



H_∞ Direct Output Feedback Control of Structures

C.C. Lin & M.C. Pan & J.Y. Wei

Department of Civil Engineering, National Chung-Hsing University, Taichung, Taiwan 40227, Republic of China

ABSTRACT: In this paper, a H_∞ direct output feedback control algorithm is developed to reduce the structural responses due to seismic loads. It is done through minimizing the entropy, a performance index measuring the trade-off between H_∞ optimality and H_2 optimality. The control forces are calculated directly from the multiplication of output measurements by a pre-calculated time-invariant feedback gain matrix. In real active control, control force execution time delay cannot be avoided. This study derives explicit formulas to calculate the maximum allowable delay time, $(t_d)_{\max}$, to assure system stability. $(t_d)_{\max}$ can be increased with increasing structural original damping or appropriately selecting control parameters. Fewer sensors and controllers and simple on-line calculation make the proposed control algorithm favourable to real implementation.

1 INTRODUCTION

Since 1970s, remarkable progress has been made in the field of active control of civil engineering structures subjected to environmental loadings such as winds and earthquakes (Soong 2000). Among those researches and real applications, various control algorithms have been investigated in designing controllers, for instance LQ (Kobori 1991a,b), LQR optimal control (Chung 1993, Lin 1996) and H_∞ control (Kose 1996, Chase 1999). The H_∞ control theory considers the worst case of external disturbances to design the optimal controller to achieve the desired performance. The results from both numerical simulations and experimental tests indicate that H_∞ control is quite effective (Kose 1996, Chase 1999). In addition, it had been applied to an actual building in Tokyo using a pair of mass dampers to reduce the bending-torsion motions of the building due to earthquakes (Fujinami 2001).

However, there are still many problems that affect active control technique towards large-scale practical application. Time delay and limited number of sensors and controllers are two of these problems. Strictly speaking, a real structure has an infinite number of degrees of freedom. It is impossible to acquire full-state measurements and feedback to calculate the required control force. Therefore, a direct output feedback control algorithm becomes necessary from practical point of view. Moreover, in real active control systems, time is consumed in data acquisition, data processing, on-line calculation, and control force execution. There is always a delay between the time at which the control force is assumed to be applied and actually applied, and which may cause degradation in control efficiency or even render the system unstable. Therefore, a robust control with the consideration of time delay effect is necessary. In the literature, the time delay effect on the H_∞ active control system has not been investigated by researchers as much as those on the LQR optimal control (Yang 1990, Agrawal 1993, Lin 1996). Mahmoud et al. (1998) designed the H_∞ controller for a class of dynamical systems considering state and input delays. They expressed the control design procedures in the form of linear matrix inequalities. No time-delayed H_∞ output feedback control system was studied.

In this paper, a H_∞ direct output feedback control algorithm through minimizing the entropy, a performance index measuring the trade-off between the H_∞ optimality and the H_2 optimality, is

developed to reduce the structural responses due to seismic loads. To achieve optimal control performance, the strategy to select both control parameters \mathbf{g} and \mathbf{a} is extensively studied. The exact solution of output feedback gain for a single-degree-of-freedom (SDOF) damped structure is derived. It can be proved analytically that LQR control is a special case of H_∞ control. Direct velocity feedback control is effective in reducing structural responses with very small number of sensors and controllers compared with the degrees of freedom of the structure. Moreover, explicit formulas are obtained to calculate the maximum allowable delay time to avoid system instability. A formula is also derived to determine the critical control weighting factor of \mathbf{a} to assure the system stability. Finally, we propose some solutions to increase the maximum allowable delay time.

2 H_∞ DIRECT OUTPUT FEEDBACK CONTROL ALGORITHM

2.1 Control Algorithm

The equation of motion of an n-DOF discrete-parameter structure under dynamic loading $w(t)$ and active control force $u(t)$ can be written as

$$M\ddot{x}(t) + C\dot{x}(t) + Kx(t) = B_1u(t) + E_1w(t) \quad (1)$$

where $x(t)$ is $(n \times 1)$ displacement vector, M is $(n \times n)$ mass matrix, C is $(n \times n)$ viscous damping matrix, K is $(n \times n)$ stiffness matrix, B_1 is $(n \times q)$ location matrix for q control forces, and E_1 is $(n \times r)$ location matrix for r external loadings. For example, consider a n -story linear shear building with an active tendon control system installed in the first story as shown in Figure 1. The control force $u(t)$ is expressed by

$$u(t) = 4k_c U(t) \cos \mathbf{q} \quad (2)$$

where $U(t)$ is actuator stroke. k_c and \mathbf{q} are the stiffness and inclination angle of tendons, respectively. Substituting equation (2) into equation (1), the equation of motion represented in state-space form becomes

$$\dot{X}(t) = AX(t) + BU(t) + Ew(t) \quad (3)$$

where

$$X(t) = \begin{Bmatrix} x(t) \\ \dot{x}(t) \end{Bmatrix}, \quad A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 4k_c \cos \mathbf{q} M^{-1}B_1 \end{bmatrix}, \quad E = \begin{bmatrix} 0 \\ M^{-1}E_1 \end{bmatrix}$$

are $(2n \times 1)$ state vector, $(2n \times 2n)$ system matrix, $(2n \times q)$ controller location matrix and $(2n \times r)$ external loading location matrix, respectively. Define a $(p \times 1)$ control output vector $Z(t)$ and a $(s \times 1)$ output measurement vector $y(t)$ as

$$Z(t) = C_1X(t) + DU(t) \quad (4a)$$

$$y(t) = C_2X(t) \quad (4b)$$

In the absence of time delay, the actuator stroke of direct output feedback control is calculated by

$$U(t) = G y(t) \quad (5)$$

where G is a $(q \times s)$ time-invariant feedback gain matrix. From equations (2)-(5), it is derived that the transfer function of $Z(t)$ with respect to $w(t)$ is expressed by

$$T_{Zw}(j\omega) = (C_1 + DGC_2)[j\omega I - (A + BGC_2)]^{-1}E \quad (6)$$

Where $j = \sqrt{-1}$. According to the optimal H_∞ control algorithm, the controller is designed such that

the H_∞ norm of $T_{Z_w}(j\omega)$ satisfies the following constraint

$$\|T_{Z_w}(j\omega)\|_\infty = \sup \bar{\mathbf{S}}[T_{Z_w}(j\omega)] < \mathbf{g} \quad (7)$$

Where $\sup \bar{\mathbf{S}}$ is the largest singular value of $T_{Z_w}(j\omega)$, \mathbf{g} is a positive attenuation constant which denotes a measurement of control performance. It has been proved (Yaesh 1997) that an optimal H_∞ control system is asymptotically stable if there exists a matrix $P \geq 0$ that satisfies the following Riccati equation

$$(A + BGC_2)^T P + P(A + BGC_2) + \frac{1}{\mathbf{g}^2} PEE^T P + (C_1 + DGC_2)^T (C_1 + DGC_2) = 0 \quad (8)$$

The controllers that satisfy equation (7) are not unique and may have an unbounded closed-loop H_2 norm. Thus, one way to design the optimal H_∞ output feedback gain is to solve equation (8) with constraint of the H_2 norm. For example, the so-called combined H_2/H_∞ control problems which the H_∞ norm bound is imposed and the upper bound of the H_2 norm is minimized. Another example is to minimize the Entropy of transfer function $T_{Z_w}(j\omega)$

$$E_n(T_{Z_w}, \mathbf{g}) \equiv -\frac{\mathbf{g}^2}{2\mathbf{p}} \int_{-\infty}^{\infty} \ln |\det[I - \mathbf{g}^{-2} T_{Z_w}^*(j\omega) T_{Z_w}(j\omega)]| d\omega \quad (9)$$

where $T_{Z_w}^*(j\omega)$ is the conjugate transform of $T_{Z_w}(j\omega)$. It has been shown that the Entropy of a complex function is an upper bound of its H_2 norm. Therefore, the minimization of the Entropy is equivalent to limiting the magnitude of the H_2 norm. In addition, the Entropy of a function is a useful measurement of how its singular values are close to the upper bound, \mathbf{g} . By minimizing the Entropy, we push all singular values of $T_{Z_w}(j\omega)$ away from \mathbf{g} . From the result of Stoorvogel (1991), the Entropy is also expressed by

$$E_n(T_{Z_w}, \mathbf{g}) = \text{tr}\{E^T P E\} \quad (10)$$

The $\text{tr}\{-\}$ denotes trace of a square matrix. Then the optimization problem is converted to minimize the Entropy of equation (10) subject to the constraint of equation (8). Incorporated with the constraint, Lagrangian can be introduced as

$$L(G, P, \mathbf{I}) \equiv \text{tr}\{E^T P E + \mathbf{I} [(A + BGC_2)^T P + P(A + BGC_2) + \frac{1}{\mathbf{g}^2} PEE^T P + (C_1 + DGC_2)^T (C_1 + DGC_2)]\} \quad (11)$$

where \mathbf{I} is a $(2n \times 2n)$ Lagrangian multiplier matrix. Let $C_1^T D = D^T C_1 = 0$ and $D^T D = I$, the necessary and sufficient conditions for minimization are

$$(A + BGC_2)^T P + P(A + BGC_2) + \frac{1}{\mathbf{g}^2} PEE^T P + C_1^T C_1 + C_2^T G^T G C_2 = 0 \quad (12a)$$

$$(A + BGC_2 + \frac{1}{\mathbf{g}^2} EE^T P) \mathbf{I} + \mathbf{I} (A + BGC_2 + \frac{1}{\mathbf{g}^2} EE^T P)^T + EE^T = 0 \quad (12b)$$

$$B^T P \mathbf{I} C_2^T + G C_2 \mathbf{I} C_2^T = 0 \quad (12c)$$

Thus, the procedures to obtain the H_∞ direct output feedback gain matrix G are: (i) decide a control output vector of equation (4a), (ii) select a disturbance attenuation constant \mathbf{g} , (iii) solve equations (12a-12c) to obtain P , \mathbf{I} and G by any iterative scheme.

According to Yaesh (1997), the Riccati equation (8) can be rewritten as

$$A^T P + PA - P(BB^T - \frac{1}{g^2} EE^T)P + C_1^T C_1 + \mathbf{n}^T PBB^T P \mathbf{n} = 0 \quad (13)$$

where $\mathbf{n} = I - \mathbf{I}C_2^T(C_2\mathbf{I}C_2^T)^{-1}C_2$? For full-state measurement, $C_2 = I$, equation (13) reduces to

$$A^T P + PA + P(\frac{1}{g^2} EE^T - BB^T)P + C_1^T C_1 = 0 \quad (14)$$

Which is the same expression derived by Lin (2000) for the H_∞ state feedback control. Thus direct output feedback control is a general form of the state feedback control. Furthermore, the Entropy is a performance index measuring the trade-off between the H_∞ optimality and the H_2 optimality. When g approaches ∞ , the Entropy of equation (9) takes the form

$$\lim_{g \rightarrow \infty} E_n(T_{Z_w}, \mathbf{g}) = \frac{1}{2p} \int_{-\infty}^{\infty} \sum_i \mathbf{s}_i^2(T_{Z_w}(j\omega)) d\omega = \|T_{Z_w}(j\omega)\|_2^2 \quad (15)$$

and equations (12a-12c) reduce to

$$(A + BGC_2)^T P + P(A + BGC_2) + C_1^T C_1 + C_2^T G^T GC_2 = 0 \quad (16a)$$

$$(A + BGC_2)\mathbf{I} + \mathbf{I}(A + BGC_2)^T + EE^T = 0 \quad (16b)$$

$$B^T P \mathbf{I} C_2^T + G C_2 \mathbf{I} C_2^T = 0 \quad (16c)$$

Above equations have the same forms as those of the LQR optimal direct output feedback control (Chung 1993). It shows that the LQR direct output feedback control is a special case of the H_∞ direct output feedback control.

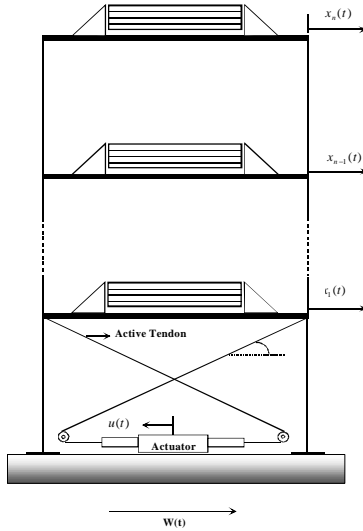


Figure 1. n -story shear building with active tendon control system

Table 1. System parameters for S1 structure

Mass , M (Kg)	2.9235×10^3
Stiffness , K (N/m)	1.3898×10^6
Damping , C (N · s/m)	1.5808×10^3
Natural frequency , w (Hz)	3.47
Tendon stiffness , k_c (N/m)	3.721×10^5
Tendon inclination , q (°)	36

2.2 Control weighting factor

Define the control output vector satisfying $D^T D = I$, $D^T C_1 = 0$ as

$$Z(t) = \mathbf{a} \begin{bmatrix} \Gamma \\ 0 \end{bmatrix} X(t) + \begin{bmatrix} 0 \\ I \end{bmatrix} U(t) \quad (17)$$

Where $Z(t)$ denotes the combination of structural displacement responses and actuator strokes and Γ is a scalar matrix with element 0 or 1. In equation (17), the control weighting factor, \mathbf{a} , determines

the relative importance between response reduction and control force requirement. The larger value of \mathbf{a} is, the greater reduction of responses. $\mathbf{a} = 0$ represents uncontrolled case.

For a SDOF structure with mass m , natural frequency \mathbf{w}_0 and damping ratio \mathbf{z}_0 , $\Gamma = [1 \ 0]$. The analytical expression of direct velocity feedback gain G is obtained by solving equations (12a-12c) as

$$G = -\frac{4k_c \cos \mathbf{q}}{m} P_v \quad (18)$$

And, the controlled frequency and damping ratio are expressed as

$$\mathbf{w}_c = \mathbf{w}_0, \quad \mathbf{z}_c = \mathbf{z}_0 + \frac{P_0^2}{P_1} \left(-\mathbf{z}_0 + \sqrt{\mathbf{z}_0^2 + \frac{P_1 P_2}{2\mathbf{w}_0^2}} \right) \quad (19)$$

In above equations,

$$P_0 = \frac{4k_c \cos \mathbf{q}}{m}, \quad P_1 = P_0^2 - \frac{1}{\mathbf{g}^2}, \quad P_2 = \mathbf{w}_0^2 \mathbf{g}^2 \left(1 - \sqrt{1 - \frac{\mathbf{a}^2}{\mathbf{w}_0^4 \mathbf{g}^2}} \right), \quad P_v = \frac{-2\mathbf{z}_0 \mathbf{w}_0 + \sqrt{4\mathbf{z}_0^2 \mathbf{w}_0^2 + 2P_1 P_2}}{P_1}$$

It is seen that, both control parameters \mathbf{g} and \mathbf{a} determine the control performance. $\mathbf{a} = 0$ leads to $G = 0$ and $\mathbf{z}_c = \mathbf{z}_0$ indicating the uncontrolled case, as expected. As \mathbf{g} decreases or \mathbf{a} increases, the controlled damping ratio \mathbf{z}_c increases. However, in order to confine P_2 and P_v to be real numbers and \mathbf{z}_c to be less than 100%, the selective ranges of \mathbf{g} and \mathbf{a} are

$$\text{If } \mathbf{g} \leq \sqrt{1 - \mathbf{z}_0^2} / P_0, \text{ then } \mathbf{w}_0^2 \mathbf{g} \sqrt{1 - \left(1 + \frac{2\mathbf{z}_0^2}{P_1 \mathbf{g}^2}\right)^2} \leq \mathbf{a} \leq \min[\mathbf{w}_0^2 \mathbf{g}, \mathbf{w}_0^2 \mathbf{g} \sqrt{2P_3 - P_3^2}] \quad (20)$$

$$\text{If } \mathbf{g} > \sqrt{1 - \mathbf{z}_0^2} / P_0, \text{ then } \quad 0 \leq \mathbf{a} \leq \min[\mathbf{w}_0^2 \mathbf{g}, \mathbf{w}_0^2 \mathbf{g} \sqrt{2P_3 - P_3^2}]$$

$$\text{where } P_3 = \frac{2(1 - \mathbf{z}_0)}{P_0^2 \mathbf{g}^2} \cdot \left[\frac{P_1(1 - \mathbf{z}_0)}{P_0^2} + 2\mathbf{z}_0 \right]$$

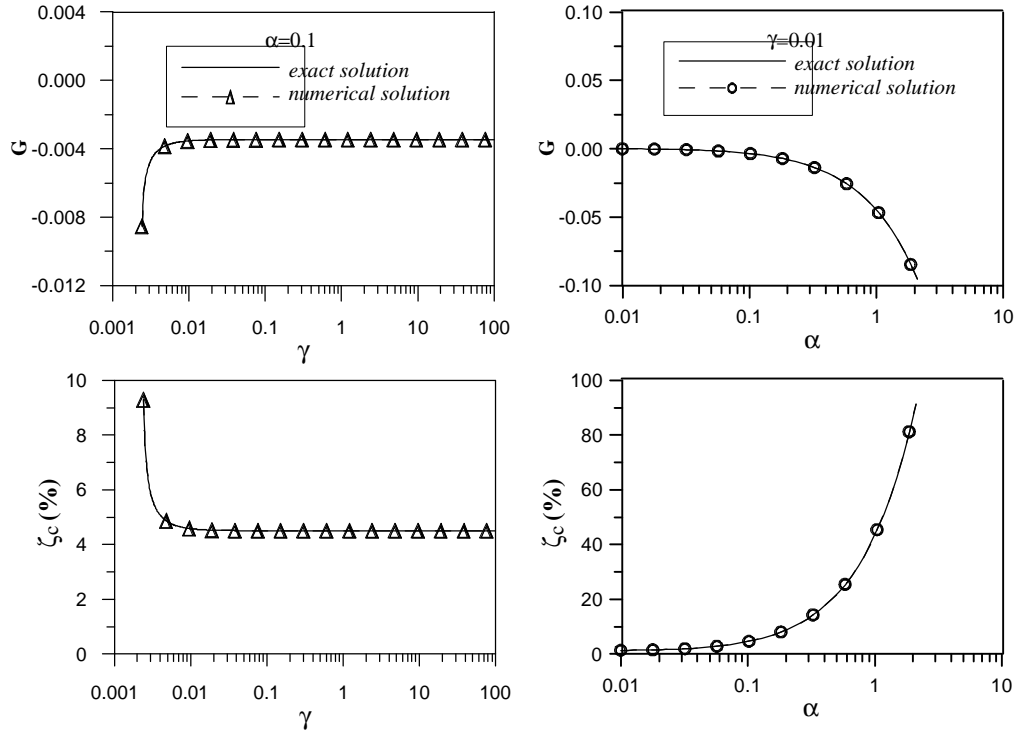


Figure 2. Variation of feedback gain G and controlled damping ratio \mathbf{z}_c for different \mathbf{g} and \mathbf{a}

2.3 Numerical simulation

A SDOF structure which system parameters listed in Table 1 is used for a parametric study to demonstrate the effectiveness of the proposed control algorithm. The analytical and numerical solutions of G and z_c of direct velocity feedback control for different g and a are shown in Figure 2. It is seen that the absolute values of G and z_c increase dramatically as g decreases or a increases. The controlled frequency w_c is always equal to the original natural frequency w_0 . Figure 3 illustrates the transfer function of absolute acceleration with respect to support acceleration for $g=0.01$ and $a=0.1, 0.5, \text{ and } 1.0$. It is found that the larger a , the smaller peak value of the transfer function. The time histories of acceleration response and control force with ($a=0.1, g=0.01$) and without control under the strong motion part of the free field ground acceleration (EW component) recorded at the campus of National Chung-Hsing University (NCHU) during the 1999 Taiwan Chi-Chi earthquake (Figure 4) are illustrated in Figure 5. Because the system damping ratio increases (from 1.24% to 4.58%), the acceleration response is significantly reduced. It is also seen that the required maximum control force is only 6% of the structural weight.

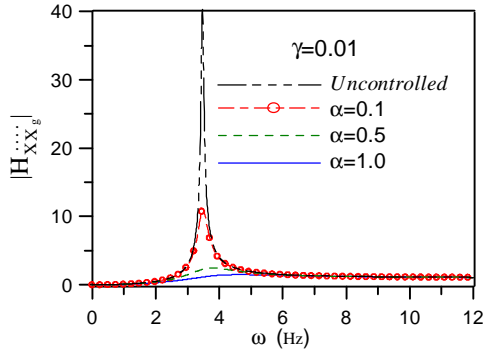


Figure 3. Absolute acceleration transfer function for different a

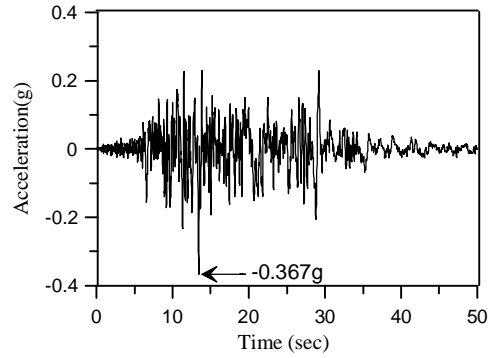


Figure 4. NCHU campus record (EW) of 1999 Taiwan Chi-Chi earthquake

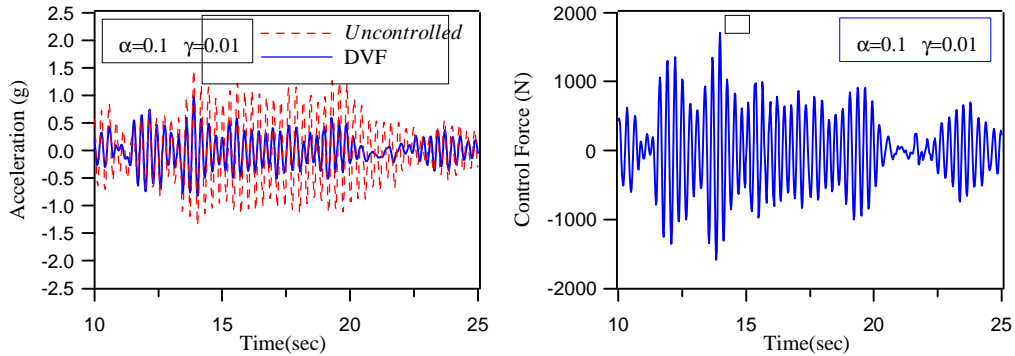


Figure 5. Time histories of acceleration response and control force

3 CONTROL FORCE EXECUTION TIME DELAY

3.1 Time delay effect

In real active control systems, time is consumed in data acquisition, data processing, on-line calculation, and control force execution. There is always a delay between the time at which the control force is assumed to be applied and really applied. That means the control force at time instant t is expressed as

$$u(t) = 4k_c \cos q G y(t - t_d) \quad (21)$$

where t_d is the delay time. Substituting equation (21) into equations (2) and (3), the system poles or eigenvalues are obtained by solving the following sets of homogeneous algebraic equations

$$\left| \bar{I} \cdot [I] - (A + e^{-t_d \bar{I}} \cdot B \cdot G) \right| = 0 \quad (22)$$

where I is the identity matrix and \bar{I} represents complex eigenvalues of the time-delayed control system. The corresponding controlled frequency and damping ratio are given as

$$\mathbf{w}_c = \left| \bar{I} \right| \quad \mathbf{z}_c = -\text{Re}(\bar{I}) / \mathbf{w}_c \quad (23)$$

When delay time t_d increases, the controlled damping ratio, \mathbf{z}_c , varies slightly for small t_d and then drops to zero very fast as shown in Figure 6 for the case of $\mathbf{g} = 0.01$ and $\mathbf{a} = 0.1, 0.5$, and 1.0 . The larger \mathbf{a} , the faster degradation of controlled damping. The maximum allowable delay time ($t_{d,\max}$) which causes system instability ($\mathbf{z}_c = 0$) will also decrease as \mathbf{a} increases.

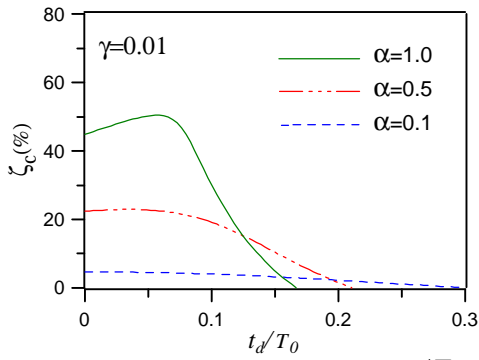


Figure 6. Variation of \mathbf{z}_c for different t_d/T_0

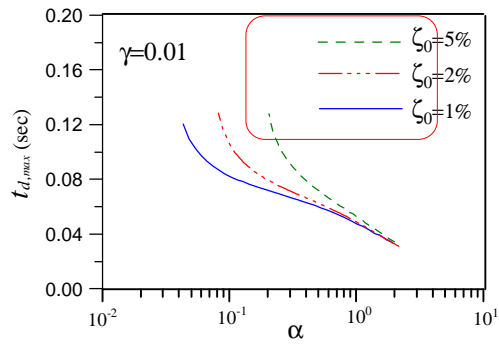


Figure 7. Variation of $t_{d,\max}$ for different \mathbf{a} and \mathbf{z}_0

3.2 Maximum delay time

For the SDOF active tendon system, the analytical expression of $t_{d,\max}$ for direct velocity feedback control is given as

$$t_{d,\max} = \sqrt{\frac{2}{2\mathbf{w}_0^2 - 4\mathbf{w}_0^2\mathbf{z}_0^2 + P_0^4 P_v^2 - \sqrt{4\mathbf{w}_0^4 + [2\mathbf{w}_0^2 - 4\mathbf{w}_0^2\mathbf{z}_0^2 + P_0^4 P_v^2]^2}}} \cos^{-1} \left\{ \frac{-2\mathbf{w}_0\mathbf{z}_0}{P_0^2 P_v} \right\} \quad (24)$$

It is observed from Figure 7 that $t_{d,\max}$ increases as system original damping ratio, \mathbf{z}_0 , increases and control weighting factor \mathbf{a} decreases. For a given damped structure, a critical (or maximum) control weighting factor, \mathbf{a}_{\max} , exists. When $\mathbf{a} < \mathbf{a}_{\max}$ is selected, the control system will remain stable even with long delay time.

3.3 Critical control weighting factor

For a given SDOF damped structure, the maximum control weighting factor (\mathbf{a}_{\max}) can be obtained as

$$\mathbf{a}_{\max} = \mathbf{w}_0^2 \mathbf{g} \sqrt{1 - \left(1 - \frac{2\mathbf{z}_0}{P_0^2 \mathbf{w}_0 \mathbf{g}^2}\right)^2} \quad (25)$$

which is derived by that the value of the arc cosine function in equation (24) equals to minus one, i.e. $(2\mathbf{w}_0\mathbf{z}_0)/(P_0^2 P_v) = 1$. The larger \mathbf{w}_0 or \mathbf{z}_0 is, the larger \mathbf{a} can be used.

4 CONCLUSIONS

This paper develops an optimal H_∞ direct output feedback control algorithm with the consideration of limited number of sensors and controllers and control force execution time delay. The closed form solutions of optimal output feedback control gains, controlled modal frequency and damping ratio are obtained. From the modal parameters, frequency domain and time domain analysis of controlled systems under earthquake excitations, it is demonstrated that direct velocity feedback is effective in reducing structural responses. The explicit formula is obtained to calculate the maximum allowable delay time. This quantity is a useful parameter for the design of control devices. Furthermore, the critical control weighting factor is also derived to avoid system instability. The allowable delay time is lengthened by adding structural dampings through passive dampers and/or selecting a control weighting factor smaller than the critical one. The control performance can thus be significantly improved even with time delay.

REFERENCES

- Agrawal, A.K. & Fujino, Y. & Bhartia, B.K. 1993. Instability due to time delay and its compensation in active control of structures, *Earthquake Engineering and Structural Dynamics*, Vol 22 211-224.
- Chase, G.J. & Breneman, S.E. & Smith, A.H. 1999. Robust H_∞ static output feedback control with actuator saturation, *Journal of Engineering Mechanics*, ASCE, Vol 125 225-233.
- Chung, L.L. & Lin, C.C. & Chu, S.Y. 1993. Optimal direct output feedback of structural control, *Journal of Engineering Mechanics*, ASCE, Vol 119 2157-2173.
- Fujinami, T. & Saito, Y. & Morishita, M. & Koike, Y. & Tanida, K. 2001. A hybrid mass damper system controlled by H_∞ control theory for reducing bending-torsion vibration of an actual building, *Earthquake Engineering and Structural Dynamics*, Vol 30 1639-1653.
- Kobori, T. & Koshika, N. & Yamada, K. & Ikeda, Y. 1991a. Seismic-response-controlled structure with active mass driver system. part 1: design, *Earthquake Engineering and Structural Dynamics*, Vol 20 133-149.
- Kobori, T. & Koshika, N. & Yamada, K. & Ikeda, Y. 1991b. Seismic-response-controlled structure with active mass driver system. part 2: verification, *Earthquake Engineering and Structural Dynamics*, Vol 20 151-166.
- Kose, I.E. & Schmitendorf, W.E. & Yang, J.N. 1996. H_∞ Active Seismic Response Control Using Static Output Feedback, *Journal of Engineering Mechanics*, ASCE, Vol 122 651-659.
- Lin, C.C. & Chung, L.L. & Lu, K.H. 1996. Optimal discrete-time structural control using direct output feedback, *Engineering Structures*, Vol 18 472-482.
- Lin, C.C. & Sheu, J.F. & Chu, S.Y. & Chung, L.L. 1996. Time delay effect and its solutions in direct output feedback control of structures, *Earthquake Engineering and Structural Dynamics*, Vol 25 547-559.
- Lin C.C. & Wei J.Y. 2000. Vibration Reduction and Stability Analysis of H_∞ Control Systems, ASCE, *Fourteenth Engineering Mechanics Conference (EM2000)*, Texas, USA.
- Mahmoud, M.S. & Terro, M.J. & Abdel-Rohman, M. 1998. An LMI approach to H_∞ -control of time-delay systems for the benchmark problem, *Earthquake Engineering and Structural Dynamics*, Vol 27 957-976.
- Soong, T.T. & Spencer Jr., B.F. 2000. Active, semi-active and hybrid control of structures, *Proc. 12th World Conference on Earthquake Engineering*, Auckland, New Zealand.
- Stoorvogel, A.A. 1991. The Singular Minimum Entropy H_∞ Control Problem, *System & Control Letter*, Vol 16 411-422.
- Yaesh, I. & Shaked, U. 1997. Minimum Entropy Static Output-Feedback Control with an H_∞ -Norm Performance Bound, *IEEE Transactions on Automatic Control*, Vol 42(6) 853-858.
- Yang, J.N. & Akbarpour, A. & Askar, G. 1990. Effect of time delay on control of seismic-excited buildings, *Journal of Structural Engineering*, ASCE, Vol 116 2801-2814.