Effective ways to model earthquake risk

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ABSTRACT: Earthquake risk assessment should address the probability of occurrence of damaging earthquakes as well as their likely effects. New tools for performing this analysis in New Zealand include a seismicity model, a model for attenuation of strong ground motion, and estimates of likely damage as a function of the severity of ground motion. A Monte Carlo methodology provides a way of combining these in order to estimate various measures of risk.

Loss curves show the annual probability that particular loss values will be equalled or exceeded. The Average Annual Loss can be determined directly from the loss curve, but it is not a good measure of the risk for phenomena such as earthquake. The Systems Engineering literature provides procedures for objective use of other measures of risk, in particular the Conditional Expected Value. Effective risk management will require all these measures, in order to assess the likely benefits of mitigation.

1 INTRODUCTION

The Australian/New Zealand standard for risk management (Standards Assn of Australia, 1999) defines risk in terms of probability and consequences. The definition is compatible with that presented by the EERI Committee on Seismic Risk (1984). A series of studies of earthquakes in New Zealand have addressed the probability of strong ground motion (e.g. Smith, 1978; Smith & Berryman 1986; Matuschka et al 1985; Peek 1980, Stirling et al 2000) but none progressed to the stage of including its consequences. They are studies of hazard rather than of risk. Assessing the risk must include consideration of the consequences of strong ground motion. The treatment need not be confined to structures, for consequences could be measured in terms of economic loss, casualties or injuries, and the methodology would be the same. There is a further goal, that of risk management. This involves putting in place mitigation measures in response to the assessment of the risk. But that demands first of all reliable risk assessment, and the present paper seeks to develop procedures for that first task.

Earthquake risk assessment can be done on a variety of scales, from a single structure to the portfolio of an insurance company or an entire city. The methodology for all applications is essentially the same.

Assessment is often done in terms of scenarios. Particular large earthquakes are identified as the major contributors to earthquake risk in a given area. The ground motion and resulting damage are then modelled, and the result is an estimate of the likely losses from those particular events. This approach is important in that it addresses significant events that pose a threat to the region of interest. It gives rise to concepts such as the Probable Maximum Loss. Much effort has been expended, for example, in modelling the likely effects of an earthquake on the Wellington fault. But it should be noted that an earthquake of characteristic magnitude on the Wellington fault.
(about 7.3) is neither the only event that will damage Wellington nor the most likely. Lessons from Kobe, Northridge and Seattle include recognition of the damage that can be inflicted by events that are smaller and more frequent than “the big one”. Further, there is always uncertainty in the characteristic magnitude. So it is important to assess risk in probabilistic terms, to complement scenario studies. This involves evaluating a whole suite of scenarios, together with the probability of each.

2 SEISMICITY MODEL

The first requirement is a model of where and how often earthquakes occur. Smith (1978) used the historical record of large earthquakes, with the assumption that it was long enough to be representative. Smith & Berryman (1986) relaxed this assumption and included some geological inference. They defined a seismicity model in terms of a number of areal sources, each of which was assumed to be spatially uniform. Source parameters (Gutenberg-Richter parameters \( a \) and \( b \), and maximum magnitude \( M_{\text{max}} \)) were estimated for each region. These parameters were derived from the historical catalogue, but were adjusted in parts of the country where geological data suggested that the historical record was not an accurate guide to the true mean seismicity rate.

Stirling (2000) and Stirling et al (2000) have compiled detailed studies of faults in New Zealand into a seismicity model that incorporates (a) fault sources and (b) distributed seismicity. Each active fault is described by its location, geometry, slip type, likely magnitude and estimated recurrence interval. Seismicity that is not known to be related to active faults is modelled in a distributed way. At each point of a grid of locations there are parameters \( a \) and \( b \), and maximum magnitude \( M_{\text{max}} \), defining a Gutenberg-Richter relationship for annual frequency \( N \) as a function of magnitude.

\[
N = \begin{cases} 
10^{a-bM} & \text{for} \ M < M_{\text{max}} \\
0 & \text{for} \ M > M_{\text{max}}
\end{cases}
\]  

(1)

The result is a complete seismicity model which represents both the historical seismicity and also geological inference about the long-term behaviour of active faults. It describes all earthquakes likely to affect the site (or sites) of interest.

3 SYNTHETIC CATALOGUE

It is common practice to incorporate the seismicity model in the process of obtaining hazard estimates by integrating over the areal and line sources, and including their frequency-magnitude parameters. This is the procedure used by the Panel on Earthquake Loss Methodology (1989) (see especially Working Paper C), and also by Stirling et al (2000).

The procedure used here for calculating the probability of occurrence of strong ground motion is instead a Monte Carlo one. This is convenient because the formula adopted for attenuation of intensity with distance has two random terms (see section 4 below). While a Monte Carlo procedure is computationally expensive, it does provide a direct way of including the random terms. Further, it provides a straightforward means of progressing from hazard modelling to risk modelling, as will be described below.

For each fault source, the return period \( T \) relates to a probability \( 1/T \) that an earthquake will occur in any given year. If a random number in the range \((0,1)\) is generated for each year of the synthetic catalogue, each time it is less than \( 1/T \) indicates an earthquake. If it is greater than \( 1/T \), no earthquake is recognized. Examining all the faults in this way determines which of them will generate earthquakes in any one year, and the process is repeated for each year of the synthetic catalogue.

The distributed seismicity can be treated by considering each grid point in the distributed seismicity model as a separate source. Parameters \( a \), \( b \) and \( M_{\text{max}} \) determine the annual frequency
of occurrence $F$ of events over threshold magnitude $M_o$ (from equation 1). As for the fault sources, a random number is generated for each year of the catalogue; if less than $F$ it implies occurrence of an earthquake. In the Stirling et al (2000) model, at no site is the annual frequency $F$ high enough to allow a significant probability of more than one event above threshold magnitude $M_o$ in any one year. (More precisely, this applies for $M_o=5.25$, the threshold used here.) That is, at any grid point there is either one event in a particular year or none. The magnitude of that event is determined from a second random number in the range $(0,1)$, according to the formula:

$$M = - \log_{10} \left[ 1 - r \left(1 - 10^{-b(M_{max} - M_o)} \right) \right] / b + M_o$$

where $r$ is the random number.

This Monte Carlo process generates a suite of event dates and magnitudes, representing an earthquake population with a magnitude distribution given by equation (1). The process is repeated for each of the grid points in the distributed seismicity model. Adding all these sequences to the synthetic catalogue of earthquakes on known faults produces the complete synthetic catalogue for the whole country. The catalogue can be of any desired duration.

### 4 ATTENUATION FUNCTION

An attenuation function is needed in order to determine the severity of the ground motion at the site of interest, due to each of the events in the synthetic catalogue. For assessment of the risk of damage to a single structure, it is likely that the McVerry et al (2000) attenuation function will be useful. It predicts the acceleration response spectrum at any given location, given the magnitude and source geometry of an earthquake elsewhere. For a portfolio of properties, such as all domestic houses in a town, the Dowrick & Rhoades (1999) model for attenuation of Modified Mercalli intensity seems to be a more appropriate choice. For ordinary structures, there is a relatively consistent relationship between intensity and damage. The Dowrick & Rhoades model has been used here, with three modifications: a random term to represent the uncertainty in fitting the model to intensity data, a second random term to position the highest intensities anywhere along the fault trace, and a near-field adjustment for long fault sources (Smith, in prep). For earthquakes on known faults, a third random term has been introduced, in order to model uncertainty in the characteristic magnitude.

For each event in the synthetic catalogue, the intensity is determined at the site (or sites) of interest. This list of strong-motion events, from a catalogue of known length (e.g. 10,000 years) gives a measure of the hazard when expressed as annual probabilities (e.g. of exceeding MM 6, 7, etc).

### 5 DAMAGE

The risk assessment methodology requires a procedure for measuring consequences, however that is defined. We estimate the amount of damage using the damage ratio, which is the cost of damage as a proportion of the replacement value of the asset. This appears to be the best tool for estimating the total cost of damage to a number of buildings, because it correlates well with intensity. A series of studies (Dowrick 1991; Dowrick & Rhoades 1993, 1995, 1997; and Dowrick et al 1995, 2001) have estimated mean damage ratios for various classes of structures in New Zealand and also the proportion of buildings that sustain damage, for various intensity levels. Combining the list of intensities likely to be experienced at a property with damage ratio information and the current replacement value of the property results in a list of damage events, over the duration of the synthetic catalogue.

For a single engineered structure, more detailed estimates of damage might be obtained by using the McVerry et al (2000) attenuation relationship, together with detailed knowledge of the design and the likely performance of the structure. Either way, it is necessary to estimate the
cost of damage, given the parameters of ground motion.

There is one problem, however, in using the Dowrick damage ratios directly. For practical reasons they are defined for intensity zones (Figure 1) which are in turn defined by contours drawn around groups of equal integer intensity values plotted in maps. Attenuation models generally give decimal values of intensity which have to be truncated to give the integer intensities as seen in the intensity maps. While it would be feasible to truncate the calculated intensities and then use the Dowrick damage ratios directly, there is information in the decimal intensities that would be lost under such a methodology. We therefore prefer to fit the experimental damage ratios with a smooth function of the form

$$D_r = A \times 10^{-\frac{B}{MMI} - C}$$

(3)

where $D_r$ is the mean damage ratio, MMI the shaking intensity and A, B and C are positive constants. The function is based primarily on the New Zealand data for intensity zones MM6 and MM7 and a combination of New Zealand and United States information (Rojahn 1985) for zones MM8 to MM10. The general form of the function is illustrated in Figure 1.

![Figure 1: Vulnerability function for single-storeyed houses.](image)

6 COMBINING ALL THE FACTORS

It is a simple step from a list of occurrences of damage, over 10,000 years say, to a compilation of statistics. However it is important first to examine the dependence of the results on the length of the synthetic catalogue. Is 10,000 years enough to give a stable result? The answer turns out to be that it is not. Successive computer runs of this length yielded figures for the total loss over that period that varied considerably. Only when the duration was extended to 100,000 years, and the total loss compared with the value from a duration twice that length, was stability demonstrated. This was a valuable lesson in Monte Carlo procedures.

The cumulative probability function is shown in Figure 2. The abscissa is the value of the damage, and the ordinate is the probability that any given damage level is exceeded in one year.
Figure 2. Annual probability of exceedence for earthquake losses to domestic buildings in Hutt City.

The data used in Figure 2 were all the domestic buildings in Hutt City (total value $4871 million). Their locations have been grouped by suburb. More detailed modelling could use a GIS formulation, specifying the actual location of each building, though the effort is substantially greater and the advantage may not be significant in this case. The methodology is the same.

The total damage due to earthquake will be much greater than Figure 2 indicates, when other factors such as damage to commercial and public buildings and to infrastructure are added, and especially business interruption. These can all be modelled, albeit with some difficulty, but the present paper is not affected by its use of only domestic buildings in its damage portfolio.

7 THE FALLACY OF THE EXPECTED VALUE

The Average Annual Loss (AAL) is the Expected Value. It can be calculated from the curve in Figure 2, or directly from the computed losses over the duration of the synthetic catalogue. But while the Expected Value is a useful parameter in assessing risk it does not tell the whole story, especially for very large events that have a low probability of occurrence. Consider, for example, how society would react if (after Haimes, 1998):

(a) Our highways were constructed to accommodate the average traffic load of vehicles of average weight

(b) Telephone lines and switchboards were sufficient in number to accommodate only the average number of phone calls per hour

(c) Emergency services provided only the average number of personnel and facilities during all hours of the day and all seasons of the year.

It is clear that we need other measures of risk, to complement the expected value. This is well understood in the insurance industry. AAL is the appropriate measure for life insurance and general insurance, but for catastrophe insurance such as earthquake, volcano and flood other measures are necessary. Haimes (1998) suggests that other useful statistics are the conditional expected values. He partitions the probability axis and calculates the expected value of the loss, given that it lies within a specified probability range.

In the present methodology, the conditional expected values can be calculated from the loss curve (Figure 2) or directly from the individual loss estimates for synthetic events.
For the Hutt City data, the results are as follows

<table>
<thead>
<tr>
<th>Probability range</th>
<th>Conditional Expected Value ($m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 to 1 (AAL)</td>
<td>2.9</td>
</tr>
<tr>
<td>0.01 to 0.1</td>
<td>4.0</td>
</tr>
<tr>
<td>0.001 to 0.01</td>
<td>166</td>
</tr>
<tr>
<td>0 to 0.001</td>
<td>1112</td>
</tr>
</tbody>
</table>

8 DEAGGREGATION

When the likely damage is calculated, event by event, from a synthetic catalogue, it is a straightforward task to deaggregate the risk estimates, to determine which are the earthquake sources that contribute most to the risk. For these sources it may be appropriate to do more detailed scenario modelling so that their effects may be estimated more precisely.

Figure 3 shows the deaggregated risk for domestic buildings in Hutt City. The 475-year loss figure (10% probability in 50 years) for this portfolio is $479 million. Faults that can cause this level of loss are shown as walls, and the height of each indicates the relative frequency of occurrence. By far the most important is the Wellington fault, which contributes 82% of these events. The Wairarapa fault contributes 12%, the Ohariu fault 4% and the Moore’s Valley fault 2%. No earthquakes in the background seismicity model can cause this level of loss.

Figure 3. Deaggregation of earthquake risk for domestic buildings in Hutt City.
9 SCENARIO STUDIES

The likely losses from individual earthquake sources can be modelled as in Figure 4. Losses from a Wellington fault earthquake have been modelled 1000 times, allowing for all the uncertainties mentioned, to produce a cumulative distribution and a histogram of likely losses. The mean loss is $890 million, and the 90-percentile $1130 million.

![Cumulative distribution and histogram of likely losses](image)

Figure 4. Hutt City losses (domestic buildings) from 1000 occurrences of a Wellington fault earthquake: cumulative distribution (left), and distribution at $100 million increments (right).

10 RISK MANAGEMENT

The risk management problem is to assess what level of mitigation is appropriate. For a proper analysis of this it will be necessary to model the various expected value parameters under several different levels of mitigation expenditure. Haimes (1998) proposes his Partitioned Multiobjective Risk Method for such problems. It involves assigning relative weights to the different conditional expected values, then optimising them jointly with the cost of mitigation. This is beyond the scope of the present study, which seeks merely to tackle the assessment methodology and to lay a necessary basis for further developments.

11 CONCLUSIONS

Probabilistic assessments of earthquake risk should be used to complement scenario studies. This requires a seismicity model, an attenuation function for strong ground motion, measures of vulnerability and assets data. A Monte Carlo technique has been used here for preparing a synthetic catalogue. It provides stable estimates of the probability of exceedance of a chosen range of damage levels, provided that a synthetic catalogue of sufficient duration is used. The Average Annual Loss and the conditional expected values of loss can be determined by integration from the probability of exceedance curve, or directly from the Monte Carlo procedure.

For risk management it will be necessary to investigate the reduction in risk that would result from proposed mitigation measures, in order to make decisions about mitigation.

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REFERENCES


Smith, W.D. in prep. *A model for MM Intensities near large earthquakes*.

