994, Chouet et al., 1998), or site effects (Chávez-García et al., 1999; Kanno et al., 2000). In terms of the SPAC method, however, the number of available instruments does not modify in the least the field procedure used to make measurements. For example, Chouet et al. (1998), used an array of stations disposed in a semi-circle. They maintained that a semi-circle allowed the retrieval of the same information as a whole circle, allowing a decrease of the station spacing in their array. However, the analysis of the data continue to follow the guidelines established by Aki (1957).

In this paper we have taken advantage of available noise records from a dense temporary digital seismograph network. The purpose of its installation was to better understand the relation between local geology and recorded ground motion in an alluvial basin. As part of its operation, the instruments of the network were programmed to trigger periodically and record ambient vibration simultaneously. We have analysed these data using the SPAC method. However, given that the installation of the network was guided by different considerations, the spatial distribution of the array is very far from the ideal geometry (concentric circles with stations covering regularly all azimuths). For this reason, in order to use the SPAC method, we must substitute temporal stationarity for spatial stationarity. Our hypothesis is that, given long enough records of ambient vibration, the time average crosscorrelation between any pair of stations is equivalent to crosscorrelation between an azimuthal distribution of stations on a circle of given radius. Thus, instead of recording microtremors simultaneously at several stations on a circle of fixed radius, we assume that the predominant direction of propagation of the microtremor field rotates with time. In this case, a pair of stations recording microtremors for a long time samples a large number of azimuths relative to the propagation direction of the microtremor field. If our hypothesis is correct, then the SPAC method is applicable even in the case of a very irregular network. Our results allow us to determine an average shear-wave profile for the basin. The results are in very good agreement with previous estimates using very different techniques, and our hypothesis is sustained. Our results show that it is possible to use the SPAC method with different array configurations, thereby greatly expanding its possible application.

2 DATA

The field experiment where the data was recorded has been described in previous papers (Chávez-García et al., 1999, 2002). We will recall only the most important points. A temporary network of 24 digital seismographs was installed from 1st August until 12th October, 1995, in Parkway valley, Wainuiomata, New Zealand. Each station consisted of a 1 Hz seismometer coupled to a EARSS seismograph (Gledhill et al., 1991). Time was received by each station from the official time broadcast in New Zealand. Station 1 was installed about 2 km to the NE from the basin, on firm rock. Four of the remainder stations were installed on the soft rock (weathered greywacke) surrounding and underlying Parkway basin, while 19 others were installed on the soft sediments filling the valley. A description of the local geology around Parkway is given in our previous papers. The average distance between stations was 40 m. Figure 1 shows the distribution of the stations.

The main purpose of the network was to record earthquakes. However, as part of its operation, the network was programmed to trigger periodically to record ambient noise, and to verify its status. Thus, 1-minute time windows of noise were recorded simultaneouly at each station every hour, each day the network operated. This system generated a very large amount of noise data. Clearly, however, the spatial distribution of stations shown in Figure 1 is far from concentric circles. However the distances between stations on the soft sediments sample in detail the distance range between 22.8 m (distance between stations 15 and 16) and 501 m (distance between stations 02 and 12). Not all station couples could be used, as some stations did not always trigger. However, a total of 91 station couples yielded enough data to obtain reliable results. The distance range covered was from 57 to 427 m.

3 ANALYSIS

We did not use all records of microtremors made by the network. Instead, we selected three different days and used the 24 1-minute windows recorded by each station each day. We recall that the SPAC method established that the spatial crosscorrelation coefficients of ambient vibration at a given inter-station distance, *r*, and angular frequency, T, $r(r,w)$, averaged over many different azimuths can be written as

$$r(r,w) = \frac{1}{2pf(0,w)} \int_0^{2p} f(r,q,w)dq = J_0\left(\frac{rw}{c(w)}\right) \quad (1)$$

where $f(0,w)$ is the autocorrelation function at the centre of the array, $f(r,q,w)$ is the crosscorrelation function between a site at coordinates (r,q) and the station at the centre of the circle, c(T) is the phase velocity at the site, and J_0 is the Bessel function of first kind and order zero. The details have been published several times (e.g. Chouet et al., 1998).

This procedure assumes that there exists a predominant direction of propagation and therefore, that azimuthal averaging is required to eliminate the unknown angle between that direction and the orientation of any given couple of stations. We do not agree with this. Our experience in microtremor measurements indicates that ambient vibration is the result of a complex mixture of waves coming from different sources. Moreover, the predominance of any one source will not be permanent, but rather will fluctuate with time. Thus, the dominant direction of the microtremor wavefield will rotate with time around a given pair of static seismic stations. In this paper we test this hypothesis. In our procedure we exchange the spatial averaging (for a given dominant direction in the microtremor wavefield) with temporal averaging. We leave our stations static and let the microtremor wavefield rotate around them.

The computations are straightforward. We took three days worth of simultaneous recordings of microtremors at the soft soil stations. At this stage, we considered only vertical components. Due to some malfunctions, not all stations recorded the same number of microtremor windows. However, the average number of windows for a given station was around 72. Each trace was filtered using a series of 30 bandpass filters, 0.5 Hz wide, centred at frequencies between 0.75 and 8 Hz, with a 0.25 Hz frequency step. The filters used were Butterworth. We computed the crosscorrelation between each possible combination of two filtered traces for each recording window and saved the crosscorrelation coefficients. Finally, we computed the average crosscorrelation between all crosscorrelation coefficients for each pair of stations, and its standard deviation. For some distances, the crosscorrelation value at the lowest frequency was very low (<0.75). These coefficients were discarded in the following processing, leaving us with “only” 71 different distances for which we have a reliable crosscorrelation coefficient.

4 RESULTS

Figure 2 shows the results as a function of frequency and distance. The left diagram shows the average crosscorrelation, while the right diagram indicates the corresponding standard deviation. We observe that the frequency of the first zero crossing of the crosscorrelation coefficients shifts to lower values with increasing distance between stations. This is exactly what is to be expected. The figure for the standard deviation does not show a significant dependence on either frequency or distance**.** The standard deviation is smaller than the magnitude of the coefficients at low frequencies, indicating that the resulting surface is meaningful. At higher frequencies, the standard deviation becomes comparable with the values of the crosscorrelation coefficients.

Next, we wrote an inversion program to retrieve $c(w)$ from the correlation coefficients, taking advantage of its known shape. This inversion in fact is equivalent to finding the argument of the Bessel function that makes smallest the difference between observed crosscorrelation coefficients and the Bessel function. Dispersion values at low frequencies are considered more reliable using data from larger distances, while phase velocities at higher frequencies are drawn from smaller distances. In our case, we are able to make a joint inversion of the whole set of crosscorrelation coefficients simultaneouly for 71 different distances. In the inversion program we have weighted each observation by its corresponding standard deviation. After making several tests, we found that smaller inter-station distances contribute results to the higher frequency range, while the larger distances constrain the dispersion curve at lower frequencies. This is a natural outcome because the more important value governing the Bessel function is the argument of its first zero crossing. The argument of the Bessel function is determined once the abscissa of its first zero crossing is constrained. The values to the left and right of this zero crossing are the largest for the Bessel function, and in our data they are larger than the corresponding standard deviation. For larger values of the argument, the Bessel function oscillates and takes smaller values, which in our data are often smaller than the corresponding standard deviation. Larger distances impose a zero crossing at smaller values of the argument, while smaller distances constrain it at larger values of the argument.

Figure 3 shows an example of the results of the inversion for two crosscorrelation curves at distances of 44 and 206 m. We observe that our average crosscorrelation has the expected shape and that the inverted Bessel function shows a very close fit to the observations. Figure 3 shows clearly the difference between the abscissae at the first zero crossing of the two functions plotted. The crosscorrelation function at 44 m goes through zero at 3.5 Hz, while that at 206 m goes through zero at 1.5 Hz. Thus, the information that really affects the inversion is the abscissa of the first zero crossing. This can be used to put limits on the validity of the dispersion curve obtained. Figure 4 shows the value of the first zero crossing in our correlation functions as a function of distance. We observe the steady increase of that value with decreasing distance. We also observe that for distances larger than 210 m this value remains constant. This means that there is no additional information at larger distances**.** Other studies (e.g. Morikawa et al., 1998) usually report crosscorrelation coefficients for only 2 different distances. Therefore, the inversion of the arguments of Bessel functions has only two different results, which are later mixed together with some consideration of the frequency range of validity from each of the two arrays. In our case we were able to use 71 different inter-station distances. Our inversion program used all these data simultaneously to obtain the arguments of all the Bessel functions at the same time.

Figure 5 shows our final phase velocity dispersion curves. This curve is well constrained in the frequency range 1.5 to 4.5 Hz. The stations used are all on the soft sediments of Parkway valley, and thus the surface waves that compose the microtremor wavefield have information only on the shear velocity profile within the valley. Those surface waves identify a strong velocity contact at the base of the sediments, but as no energy escapes from the sediments, they have no way of knowing the shear-wave velocity of the rocks below the valley. Moreover, the maximum distance between our stations is about 400 m. The phase velocity dispersion curve indicates a value of about 650 m/s at 1.5 Hz. Thus, the wavelength at this frequency is 433 m. Hence at low frequencies, our results are bounded by the largest inter-station distances in our data. At the high frequency end (4.5 Hz), the wavelength is about 55 m, close to the minimum distances for which we have reliable data. Therefore, the resolution of our array is bounded by the available distances for the computation of crosscorrelations, as it should be. Figure 5 shows the observed dispersion curve using three different symbols. We tested inverting only the coefficients computed for inter-station distances less than 170 m, independently of those for larger distances. Different symbols show the results using all distances together. Our inversion procedure for frequencies larger than 2 Hz is governed by crosscorrelation coefficients for smaller distances. At lower frequencies, results for larger distances contribute most of the information. Thus, inverting simultaneously for many crosscorrelation distances automatically takes into account the different resolution in frequency of the observations. The observed phase velocity dispersion curve was inverted using standard methods (Herrmann, 1987) to obtain the S-wave velocity profile at Parkway. The dispersion curve computed from the final model is also shown in Figure 5. The fit between observed and modelled dispersion curves is very good.

5 CONCLUSIONS

We made the hypothesis that the dominant direction of propagation of surface waves in a microtremor wavefield rotates with time. Thus, we can use the SPAC method with a fixed pair of stations, provided that they record ambient vibration for a long enough time interval. We have tested this hypothesis using data from the temporary seismograph network installed at Parkway, Wainuiomata, New Zealand. The results show that our hypothesis is sustained. We obtained crosscorrelation coefficients for 71 distance spacings. Those were inverted to obtain a phase velocity dispersion curve, which in turn was inverted to recover a shear-wave velocity profile. The results are in very good agreement with previous estimates using very different techniques, and our hypothesis is sustained. We obtain good results in the frequency band 1.5 to 4.5 Hz. These limiting frequencies are related to the maximum and minimum distances for which we are able to compute reliable crosscorrelations between stations. Our results show that it is possible to use the SPAC method with different array configurations, thereby expanding greatly its possible application. Future work using this technique will include the analysis of the horizontal components recorded at this array. Also, we will explore the use of the rock stations at Parkway, and finally, we envisage the application of this technique of analysis to datasets from other arrays.

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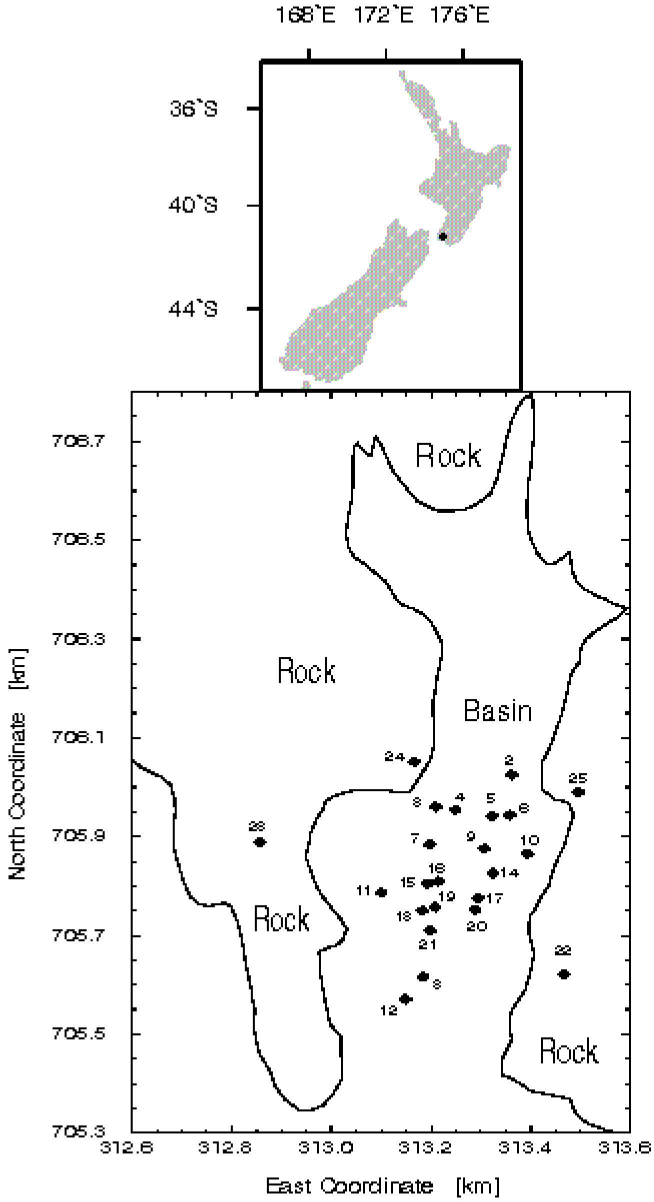

Figure 1. Distribution of stations in Parkway valley, Wainuiomata, New Zealand. The solid line shows the limit between soft soil sediments and underlying bedrock.

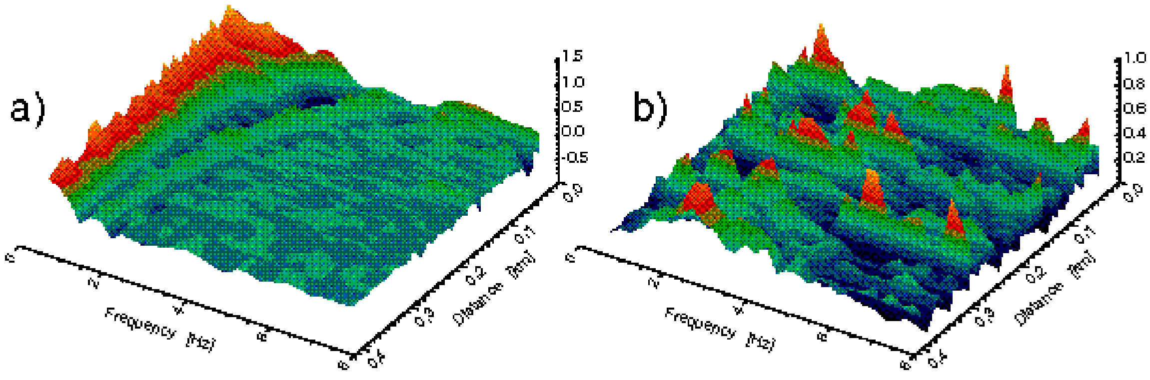

Figure 2. a) Average of crosscorrelation coefficients computed from all pairs of stations on soft soil in Parkway basin, as a function of frequency and distance. b) Corresponding standard deviation.

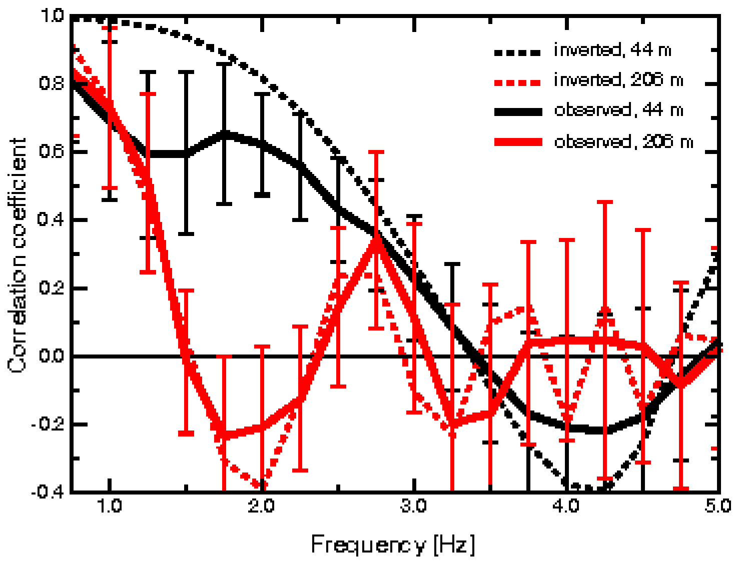

Figure 3. Example of the inversion of two cross correlation coefficients (solid lines) to obtain the corresponding Bessel function (dotted lines). Results are shown for two individual inter-station distances, 44 and 206 m. Note the good fit between observations and the inverted Bessel functions at its first zero crossing. This value governs the results.

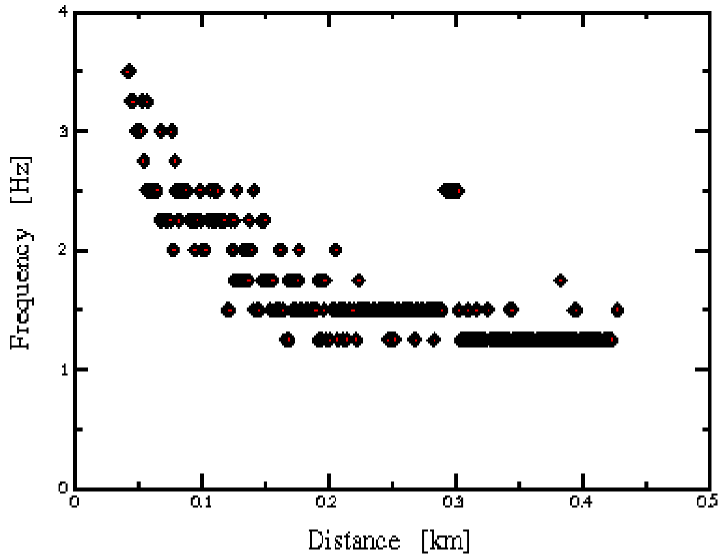

Figure 4. Frequency of the first zero crossing of all our crosscorrelation coefficients (average values) as a function of inter-station distance. This value should decrease monotonically with increasing distance. At about 200 m, the frequency of the first zero crossing becomes constant. This indicates that there is no additional information contributed by station pairs at larger distances.

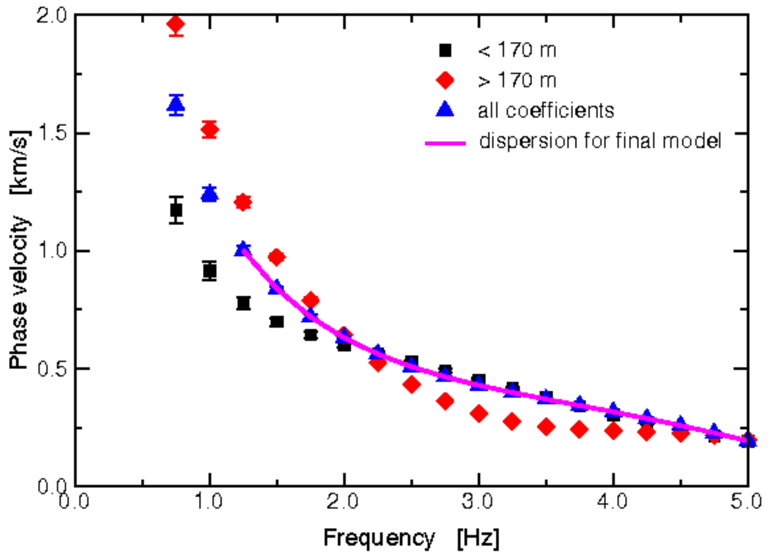

Figure 5. The symbols show the final phase velocity dispersion curves inverted from the data. The blue, point-up triangles, show the result obtained from the inversion of coefficients for all distances together. The black squares show the result obtained from the inversion of the crosscorrelation coefficients computed for station pairs at less than 170 m distance apart. Red diamonds correspond to the results of considering only station pairs at more than 170 m distance. The solid line shows the dispersion curve obtained from the inverted soil profile. We observe that small distances contribute most of the information at higher frequencies, while larger distances contribute more at lower frequencies, as intuitively expected.