2003 Pacific Conference on Earthquake Engineering

Earthquake duration effects on very low-cycle structural damage estimates

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ABSTRACT: The duration of strong ground motion has been shown to have a significant effect on the level of damage sustained by engineered structures during moderate to severe earthquakes. In this study, the authors develop and make use of the regressive relationship between the fundamental structural period of a system and the number of deformation response cycles during an earthquake of a specified duration. Damage is assumed to accumulate linearly in a structure or component based on the well-known Park-Ang damage model. The deformation response peaks for non-linear systems can be shown to closely fit a two-parameter Weibull distribution. Knowledge of the statistics of the response peaks for non-linear systems was combined with the theory of order statistics to quantify the effect of duration on damage estimates. It is concluded based on the mean structural reliability indices for a suite of non-linear systems that earthquake duration has a significant effect that it should be considered in seismic reliability analyses that focus on low-cycle damage.

1 INTRODUCTION

Response spectra have been widely accepted as a reasonable measure of seismic design kinematics enabling designers to estimate probable seismic loads. However, previous research by Jeong and Iwan (1988) revealed that the safety of a structure may depend on more than just the peak response. Specifically, structural failure under cyclic loading necessitates the need for further study of very low cycle failure based on the time history of the deformation, i.e. hysteresis. Thus, the use of the response spectrum alone to specify a design input ground motion neglects the effect of earthquake duration, or more specifically, cycling, on the damage of structures. Their results also found that the expected damage is highly dependant on both the ductility of the response and duration of the excitation. Rahnama and Manuel (1996) found that strong motion duration has no effect on strength demands but they state that duration should have a significant effect on cumulative damage measures.

The present paper focuses on quantifying the effect of cumulative, low-cycle, damage in a hypothetical suite of structures. An existing suite of ground motion records was scaled using linear response spectrum scaling from the United States Geological Survey map (Frankel et al. 1996). The structures were idealized as elasto-plastic (E-P) oscillators in order to simplify the computation and allow generalization of the results. Initially, the suite of E-P oscillators were excited using each realization in the suite of ground motion records. The resulting one-sided (positive) peaks for each response were fit to an extreme value Type III distribution (Weibull). Then, treating the Weibull parameters of the E-P response as random variables, the theory of order statistics was used to model the probability distribution of the m^{th} highest peaks for the entire family of ground motions within a Monte Carlo framework. By varying the number of trials in the order statistic, the probability distributions for the m^{th} highest distribution varied. The Park-Ang (1985) damage model was used to estimate the probability of failure and subsequently the mean reliability indices, based on a damage limit state, as a function of the duration of the earthquakes, as previously described. The mean reliability index for the entire suite of structures at each earthquake duration was selected as an arbitrary, but logical, way to measure the effect of duration. Also included was the 84th percentile.

2 DESCRIPTION OF APPROACH

2.1 Earthquake Demand

Ground motions suites corresponding to three different seismic zones were provided for the U.S. cities of Boston, Seattle and Los Angeles (Somerville et al. 1997). The ground motion suites for Los Angeles, California, were selected for presentation in this paper. Suites of time histories are provided at two probabilities of occurrence (2% in 50 years and 10% in 50 years) and are for the soil type B /C boundary, i.e. soft rock. For the present study the spectral acceleration levels were not modified to soil since the exact location of the site in Los Angeles is not specified. Uniform hazard levels that correspond to the seismic hazard levels used in codified design in the United States were selected, i.e. 2% and 10% exceedance in 50 years. A set of response spectra with periods ranging from 0.1 to 2.0 seconds at 5% damping (Somerville et al. 1997) were used in this study and are presented in Table 1.

Table 1. USGS 5% Damped Response Spectra for Site Category SB/SC for soft rock (Somerville et al. 1997)

Probability	**Location**	**Structural Period**					
		0.1	0.2	0.3	0.5	1.0	2.0
10% in 50 years	L.A.	0.905	1.121	1.07	0.705	0.381	0.186
2% in 50 years	L.A.	1.288	1.688	1.609	1.257	0.667	0.304

Once the target spectral accelerations were determined as shown in Table 1, and the suite of earthquake ground motions scaled to excite a linear oscillator with the appropriate structural period to the pseudo acceleration level, the ground motions were coupled with the structural models. The shaded values in Table correspond to the two structural periods making up the suite of structures in this study. For those readers unfamiliar with spectral scaling please refer to Chopra (2001), or any other earthquake dynamics text.

An elasto-plastic (E-P) oscillator is the simplest hysteretic oscillator used in structural dynamics but it possesses many properties that lend themselves well to sensitivity investigations such as the one presented here. The load-deformation relationship for an E-P oscillator is shown in Figure 1.

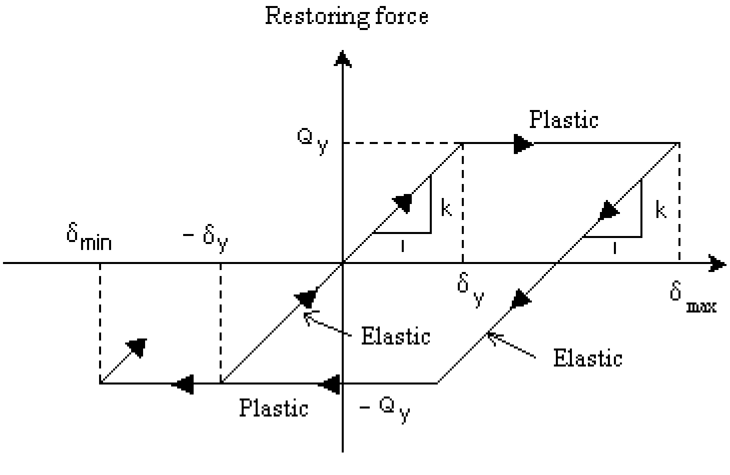

Figure 1: Illustration of Elasto-plastic (EP) hysteretic behavior

Interestingly, the peaks of a Gaussian process have been shown to be well-modelled by the Rayleigh distribution (Cartwright and Longuet-Higgins 1956). The response of a non-Gaussian process, such as the peaks of structural response during earthquakes, has been shown to be well-modelled by a Weibull distribution (Niedzwecki et al. 2000). Each suite of ground motions in the present study consisted of ten earthquake records for which details can be found in Somerville et al. (1997). The two-parameter Weibull distribution can be expressed as

$$F(x) = 1 - e^{\left(\frac{x}{l}\right)^{k}} \qquad (1)$$

where *x* is the random variable and l and k are the scale and shape parameters respectively. In the present study, l and k were treated as random variables in order to describe an entire family of earthquake responses representative of the suite of earthquakes, from which the responses were derived. For the E-P deformation peaks from the suites of ground motions, both parameters were found to fit a lognormal distribution with 95% confidence using a K-S test, and it was therefore felt to be an adequate model. However, the left and right tails of the lognormally distributed k and l variables were not included, i.e. an upper and lower truncated lognormal distribution was used. The authors reasoned that this was conceptually appropriate since the lognormal distribution is defined on the interval $\left[0, \infty\right]$, but the range of realistic structural responses is not. Finally, the earthquake demand in terms of the one-sided peaks of the E-P response is expressed in terms of l and k for each family, or suite, of earthquakes.

2.2 Damage accumulation during earthquakes

There are many different damage indices in existence. This study uses a damage model developed for reinforced concrete (Park and Ang 1985). The model expresses the damage caused by excessive deformation and the damage caused by repeated cyclic loading as a linear combination of the two. The damage index, *D*, can be represented as

$$D = \frac{d_M}{d_u} + \frac{y}{Q_y d_u} \int dE \qquad (2)$$

where δ_M is the maximum deformation of the oscillator under earthquake excitation, δ_u is the ultimate deformation of the oscillator under monotonic loading, Q_y is the yield strength, $\int dE$ is the incremental hysteretic energy, and y is the (experimental) calibration parameter. Values of the damage index, *D*, are such that $D \geq 1.0$ signifies complete collapse or total damage, depending on the details of the calibration. The model can be calibrated easily by setting $D = 1$ at failure for experimental data and solving for y. However, this will not be done in this study, but rather four different hypothetical values of y will be used. Each of those values correspond to structures with varying levels of sensitivity to damage from cycling. In order to determine the ultimate deformation, d_u, for an E-P oscillator, one only needs to multiply the estimated yield deformation, d_y, by a ductility factor, *m*, which gives

$$d_u = m_u d_y \qquad (3)$$

Substitution of equation (3) into equation (2) provides the E-P oscillator special case for the Park-Ang damage model

$$D = \frac{d_M}{m d_y} + \frac{y}{Q_y m d_y} \sum_{i=1}^{k} \mathrm{U}\left[d_i - d_y\right]\left(d_i - d_y\right) Q_y \qquad (4)$$

where d_i is the deformation for the i^{th} peak. The present analysis focuses on very low-cycle damage which the authors assume to be $n \leq 4$ in equation (4). It follows that the damage accumulation is a function of the statistical distribution of the response peaks. In turn, the probability density function (pdf) of the response peaks can be expressed by applying the theory of order statistics (Madsen et al. 1986) as

$$f_m(x) = \frac{n!}{(m-1)!(n-m)!}\{F(x)\}^{m-1}[1-F(x)]^{n-m} f(x) \tag{5}$$

where *F(x)* is the cumulative distribution function given in equation (1) and *f(x)* is its pdf. In other words, each peak, e.g. 1st highest peak, has an associated probability distribution which is related to k and l and the number of cycles the structure undergoes. The number of cycles, *N*, in turn, is related to the structural parameters and the earthquake duration.

2.3 Calculation of damage probabilities

The probability of failure can be defined using the damage model given in equation (4) as

$$\text{Failure probability} = P_f = \text{ Probability } \left[1 - D < 0\right] \tag{6}$$

or in terms of the E-P parameters and peaks as

$$P_f = P\left[1-\left\{\frac{d_m}{md_y}+\frac{y}{Q_y md_y}\sum_{i=1}^{k}\mathrm{U}\left[d_i-d_y\right]\left(d_i-d_y\right)Q_y\right\}<0\right] \tag{7}$$

Based on knowledge of only l, k, *n*, and *m* in equation (5), the distribution for the m^{th} highest peak in an earthquake can be generated. The damage index, *D*, can be readily calculated using equation (4) for *k* = 4 given a value of *F(x)*, and its derivative *f(x)*. Recall that the l and k parameters were modelled as truncated lognormal, thus by randomly generating variates of the appropriate lognormal cdf and holding *n* constant, the P_f can be calculated using equation (7). This well known approach of using the inverse of the cdf is known as Monte Carlo simulation (MCS) (Ayyub and McCuen 1997) and served as the framework for calculation of the P_f in the present study. It is of particular interest to use this method to investigate the effect of earthquake duration on the probability of failure, i.e. $P\left[D-1<0\right]$.

2.4 Quantifying the effect of earthquake duration for the E-P oscillator

Recall that the value *n* in equation (5) is analogous to the number of trials, or essentially the number of peaks during an earthquake. In general, a linear oscillator will oscillate at its natural period of vibration regardless of the frequency content of the excitation signal. This means, for an earthquake of duration, *d*, a linear oscillator will have d/T_n one-sided peaks, where T_n is the natural period of vibration of the linear oscillator. However, for a nonlinear hysteretic oscillator such as the E-P oscillator, this number can vary significantly depending on the properties of the oscillator. One characteristic of the E-P oscillator is that it can be fully described by two variables; T_n and Q_y. Of course, Q_y must be expressed in units of force and so some percentage of *mg*, where *m* is mass and *g* is gravity. Numerous earthquake ground motion records from around the United States were used to excite E-P oscillators having different structural periods and yield strengths in order to develop an E-P database that relates the number of peaks for an E-P oscillator to d/T_n for linear oscillators. In turn, this relates to the earthquake duration, *d*. The relationship between *n* for the E-P oscillators and earthquake duration was idealized using linear regression and it should be noted that the scatter was not considered in the present analysis. Figure 2 presents the relationship for E-P oscillators with varying yield strengths having T_n = 0.2 sec. The ordinate (labelled $\mathrm{N_{elastic}}$) presents the number of peaks calculated for a linear oscillator as described above and the abscissa (labelled $\mathrm{N_{elasto}}$) provides the number of peaks counted from the numerical results for the E-P oscillator.

3 ILLUSTRATIVE EXAMPLES

In order to quantify the effect of earthquake duration on very low cycle damage probabilities it was necessary to consider multiple examples with varying properties, i.e. a suite of oscillators. The probabilities for each of the earthquake families considered in Table 1 correspond to 475 and 2,475 year return periods. Thirty-two different structural models having varying ductility levels, damage calibration parameters, and yield strengths were used. Also calculated were the failure probabilities for a one-time threshold excursion set at the displacement corresponding to the ultimate strength given in equation (3). Mathematically, that failure probability can be expressed as

$$P_f = P\left[1 - \frac{d_M}{d_u} < 0\right] \tag{8}$$

which reduces to a simple level crossing problem. In order to investigate the effect of earthquake duration on damage probabilities, the earthquake duration was varied from 30 seconds to 90 seconds in increments of 5 seconds. This was accomplished without time domain simulation by varying the value of n (based on Figure 2) for generation of the m^{th} highest distribution, as shown by equation (5). All failure probabilities were calculated using Monte Carlo simulation (MCS). One thousand random variates were used to generate one thousand random values or k, and another one thousand random variates used to generate values of l. Hence, probability distributions for the m^{th} highest peak were different for each simulation within the Monte Carlo procedure.

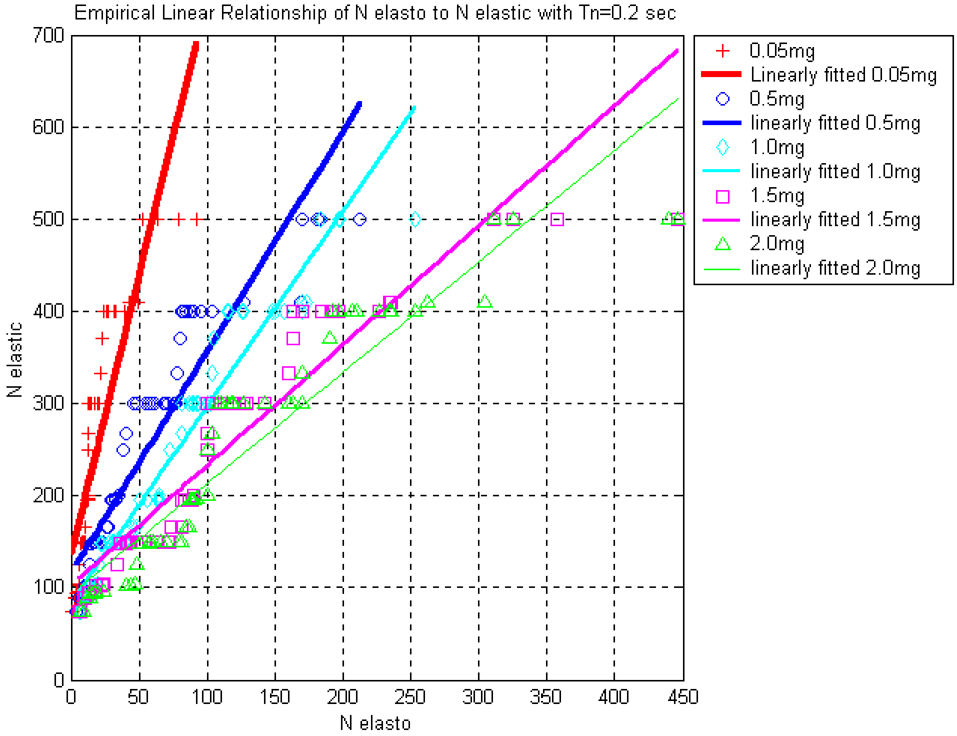

Figure 2: Regressive relationship between the numbers of peaks for a linear versus an E-P oscillator

Figure 3 presents the results of the analysis in terms of the reliability index, b , which can be calculated as

$$b = \Phi^{-1}\left\{1 - P_f\right\} \tag{9}$$

where $\Phi^{-1}\{\cdot\}$ is the inverse of the standard normal distribution function. The left window in Figure 3 shows the distribution of reliability indices, as calculated using equation (9), for the suite of oscillators. Notice that each line does not have 16 data points. This is because for some of the oscillators with combinations of low ductility and low yield strength, i.e. brittle and weak, the failure probability was equal to unity. Of course, this is only an approximation since 1,000 simulations were used and the failure probability is between zero and unity, not inclusive of them. The dashed line connecting the left hand plot and the right hand plot illustrates the connection between the two. For each duration on the right hand plot, i.e. every 5 seconds, it was necessary to calculate the values used for the left hand figure.

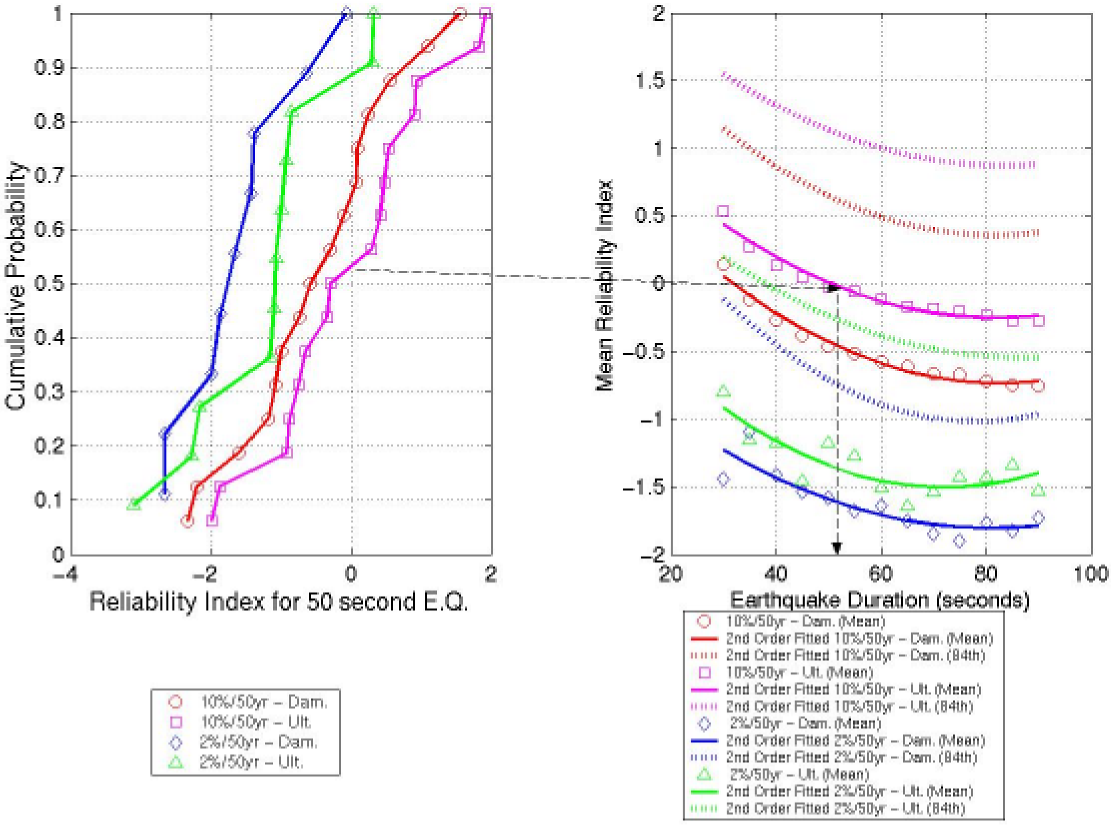

Figure 3 Reliability indices of the E-P suites for various limit states and the effect of earthquake duration

4 SUMMARY AND CONCLUSIONS

A combination of nonlinear structural dynamics and the theory of order statistics was applied to investigate the effect of duration on very low-cycle damage estimates. With the implementation of performance-based seismic design on the near horizon it was felt to be appropriate to use reliability indices as a measure. It can be concluded based on the results presented and supporting analyses (Goh 2002) that earthquake duration has a significant effect on reliability, that it should be included when low-cycle, and even very low-cycle, damage is under consideration.

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