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Comparison of pseudo-dynamic test and inelastic time history computer analysis

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ABSTRACT: The paper discusses the reasoning behind the performance of pseudo-dynamic tests, the background and the theory. The pseudo-dynamic testing facilities developed at BRANZ were verified by comparing with inelastic time history analysis.

1 INTRODUCTION

The pseudo-dynamic (PD) test procedure is a method of imposing seismic deformations to a test specimen. It was first described by Takanashi (1974). It is a combination of a conventional cyclic test and analytical modelling in real-time. A computer determines the displacements imposed on a test specimen based on the measured specimen resistance at each instance, the analytically calculated inertia and damping forces and the digitised earthquake acceleration record. It has the advantage over analytical modelling in that the computations at each instance are based on the measured resistance and the uncertainties associated in calculating this force after a specific deflection time history are not present in a PD test. The faster modern computers and software have largely overcome the speed limitations of the earlier PD programs. BRANZ used LabView Version 6 (National Instruments 2000) software on a computer with a clock speed of 800 MHz.

Whereas the seismic mass must be present in a shake table test, it is simulated in a PD test. The damping in a shake table test is that actually present in the test specimen, whereas it is simulated in a PD test. These differences lead to many advantages, and some disadvantages as discussed by Thurston (2002). One of the major differences is that the PD test can be performed at a slower rate (say 1/16th real time speed). This provides greater opportunity for viewing the test. However, greater loading speed usually results in greater specimen strength and in this regard the PD test results are expected to be slightly conservative.

A PD test consists of a test specimen with one or more servo-controlled actuators used to apply forces at specific locations. For instance, a multi-storey wall specimen would normally use actuators at each floor level. At the end of each time step applied forces and specimen deflections are recorded via computer and based on these values the deflections to be imposed during the next time step are calculated.

To verify the PD method comparisons were made of PD test results, with an established Inelastic Time History Analysis software package (ITHA) software package called Ruaumoko (Carr, 2000). Thurston (2002) presents a full set of comparative plots. However, due to space limitations only some typical comparisons are presented here. Thurston (2002) reported that the agreement between shake table tests and non-linear dynamic analysis software by others has also been good. Hence, where a structure can be adequately modelled then ITHA software can be used to predict the seismic performance.

The seismic performance of houses is usually greatly enhanced by load sharing and composite action of both the structural and non-structural elements and the lateral restraint due to gravity load resisting wall "rocking action". Thurston and Park (2002) showed that it was necessary to consider this enhanced stiffness and strength (hereafter called "systems effects"), to avoid undue conservatism when designing to NZS3604 (SNZ 1999) bracing wall provision requirements. These influences are difficult to model. Shake table tests of a total house requires a very large shake table and relies on the fidelity of the applied ground accelerations. It is considered that the PD test method is the most suitable vehicle to examine ‘system’ effects and total house performance.

Most New Zealand houses are designed and constructed using the non-specific design procedures in the New Zealand standard for Timber Framed Buildings, NZS 3604 (SNZ 1999). The PD facilities have been developed to provide BRANZ with an alternative means of assessing the racking resistance of bracing walls. The BRANZ P21 test method is used to obtain the bracing ratings of timber framed walls, to meet the wind and seismic demand stipulated in the light timber framing standard, NZS 3604 (SNZ 1999). By modelling hysteresis loops from P21 tests, Thurston and Park (2002) performed ITHA with earthquakes corresponding to NZS 4203 (SNZ 1992) design earthquake spectra to derive a revised evaluation method for determining bracing ratings from P21 test results. By getting agreement with PD and ITHA method as described in this paper, BRANZ derived confidence that the ITHA approach used by Thurston and Park was valid.

2 TYPES OF PD TESTS CURRENTLY PROGRAMMED.

This paper discusses the development of the BRANZ pseudo-dynamic testing facility for evaluation of single and two storey structures without torsion effects. Thurston (2002) showed that the basic PD algorithms could be modified to allow two storey structures to be tested as a combination of two single storey structures. The input can be specified as one of a selection of earthquakes, a pulse of given magnitude and time, or a continuous Sinusoidal excitation. Thurston developed the package further to include the effects of torsion but due to space limitations this cannot be included in this paper. Results indicated that torsional effects are unlikely to increase deflection demand on house walls by more than 20%. Further, the response of houses under bi-axial earthquakes where weaker walls in both directions are yielding is unlikely to be significantly greater than from a uni-axial earthquake.

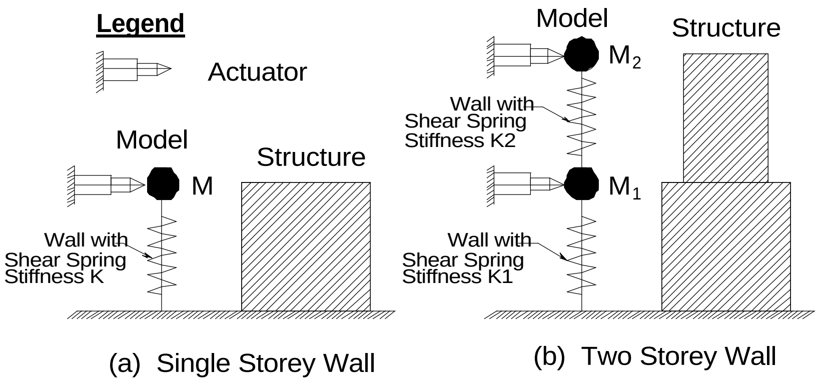

Figure 1 Sketch of Structures Analysed

3 DAMPING IN SDOF SYSTEMS

Consider the forces on a single degree of freedom (SDOF) wall subjected to a ground acceleration $\ddot{G}_X$ as shown in Figure 1(a). The seismic mass at the top of the wall of elastic stiffness K is taken to be M and the deflection at the top of the wall is taken to be X. (K is a function of X in non-linear systems.) A force balance at the mass provides the following equation (Chopra 2001):

$$M\ddot{X} + C\dot{X} + KX = -M\ddot{G}_X$$

The term C is the damping restoring force per unit velocity and is often referred to as Coulomb damping. Damping is not well understood and is thought to be the cumulative friction (parts rubbing together) including air resistance to motion. Note that hysteretic damping is the energy absorbed by the inelastic action of the structural element and is effectively simulated from the shape of the hysteresis loop used in ITHA.

An example of the sensitivity of ITHA to the level of damping is presented in Figure 2. The analysis was on a plasterboard wall subjected to the 1940 N-S earthquake modified to approximately fit the NZS4203 (SNZ 1992) spectra. As the plasterboard wall model reaches 70% of peak load at 6 mm deflection the deflections of Figure 2 indicate large hysteretic damping had occurred. However, Figure 2 still shows significant sensitivity to coulomb damping.

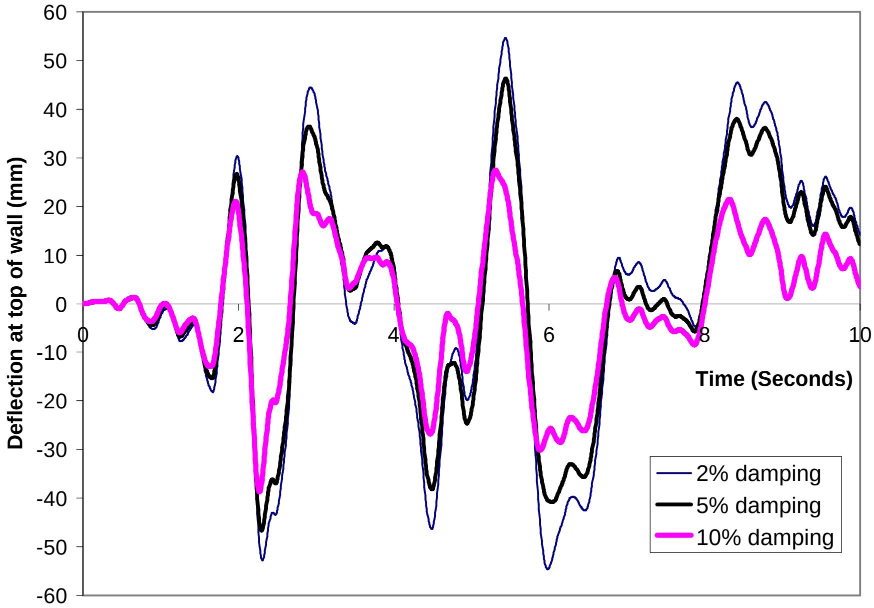

Figure 2. Influence of damping on seismic deflection of a plasterboard wall

Damping is usually expressed in terms of the ratio of critical damping, l , and for a SDOF structure it can be shown (Chopra 2001) that C = $2 l \sqrt{K.M}$ …… (1)

Thurston (2002) discusses how the damping ratio can be derived from free vibration tests and (for house wall systems which exhibit non-linearity from the onset) the importance of using the same deflection for the free vibration test as used for determining the initial stiffness for use in ITHA.

4 DAMPING IN MULTI DEGREE OF FREEDOM SYSTEMS

The damping used in this paper is the Rayleigh initial stiffness damping whereby the matrix [C] is obtained from the mass matrix [M] and the stiffness matrix [K] using (Carr 2000):

[C] = α[M] +β[K] ……….(2)

Where $\alpha = \dfrac{2.w_i.w_j(w_i.l_j - w_j.l_i)}{w_i^2 - w_j^2}$ and $\beta = \dfrac{2.(w_i.l_i - w_j.l_j)}{w_i^2 - w_j^2}$ …….. (3)

l_i and l_j are damping ratios for modes *i*,*j* with natural frequencies w_i and w_j respectively.

5 PSEUDO-DYNAMIC PROGRAMMING EQUATIONS AND THEORY

From a force balance, Chopra (2001) gave the matrix form of the generalised equations of motion of an elastic structure subjected to earthquake excitation, $[G]$:

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = -[M]\{\ddot{G}\} \quad \text{...... (4)}$$

where $u, \dot{u}, \ddot{u}$ represents the displacement, velocity and acceleration respectively.

5.1 *SDOF Wall*

The structure to be analysed under earthquake excitation is shown in Figure 1(a). The equations derived below are for an earthquake acceleration applied over a small time step. During this time step the load-deflection relationship for the wall is considered to be perfectly elastic. At the end of the time increment the wall stiffness is reassessed. Thus, the methodology can be applied to non-linear systems.

For a SDOF system Eqn. (4) reduces to:

$$M\ddot{X} + C\dot{X} + KX = -M\ddot{G} \quad \text{...... (5)}$$

A PD test is run at a slower rate than real time (generally PD Time/Real Time >10). Thus, the accelerations and velocities experienced in a PD test are low. Therefore, the inertia and viscous forces can be taken to be zero. In a PD test, the applied force F = spring force (K.X). Thus:

$$\ddot{X} = -(C\dot{X} + F + M\ddot{G}) \quad \text{......(6)}$$

The damping, C, can be found from Equations (4) and (5).

Put $w_j = 0$ and $w_i = \sqrt{\frac{K}{M}}$. Thus, α = 0., β = $2l\sqrt{\frac{K}{M}}$, and C = $2l\sqrt{K.M}$

Hence, if all values on the right hand side of Eqn (6) are known at time T, then $\ddot{X}$ can be calculated.

By integrating over the next time step $\dot{X}$ and X at the end of the time step can be calculated. The computer controlling the pseudo-dynamic process then sends a signal to make the actuator move the top of the wall to the new calculated X displacement. When this movement is completed at the end of the time step then the applied force F is measured. Therefore, all the information assumed known at time T is now known at time T+ΔT and so the process can continue.

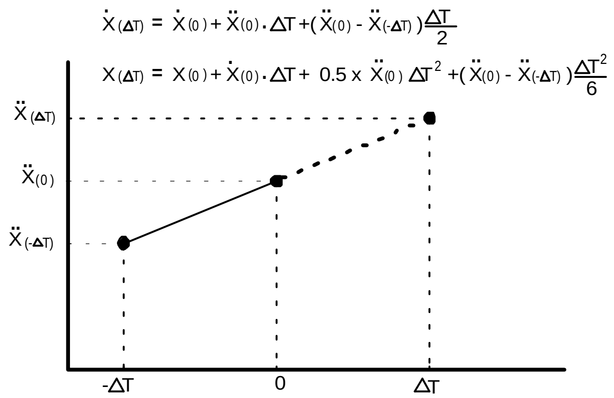

Figure 3 Equations of Motion for Constant Slope Acceleration.

Many integration processes are available (Chopra 2001). Each have specific problems and advantages. The simplest processes is to assume the acceleration is constant over each time step. The time step needs to be small to obtain accuracy. A larger time step may be used if the acceleration slope over the next time increment is assumed to be equal to that of the preceding time increment. The mathematics to calculate the velocity and deflection at the end of the time increment for this assumption is given in Figure 3 and were obtained by single and double integration of the assumed accelerations.

5.2 *2DOF System.*

The nomenclature of the structure to be analysed is shown in Figure 1(b). The wall deflections are X_1 and X_2 at Level 1 and 2 respectively. The mass and stiffness matrices are:

$$[M]=\begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix} \quad ; \quad [K]=\begin{bmatrix} K_1+K_1 & -K_2 \\ -K_2 & K_2 \end{bmatrix}$$

The damping coefficients, C_{ij}, can be found from Equations (2) and (3). Namely:

$C_{11} = \alpha.M_1+\beta.(K_1+K_2)$; $C_{12} = -\beta.K_2$; $C_{21} = -\beta.K_2$ and $C_{22} = \alpha.M_2+\beta.K_2$

Hence, for an earthquake $\ddot{G}_X$ in the direction (X) of the Wall1 and Wall2 eqn.(4) reduces to:

At mass M_1: $M_1.\ddot{X}_1 + C_{11}.\dot{X}_1 + C_{12}\dot{X}_1 + K_1.X_1-K_2.(X_2-X_1) = -M_1.\ddot{G}_X$ ….. (7)

At mass M_2: $M_2.\ddot{X}_2 + C_{21}.\dot{X}_1 + C_{22}\dot{X}_2 + K_2.(X_2-X_1) = -M_2.\ddot{G}_X$ ……..…. (8)

Assume that the force applied by the actuator at Level 1 is F_1 and the force applied by the actuator at Level 2 is F_2. In a PD test, (PD time), because the actual inertia and viscous terms are negligible, then a force balance at the masses provides the following equations:

At Level 1: $K_1.X_1= F_1+F_2$. **At Level 2**: $K_2*(X_2-X_1) = F_2$. Thus, $F_1= K_1.X_1-K_2.(X_2-X_1)$

Hence, equations (7) and (8) reduce to:

At mass M_1: $M_1.\ddot{X}_1$ =-(($C_{11}.\dot{X}_1 + C_{12}\dot{X}_1 + F_1)/M_1 + \ddot{G}_X$) …… (9)

At mass M_2: $M_2.\ddot{X}_2$ =-(($C_{21}.\dot{X}_1 + C_{22}\dot{X}_2 + F_2)/M_2 + \ddot{G}_X$) …….(10)

Therefore, if all values on the right hand side of Eqns (9) and (10) are known at time T, then $\ddot{X}_1$ *and* $\ddot{X}_2$ can be calculated. By integrating over the next time step the velocities and displacements at X_1 and X_2 can be calculated. The computer controlling the pseudo-dynamic process then sends a signal to make the actuators move the tops of Wall1 and Wall2 to the new calculated X_1 and X_2 displacements. Note, that it is important that the new positions are obtained simultaneously and this is almost achieved by subdividing the time step into 10 equal time increments and sending the actuators to $X_{old} + n*(X_{new}-X_{old})/10$ at each increment where n increases from 1 to 10 at corresponding increments. At the end of the time increment the forces F_1 and F_2 are measured. Thus, all the information assumed known at time T is now known at time $T+\Delta T$ and so the process can continue.

6 VERIFICATION OF PROGRAM

Thurston (2002) performed a series of PD tests on single and two storey plywood and plasterboard walls of various lengths and compared the results with predictions from ITHA. The ITHA modelled pinched loop hysteretic data from slow cyclic testing of the same walls. Generally very good agreement was obtained between the ITHA and PD test where the excitation level resulted in the maximum single or inter-storey wall deflection in the range 20-40 mm. Agreement was better for plywood walls than plasterboard walls which sometimes exhibited brittle behaviour in the PD test (particularly the shorter walls). Agreement was only moderate at low deflections where the ITHA model remained elastic but the PD exhibited non-linearity and exhibited a more sluggish response. At large deflections (where the ITHA hysteretic model was poor) the PD test specimen tended to drift to one side (particularly for plywood walls) whereas the ITHA gave a more symmetrical response. The seismic deflections in a 2DOF structure were found to be sensitive to the degree of base isolation – ie the relative strength/stiffness of the upper and lower floors.

A sample time history deflection plots for (a) a plasterboard and (b) a plywood single storey wall is shown in Figure 4 and for a two-storey plywood wall in Figure 5. Excitation in all instances was from the 1940 N-S El Centro earthquake modified to approximately fit the NZS4203 (SNZ 1992) spectra. These illustrate some of the trends discussed above.

7 CONCLUSIONS

For single and two-storey timber framed sheathed shear walls, the pseudo-dynamic (PD) test method was shown to give good agreement with Inelastic Time History Analysis (ITHA) using the Ruaumoko software package. A similar agreement was found for single storey torsionally prone structures. The PD test method is likely to be better than the computer simulation because it reflects actual specimen performance, including non-symmetry and hence is likely to better predict drift to one direction.

Others have shown agreement between shake table and computer analysis. Good agreement between computer analysis and PD test has been found in this paper. Therefore, for the structures considered in this paper, it is believed that the pseudo-dynamic (PD) test method has been proven.

8 REFERENCES

Carr A.J., Ruamoko. 2000. Computer Program User Manual. University of Canterbury, New Zealand.

Chopra Anil K. 2001. Dynamics of Structures. Theory and Application to Earthquake Engineering. Second Edition. Prentice Hall. New Jersey. 844 pages.

Standards New Zealand 1992. General Structural Design and Design Loadings For Buildings. NZS 4203. Wellington, New Zealand.

Standards New Zealand 1999. Timber Framed Buildings. NZS 3604. Wellington, New Zealand

National Instruments Corp. 2000, LabView Version 6. Austin, Texas

Takanashi, K. et al. 1974. Seismic failure analysis of structures by computer pulsator on-line system. *Journal of the Institute of Industrial Science*, University of Tokyo. 26(11): 13-25.

Thurston, S. J. 2002. Pseudo-dynamic testing at BRANZ. *Study Report SR109.* BRANZ, Judgeford, New Zealand.

Thurston S J and Park S G 2002 Seismic Design of New Zealand Houses. Paper submitted to NZNSEE

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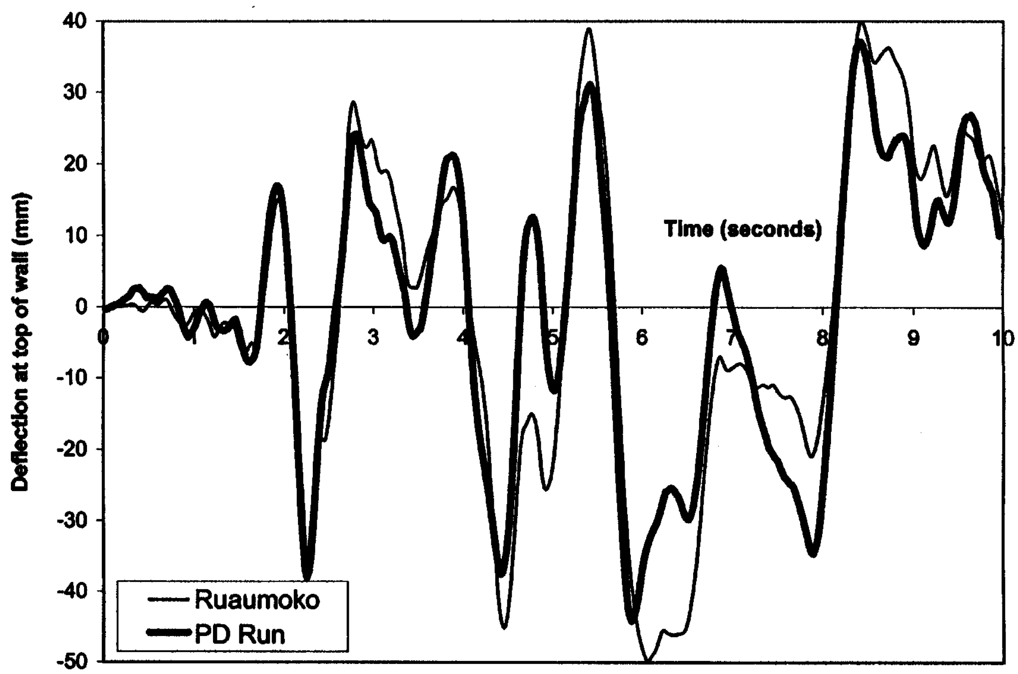
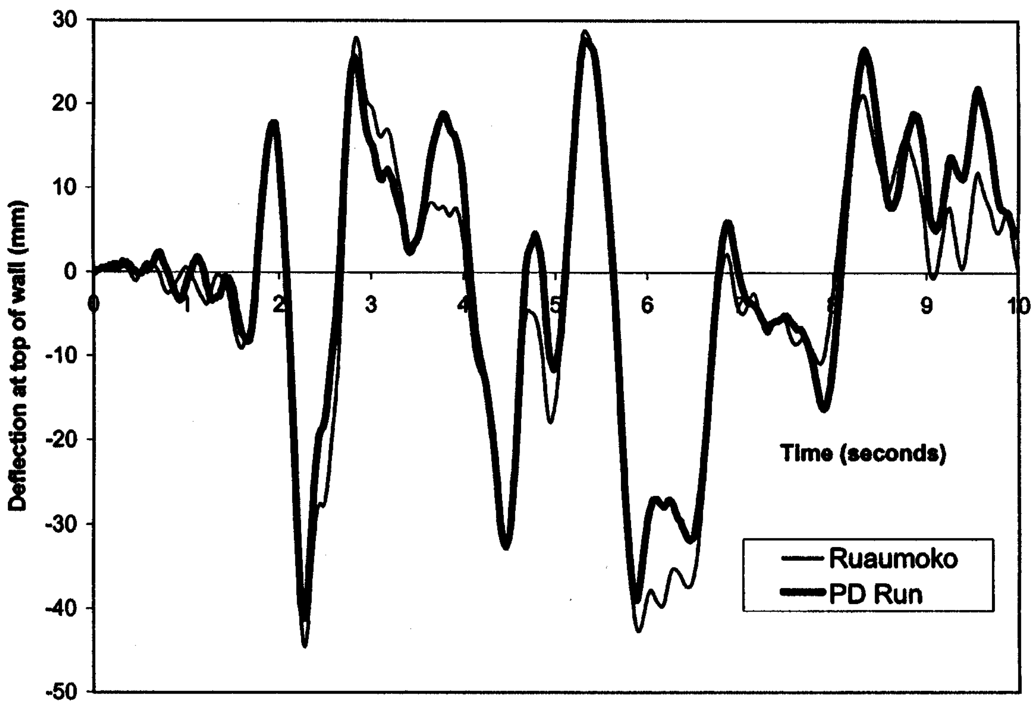
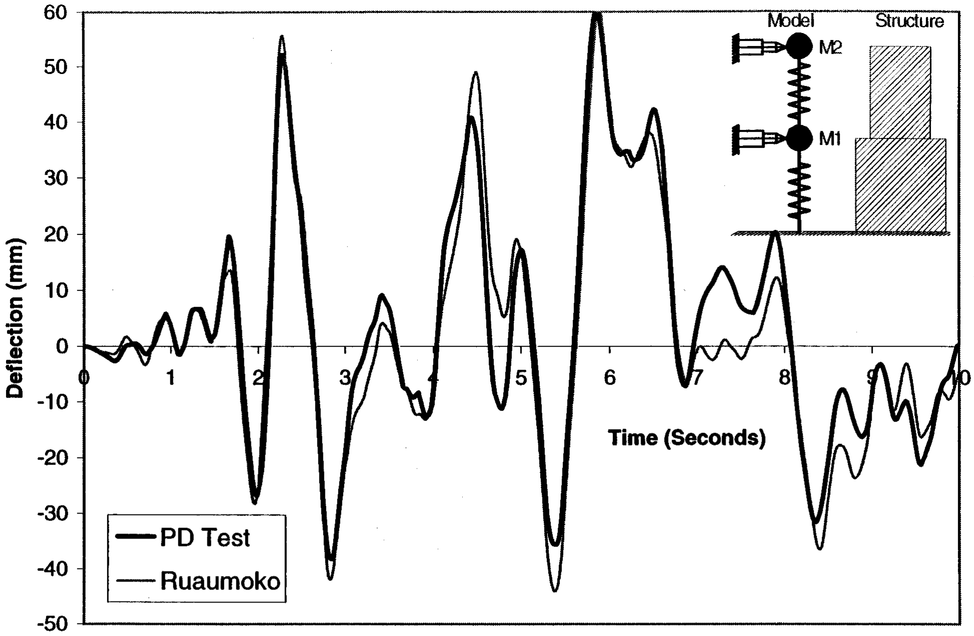


(a) 2.4m long plasterboard wall

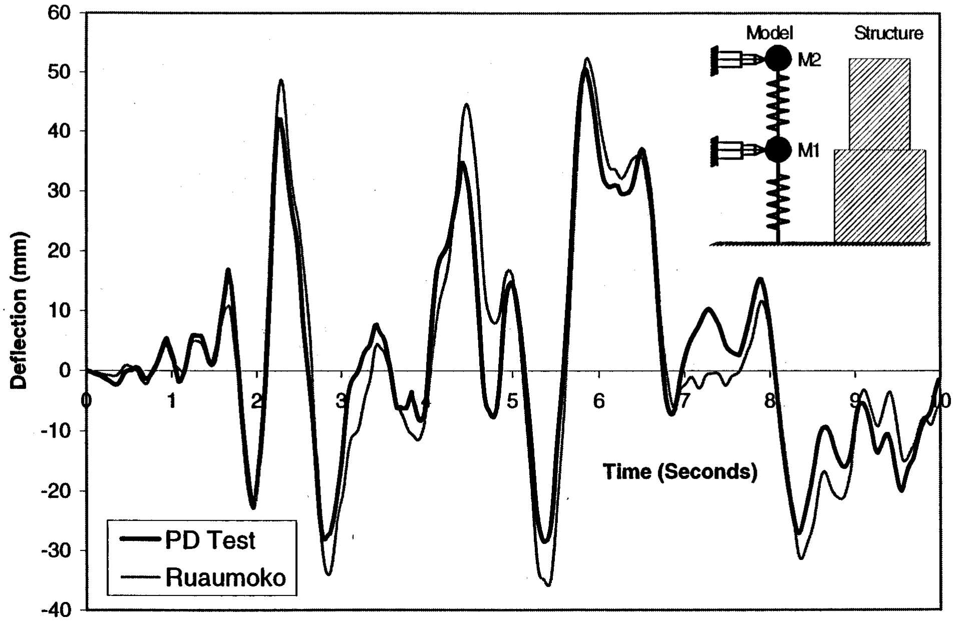


(b) 2.4m long plywood wall

Figure 4. Comparison of PD and ITHA for single storey walls



(a) Level 2 Deflections



(b Level 1 Deflections

Figure 5 Comparison of PD and ITHA for a Plywood Two Storey Wall