

# Influence of hysteretic form on seismic behaviour of structures



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H. Judi, R. C. Fenwick and B. J. Davidson

*Civil and Environmental Engineering, University of Auckland*

**ABSTRACT:** In the seismic design spectra in the Loadings Standard no recognition is given for the influence of differing hysteretic behaviour associated with different materials, structural types and detailing standards. In practice hysteretic behaviour varies widely, from the near bilinear typical of eccentrically braced frames to the pinched forms associated with masonry.

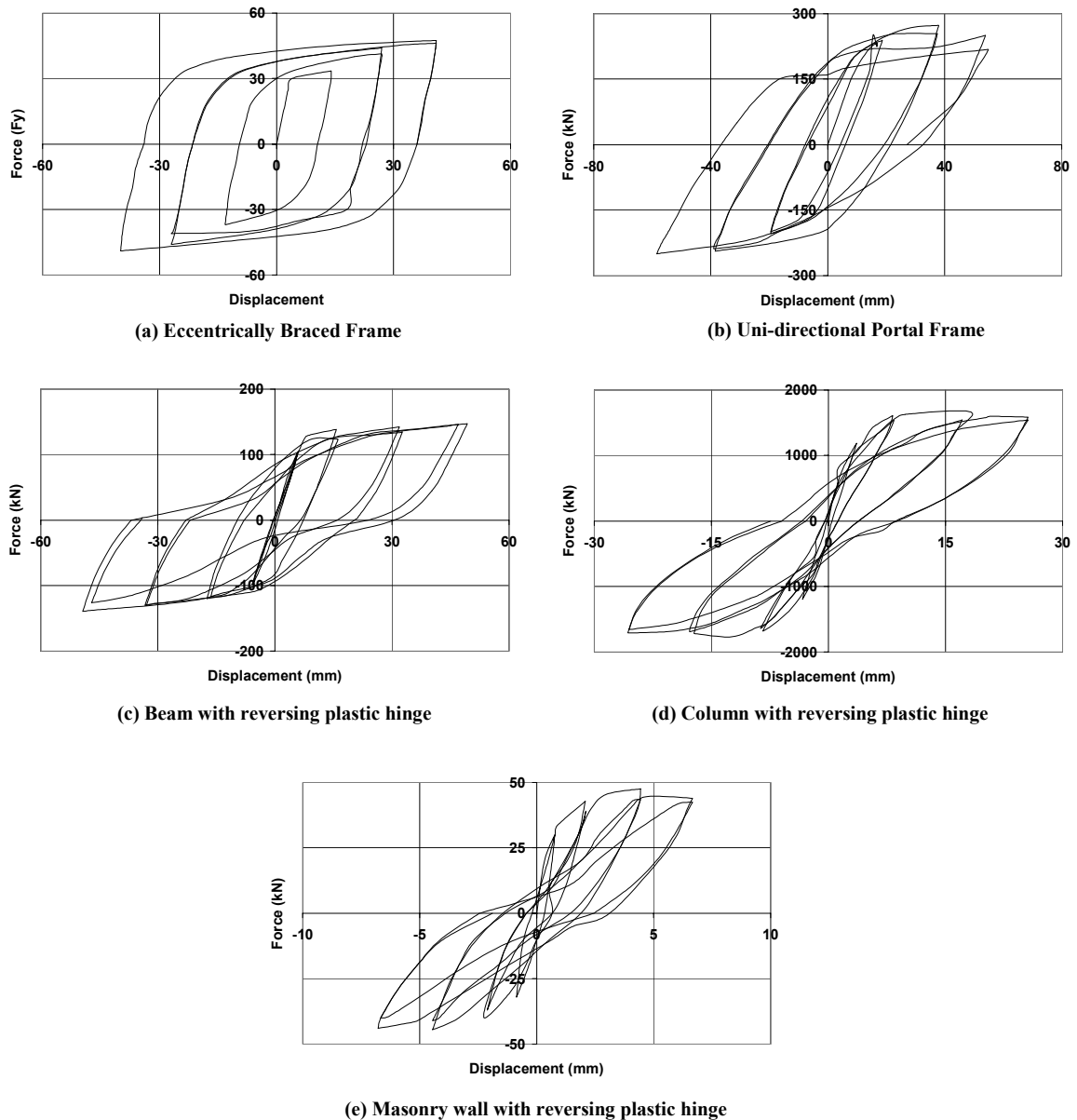
To investigate this a series of time history analyses were made with different ground motions and a range of hysteretic models. It is shown that the hysteretic form, provided that appreciable energy can be dissipated, has only a small influence on the maximum displacement that is sustained. Varying the viscous damping level was found to have a significant influence on elastically responding structures but a smaller effect on structures with some ductile capacity. Increasing the strain-hardening ratio was found to have a small influence on the required strength for a given ductility.

## 1 INTRODUCTION

The form of hysteretic response of structures varies very significantly with the structural form, materials, detailing and characteristics of the foundation. However, in the Loadings Standard [SANZ, 1992], and many other seismic codes of practice, only one set of response spectra are given to cover all hysteretic forms. Recently a number of proposals have been made to adopt a different method of seismic design, namely Displacement Based Design. These approaches, of which Direct Displacement Based Design [Priestley and Kowalsky, 2000] is one, recognise the influence of hysteretic form on seismic response. With this method structures which develop pinched load deflection response under cyclic loading are designed for a higher strength level than structures with a near bilinear response. This paper sets out to examine the influence of hysteretic form, viscous damping and strain hardening percentages on seismic response.

## 2 HYSTERETIC FORMS

As illustrated in Fig. 1 the hysteretic form varies with the materials and structural type. Fig. 1 (a) shows a near bilinear response, which was obtained from a test of a shear-yielding element for an eccentrically braced frame [Popov et al, 1987]. There was very little degradation on repeated loading cycles. As larger displacements were applied strain hardening increased the maximum load sustained. A bilinear model provides a conservative representation for this behaviour.



**Fig. 1 Force displacement relationships for different structural elements**

A similar hysteretic shape is obtained from reinforced concrete frames, which develop unidirectional plastic hinges [Megget and Fenwick, 1989] when subjected to inelastic cyclic loading. The lateral force versus displacement for such a case is shown in Fig. 1 (b). As the displacements are increased there is a limited amount of stiffness degradation. However, until the reinforcement buckles the response can be realistically represented by a bilinear model.

Fig. 1 (c) shows the force versus displacement response of a reinforced concrete beam subjected to cyclic inelastic loading [Fenwick et al, 1981]. In this case a reversing plastic hinge formed. The shear reversal results in a pinched response, which arises due to the yielding of the stirrups and the opening and closing of diagonal cracks in the plastic hinge. Under repeated loading to the same displacement some stiffness degradation occurs as additional yielding of the stirrups takes place. It should be noted that the unloading curves remain relatively steep, with little recovery in deflection when the load is removed.

Fig. 1 (d) shows the lateral force versus displacement for a reinforced concrete column subjected to cyclic loading [Gill, 1979]. In this case the column sustained an axial load of

" $0.21A_g f'_c$ ". There are two marked differences in the behaviour of columns as compared to beams. The first is that the axial load reduces the magnitude of the shear resisted by the stirrups. This limits the yielding of the stirrups and the width of the diagonal cracks. Consequently pinching of the response curve is greatly reduced. The second difference is in the stiffness of the unloading curves. With beams this stiffness is high, as the cracks that form due to tension yield of the reinforcement remain open when the load is removed. With the column the axial load helps to close these cracks when the lateral force is removed, and as a result there is greater recovery in displacement.

Fig.1 (e) shows the lateral force displacement response for a masonry wall [Brammer, 1995]. The relationship has some similarity to that of the reinforced concrete beam, which forms a reversing plastic hinge. However, with the masonry wall the stiffness degradation of the loading curves is higher as a result of greater shear deformation.

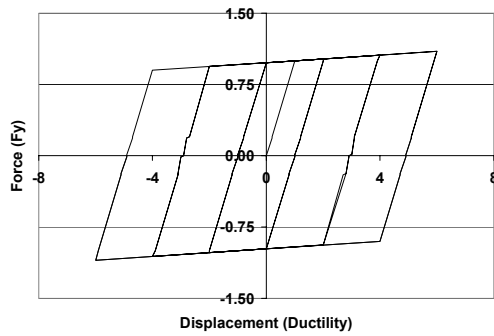
All the load deflection relationships shown in Fig. 1 dissipate appreciable energy by hysteretic behaviour of the materials. In this respect they are different from two further extreme cases. The first of these is for prestressed concrete members reinforced with unbonded cables. The response in this case is typically S shaped, with the loading and unloading curves lying very close to each other. Analyses using analytical models representing this behaviour indicate that the lack of energy dissipation increases the ductility demand typically by 50 percent when compared to a bilinear response [Priestley and Tao, 1993, Ouzounova, 1998]. The other extreme is for a tension braced pin jointed frame. A limited amount of energy may be dissipated by these structures with the diagonal bars yielding in tension. However, when load reversals occur the diagonal bars buckle and the stiffness drops to close to zero.

### 3 HYSTERETIC MODELS

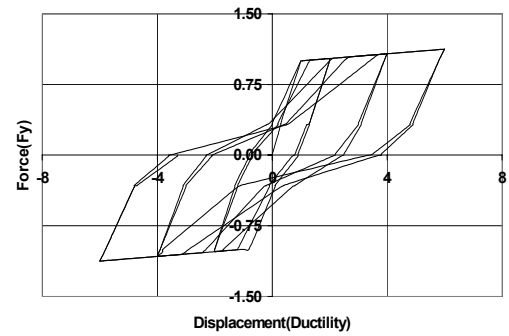
A basic hysteretic model has been developed for a single degree of freedom analysis program [Fenwick and Davidson 1994]. The loading relationship is represented by three straight lines, of which the third line represents strain hardening relationship. A further two lines represent the unloading relationship, so that a half cycle of loading involving inelastic deformation is represented by five changes of stiffness. The gradient of each line is controlled by coefficients, which vary with both the maximum displacement that is reached and the sum of the inelastic displacements sustained along the strain hardening lines. This basic model allows a wide range of hysteretic relationships to be represented. In particular it allows stiffness degradation to be modelled under cyclic loading conditions where the previous maximum displacements sustained in each direction are not exceeded.

The coefficients defining the gradients of the five lines have been chosen to give four different models for the analyses described in this paper. They are described below.

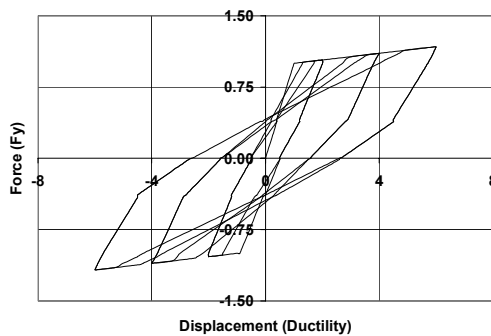
1. Bilinear model is illustrated in Fig. 2 (a). This is a standard model.
2. Beam model is illustrated in Fig. 2(b). The coefficients were derived from the results of reinforced concrete beams tested under cyclic loading such that they formed reversing plastic hinges [Fenwick et al 1981]. This relationship is representative of the lateral force displacement response of well detailed ductile concrete moment resisting frame structures, which form reversing plastic hinges. Pinching of the load deflection response and high unloading stiffness values are characteristics of this model.
3. Column model is illustrated in Fig.2(c). The coefficients were derived from test results of a reinforced concrete column [Ang, 1981], which sustained an axial load of  $0.2 A_g f'_c$ . As previously explained this gives unloading stiffness values that decrease as the magnitude of inelastic displacement increases.



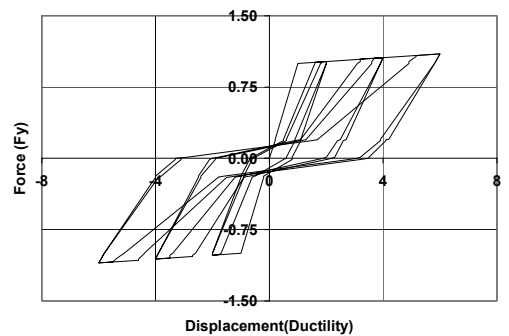
(a) Bilinear model



(b) Beam model



(c) Column model



(d) Masonry Model

**Fig. 2 Analytical hysteretic models**

4. The masonry model was developed from test results obtained from a wall test [Brammer, 1995]. As illustrated in Fig.2 (d) this is similar to the beam model but with greater pinching of the load deflection relationship.

These four models cover a wide range of hysteretic shapes met in practice. However, the extremes of prestressed concrete members with un-grouted cables and pin jointed tension braced frames have been excluded.

#### 4 ANALYSES

The analyses were made using a series of twelve earthquake ground motions, which are identified in ATC 40 [1996] as suitable candidates for time history analyses. All these records were recorded at sites with stiff to medium ground conditions located at least 10km from fault rupture. The magnitude of each event was not less than 6.5 and the peak ground acceleration was at least 0.2g. There are several records for each earthquake event.

##### ***Loma Prieta Earthquake, Oct. 17<sup>th</sup>, 1989***

1. Hollister, South Street and Pine drive, channel 1-90°
2. Hollister, South Street and Pine drive, channel 3-0°
3. Gilroy #2 Hwy, Bolsa Road Motel, Channel 1-90°, Gavilan, College Water Tank
4. Gilroy #2 Hwy, Bolsa Road Motel, Channel 3-0°, Gavilan College Water Tank

**Landers Earthquake, June 28th. 1992.**

- 4 Joshua Tree fire Station, channel 1-90°
- 5 Joshua Tree fire Station, channel 3-0°
- 6 Yermo Fire station, channel 1-360°
- 7 Yermo Fire station, channel 3-270°

**Northridge Earthquake, 17<sup>th</sup>. Jan., 1994**

- 9 Moorpark, channel 1-180°
- 10 Moorpark, channel 1-90°
- 11 Century City, Lacc North, channel 1-90°
- 12 Century City, Lacc North, channel 3-360°

In each group single degree of freedom structures were analysed for the 12 earthquake ground motions for 48 different structures. These had initial elastic periods of 0.3s to 5s with 0.1s steps. The analyses were made to determine the required strength for nominated ductility levels. In different sets of analyses the effect of changing the hysteretic model, the level of viscous damping and the strain hardening values were examined. Details are given in the next section.

## 5 RESULTS OF ANALYSES

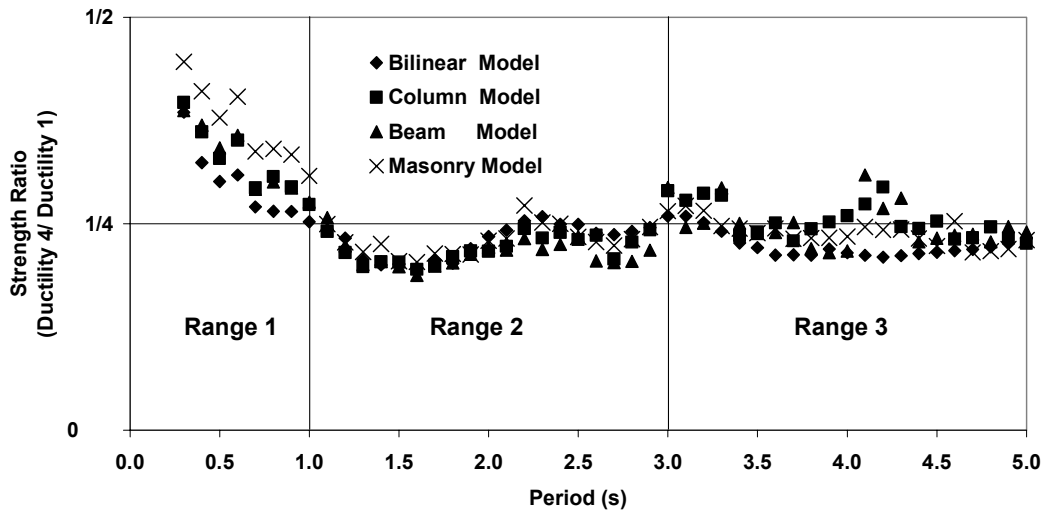
### 5.1 Influence of hysteretic form on response

In this set of analyses the strength required for ductility levels of 2, 4 and 6 were determined with all 4 hysteretic models. All the structures were given a viscous damping level,  $\xi$ , of 5 percent and a strain hardening ratio,  $\alpha$ , of 2.5 percent. The required strength for each individual period, ductility level and earthquake ground motion was divided by the corresponding strength required for elastic response. The values obtained for each period and specified hysteretic model were averaged for the 12 ground motions. Fig. 3 shows the results obtained for ductility 4. Very similar relationships were obtained for ductility levels of 2 and 6.

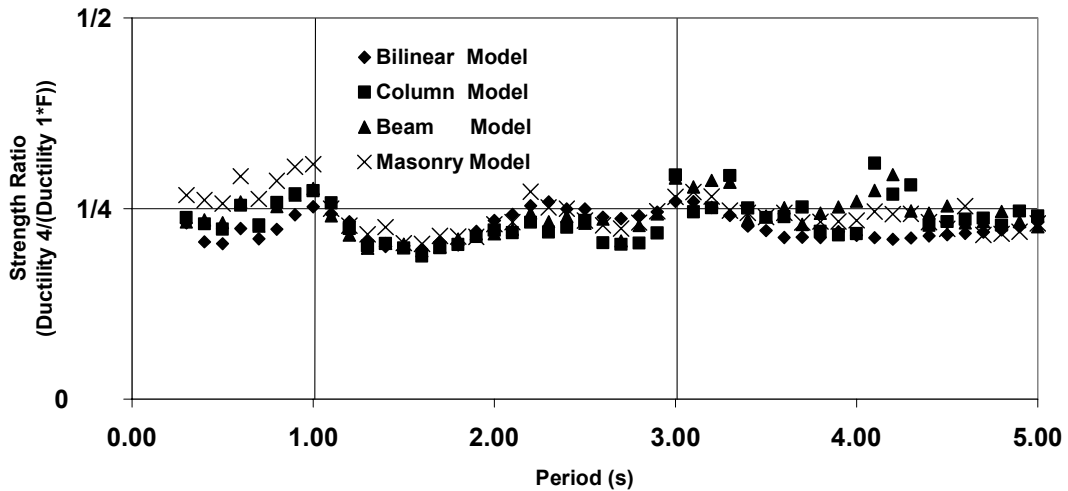
Fig. 3 (a) shows that the strength ratio for structures with a period greater than 1 second is close to the inverse of the ductility ratio, for all the hysteretic models. This corresponds to the equal displacement concept. For periods below the 1second value the required strength ratio is greater than that implied by the equal displacement concept. The current NZ Loadings Standard [SANZ, 1992] recognises this and the specified strength for structures with initial fundamental periods in the range of 0 to 0.7 seconds is increased above the level implied by the equal displacement concept.

One approach is to assume the required strength can be defined as the strength given by the equal displacement concept times a factor, F. As indicated in Fig. 3 (a) the value of "F" is close to 1 for the initial period in excess of 1 second. Values of the factor "F" are implied by the Loadings Standard in Table 4.6.4, and these have been calculated and listed in Table 1. However, an alternative set of values for "F" is also proposed. From Fig.3 (a) it can be seen that the increase in strength ratio is close to linear between a period of 1 and 0.3 seconds, with the increase corresponding to close to 1/6. A similar value occurs in the ductility 6 results and a slightly smaller increase occurs with the ductility 2 values. Assuming the value of 1/6 applies to the three ductility levels the proposed values for "F" have been found and listed in Table 1. Taking the results, such as those shown in Fig. 3 (a), and dividing by the appropriate values of "F," should ideally, lead to a uniform normalised value of strength ratio with period. The results for ductility 4 using the proposed values of "F" are shown in Fig. 3 (b). It can be seen that for practical purposes the desired result is obtained.

To enable the many analytical values to be interpreted the structures were divided into 3 groups, depending on the periods. Ranges 1, 2 and 3 were for structures with periods in 0.3 to 1s, 1.1 to 3s and 3.1 to 5s respectively. For the range 1 values the strength ratios were divided by the "F" factors and listed in Table 2, with the values in brackets corresponding to the "F" factors calculated from the Loadings Standard. The averaged values for each range are shown in Table 2.



(a) Strength ratios for different models



(b) Strength ratios normalised by factor (F)

**Fig. 3 Influence of hysteretic model on strength ratio**

**Table 1:** Values of factor F as defined by the Standard and as proposed

Period (s)	From Standard Ductility			As Proposed Ductility		
	2	4	6	2	4	6
0.3	1.22	1.36	1.44	1.33	1.67	2.00
0.4	1.22	1.36	1.44	1.29	1.57	1.85
0.5	1.16	1.28	1.31	1.24	1.48	1.71
0.6	1.08	1.12	1.14	1.19	1.38	1.57
0.7	1.00	1.00	1.00	1.14	1.29	1.43
0.8	1.00	1.00	1.00	1.09	1.19	1.29
0.9	1.00	1.00	1.00	1.05	1.09	1.14
1.0	1.00	1.00	1.00	1.00	1.00	1.00

From Table 2 and Fig. 3 it can be seen that for the three ductility ratios and the bilinear, beam

and column models the strength ratios were almost identical. On average the masonry model requires an increase of 8 percent more than for the other models. The general conclusion is that the form of hysteretic model made little difference to the strength.

**Table 2:** Average strength ratios normalised by “F”

$\xi\%$	$\alpha$ %	$\mu$	Range *	Bi-linear	Beam	Column	Masonry	Av. All	Code
5.0	2.5	2	1	0.434 (0.443)	0.453 (0.463)	0.454 (0.462)	0.486 (0.495)	0.457 (0.466)	0.5
			2	0.449	0.433	0.432	0.460	0.444	0.5
			3	0.469	0.459	0.465	0.480	0.468	0.5
5.0	2.5	4	1	0.244 (0.228)	0.247 (0.253)	0.248 (0.252)	0.280 (0.286)	0.255 (0.255)	0.25
			2	0.231	0.218	0.224	0.232	0.226	0.25
			3	0.222	0.232	0.256	0.244	0.239	0.25
5.0	2.5	6	1	0.147 (0.149)	0.169 (0.176)	0.169 (0.175)	0.192 (0.200)	0.169 (0.175)	0.167
5.0	2.5		2	0.156	0.147	0.159	0.167	0.155	0.167
5.0	2.5		3	0.143	0.176	0.188	0.177	0.171	0.167

\* Range 1 = 0.3 – 1s; Range 2 = 1.1-3s and Range 3 = 3.1 to 5s

### 5.2 Influence of damping ratio on strength

The analyses described in the first set were repeated but the viscous damping level,  $\xi$ , was set first to 0.5 percent and then to 2 percent. The required strength found in each analysis was divided by the corresponding strength for the same earthquake record and ductility level obtained with 5 percent viscous damping. The results are summarised in Table 3, together with values calculated from equations given by both Kawashima [1995] and Eurocode 8 [1996].

From Table 3 it can be seen that decreasing the damping below 5% increases the strength required. For the elastically responding structures in the period range 1 (0.3 to 1.0s) the strength increase is similar to that predicted by Kawashima and Eurocode 8. However, for ranges 2 and 3 the average strength increase is close to 7/8 and 2/3 of that in range 1 respectively. With a ductility of 2 or more the effect of a reduction in viscous damping is smaller. In this case the strength increase is typically 35% of that given by the Eurocode 8 and Kawashima equations.

**Table 3: Influence of viscous damping on ratio of strength required to 5%damping**

$\xi$ %	$\alpha$ %	$\mu$	Range	Bi-linear	Beam	Column	Masonry	Av. All	Std. Equ.
0.5	2.5	1	1	1.72	1.72	1.72	1.72	1.72	Kawashima 1.75 EC8 1.67
	2.5		2	1.60	1.60	1.60	1.60	1.60	
	2.5		3	1.46	1.46	1.46	1.46	1.46	
0.5	2.5	2	1	1.22	1.26	1.27	1.32	1.27	
	2.5		2	1.23	1.24	1.25	1.31	1.26	
	2.5		3	1.28	1.18	1.19	1.23	1.22	
0.5	2.5	4	1	1.12	1.18	1.23	1.27	1.20	
	2.5		2	1.18	1.20	1.19	1.25	1.20	
	2.5		3	1.21	1.17	1.16	1.25	1.20	
0.5	2.5	6	1	1.14	1.19	1.18	1.28	1.20	
	2.5		2	1.24	1.27	1.21	1.29	1.25	
	2.5		3	1.22	1.21	1.23	1.36	1.26	
2	2.5	1	1	1.31	1.31	1.31	1.31	1.31	Kawashima 1.33 EC8 1.32
	2.5		2	1.28	1.28	1.28	1.28	1.28	
	2.5		3	1.21	1.21	1.21	1.21	1.21	
2	2.5	2	1	1.14	1.15	1.16	1.19	1.16	
	2.5		2	1.11	1.14	1.14	1.18	1.14	
	2.5		3	1.13	1.10	1.10	1.12	1.11	
2	2.5	4	1	1.08	1.12	1.15	1.17	1.13	
	2.5		2	1.07	1.10	1.11	1.15	1.11	
	2.5		3	1.09	1.03	1.05	1.15	1.08	
2	2.5	6	1	1.09	1.13	1.13	1.16	1.13	
	2.5		2	1.09	1.13	1.10	1.19	1.11	
	2.5		3	1.09	1.07	1.05	1.26	1.12	

### 5.3 Influence of strain-hardening ratio

This set of analyses was similar to those in the previous section except the strain hardening ratio,  $\alpha$ , was first decreased from 2.5% to 0% and then increased to 5%. The viscous damping level was maintained at 5 percent. In this case only the bilinear and masonry models were analysed. The individual strengths with each earthquake record and ductility level were divided by the corresponding value found for the structure with 2.5% strain hardening. The results of these analyses are given in Table 4.

**Table 4: Influence of strain hardening level on strength ratio to 2.5% strain hardening**

$\alpha$	$\mu$	Bilinear Model				Masonry model			
		Period range				Period range			
		0.3 – 1s	1.1 – 3s	3.1 – 5s	Av.	0.3 – 1s	1.1 – 3s	3.1 – 5s	Av.
0	2	1.01	1.03	1.02	1.02	1.00	1.00	1.00	1.00
	4	1.09	1.09	1.10	1.09	1.01	1.00	1.01	1.01
	6	1.13	1.16	1.17	1.15	1.01	1.02	1.02	1.02
5	2	0.98	0.98	0.99	0.98	1.00	1.00	1.00	1.00
	4	0.95	0.95	0.94	0.95	0.99	1.00	1.02	1.00
	6	0.94	0.93	0.93	0.93	0.99	0.99	1.04	1.01

From Table 4 it can be seen that changing the strain-hardening ratio percentage has no appreciable influence on the masonry model. However, there is a significant influence with the bilinear model when the ductility level exceeds 2. For ductility levels of 4 and 6 reducing the strain-hardening percentage to zero increases the required strength by an average of 9 and 15 percent respectively. The corresponding change when the strain hardening ratio was increased to 5 percent was to give an average decrease in strength of 5 and 7 percent respectively.



## 6 CONCLUSIONS

- 1 Approximately 36 000 single degree of freedom analyses have been made using a range of earthquake ground motions to examine the influence of hysteretic form, viscous damping level and strain hardening characteristics, on the strength required for elastic and ductile structures. Structures which have load displacement responses, which can dissipate very little energy hysterically (ie prestressed concrete with unbonded tendons), were excluded from the investigation.
- 2 It is shown that with hysteretic form has only a minor influence on the strength required for a given level of ductility. This finding agrees with previous smaller studies [Anaganostopoulos and Roesset, 1974, Otani, 1980]
- 3 Reducing the viscous damping level below 5 percent is shown to have a significant influence on elastic response but an appreciably smaller effect on the strength required for structures with a ductility of 2 or more.
- 4 Changing the strain-hardening ratio from 2.5 percent to zero, or 5 percent, is shown to have only a minor influence on the required strength with the masonry model. However, with the bilinear model the significance of these changes increases with ductility. For a ductility of 6 reducing the strain-hardening percentage to zero increases the required strength by an average of 15 percent. The corresponding decrease when the strain hardening ratio is increased to 5 percent is 7 percent.

## 7 ACKNOWLEDGEMENTS

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