# Dynamic Analysis of RC Frames Including Buckling of Longitudinal Steel Reinforcement

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ABSTRACT: The overload behaviour and mode of collapse of a reinforced concrete frame is strongly influenced by the ability, or otherwise, of critical regions in the beams and columns to undergo plastic deformations. In some circumstances outward buckling of the compressive reinforcing bars can follow spalling of the compressive cover concrete and lead to local failure and overall system collapse. In this paper compressive bar buckling is treated in a simplified manner by modifying the constitutive relations for the bar. This simplified treatment is incorporated into a finite-element based analysis procedure which predicts overload behaviour and collapse of concrete frames under severe dynamic loads.

## 1 INTRODUCTION

In recent studies of concrete frames under dynamic loads (Kawano et al., 1998), a non-linear finite-element simulation procedure was developed to predict the full range behaviour and collapse of the frame. The analysis considered only flexural behaviour of the components, and ignored secondary effects such as shear deformations, local shear failure, local joint deformations, joint failure, and local collapse in high moment regions as the result of effects such as compressive bar buckling.

Simulation studies undertaken with the computer program based on the analytical technique have shown that the predicted earthquake magnitude for complete collapse is unrealistic when these secondary effects are ignored. For example, system collapse was found not to occur, even though local flexural deformations had occurred in some beams of such magnitude that the concrete was completely crushed. In such situations buckling of the compressive bars would rapidly lead to system collapse.

The analysis procedure and the associated computer program are therefore being progressively upgraded to model these important second order effects. This paper gives details of the simplified treatment being considered to model the buckling of compressive reinforcing bars in overloaded beams and columns, and hence the effect of local bar buckling on the overall behaviour and collapse of the frame.

## 2 OVERLOAD BEHAVIOUR OF COMPRESSIVE REINFORCING BARS

The behaviour in the compressive face of a concrete member at overload depends on a variety of factors, including: the size and shape of the cross-section, the amount of longitudinal steel in the compressive face, the amount of transverse reinforcement providing confinement to the section, the amount of cover concrete, and the material stress-stain properties for the steel and concrete.

Figure 1 shows a short length of a beam or column, and its cross section with reinforcing bars in the compressive face. Rectangular lateral steel reinforcement is provided at spacing $s$. The tendency for the compressively loaded steel bars to buckle and deflect outwards is initially resisted by the lateral restraint provided by the surrounding cover concrete as well as the transverse steel ties (or stirrups).

As the compressive loads increase and approach the section capacity, the concrete surrounding the compressive bars carries large longitudinal compressive stress, and eventually becomes prone to longitudinal cracking, and spalling. After spalling of the cover concrete has occurred, it is left to the stirrups to provide the restraint against lateral movement and buckling.

For the corner compressive bars, the two transverse legs of a stirrup (Fig. 1c) provide a comparatively stiff support. This is not the case for an intermediate bar (Fig. 1d), where the support is provided by one lateral arm of the stirrup and relies on the flexural resistance of the stirrup to prevent outward buckling. Intermediate bars will therefore buckle first due to their reduced lateral restraint.

The effect of buckling on the load-deformation behaviour of steel reinforcing bars has been studied under monotonic (Monti & Nuti, 1992) and cyclic (Rodriguez et al., 1999; Suda et al., 1996) loads. From their results, these researchers developed modifications to steel constitutive relationships to take into account the buckling of longitudinal reinforcement. Of particular importance has been the post-buckling load-deformation behaviour of compression reinforcement.

Monti & Nuti (1992) introduced modifications to the Menegotto & Pinto (1973) steel cyclic stress-strain relationship for reinforcement based on their monotonic testing of reinforcing bar samples of varying effective lengths and four hardening rules (kinematic, isotropic, memory, and saturation). Their results indicated that for stirrup spacing, $s$, to bar diameter, $d_b$, ratios ($s/d_b$) less than 5 no buckling occurs and compressive behaviour is similar to tensile behaviour. For $s/d_b$ ratios above 11 buckling occurred as soon as the steel yielded. For $s/d_b$ ratios between 5 and 11 post-buckling softening occurred after the steel had yielded.

Suda et al. (1996) used a stress sensor to measure the strains in a reinforcing bar in a column that was subjected to cyclic load. Once the concrete had spalled, the reinforcement was able to buckle and the stress-strain curve softened with increased compressive strain. When the load was cycled again, the maximum compressive stress reached was between 10-20% of the

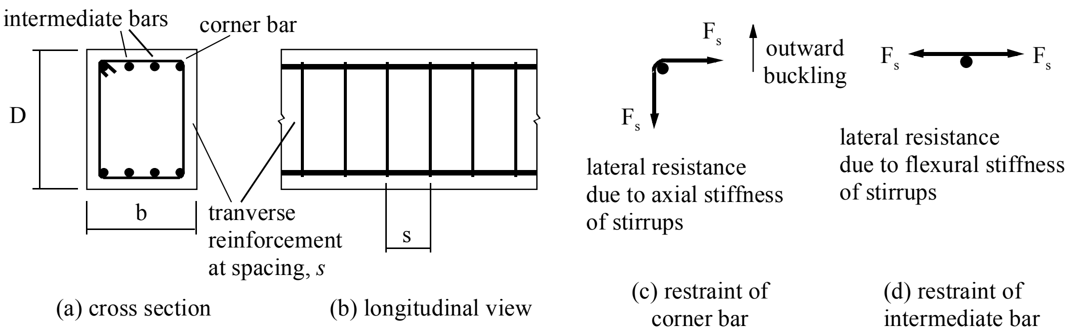


Figure 1 Restraint details for corner and intermediate bars.

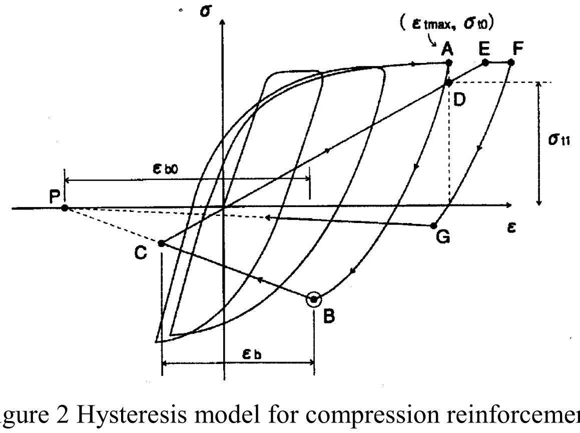


Figure 2 Hysteresis model for compression reinforcement after buckling (after Suda et al., 1996).

previous buckling stress (compare points B to G in Fig. 2). A new stress-strain relationship shown in Figure 2 was proposed for post-buckling behaviour. The authors indicated that the splitting force of concrete is important to the buckling behaviour.

Using a simple buckling model between consecutive stirrups, Gomes & Appleton (1997) developed a stress-strain diagram for the interaction of a bar under axial compression and bending. This interaction diagram was superimposed onto the Menegotto & Pinto (1973) constitutive relationship. For reversed loads from the tensile region the interaction diagram was superimposed at the point of zero stress.

## 3 USE OF A MODIFIED CONSTITUTIVE RELATION TO TREAT BAR BUCKLING

Previous attempts to model buckling at a local steel level have resulted in post-buckling softening being introduced after a critical value of stress or strain has been exceeded. These methods can be quite complex in their application and can be simplified by using a predetermined buckling strain, softening slope and stress degradation. This simplification can have a significant effect on the structural response and provide a mechanism for further strength degradation to occur and the eventual collapse of the structure. The analytical method adopted in this research is based on a modified Ohi & Akiyama model for the cyclic steel constitutive relations (Kato et al., 1973; Meng et al., 1992). The modifications were introduced to account for the buckling of compression steel reinforcement at a fibre level.

For monotonic loading, buckling and lateral displacement of the bar is assumed to begin at a critical buckling strain, $\varepsilon_{lb}$, indicated by point A in Figure 3. If, after buckling, the compressive strain in the bar continues to increase, the stress is assumed to fall-off progressively with a slope of $\tau_{lb}E_s$, where $E_s$, is the Young's modulus for longitudinal reinforcement. Softening continues until a second limiting strain, $\varepsilon_{ps}$, is reached at point B. With further increases in compressive strain (past point B), the stress in the bar is assumed to soften at a less severe rate of $-0.005E_s$.

The extension to the cyclic case is the same as for monotonic, however, the state of stress and strain must be known throughout the analysis to ensure post-buckling softening is applied at correct strains. Compressive strain may be applied after a significant period of tensile strain. With increasing tensile strain from the origin (point O) in Figure 4, the steel is strained past yield until point C. After point C, compressive strain is applied and the reversal of load is modelled using normal constitutive relationships until point D, where the buckling strain criteria has been exceeded. For this case the strain is still in the tensile range but has been sufficiently compressed to buckle the steel reinforcement. The stress at this point is equal to the $\sigma_{lb1}$. From points D to E post-buckling softening is applied to the model. Point E signifies another load reversal and tensile strain is added until point F, where the normal constitutive relationships are

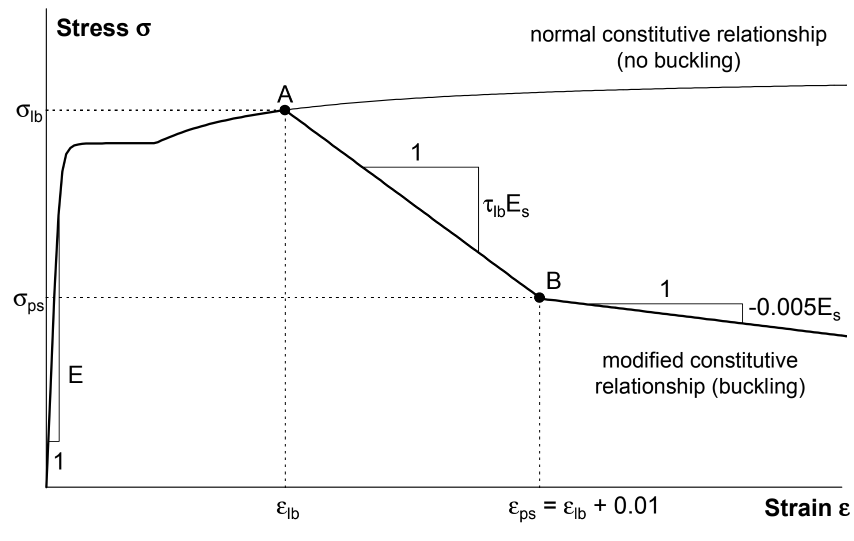


Figure 3 Buckling model for monotonic behaviour of compressive reinforcement.

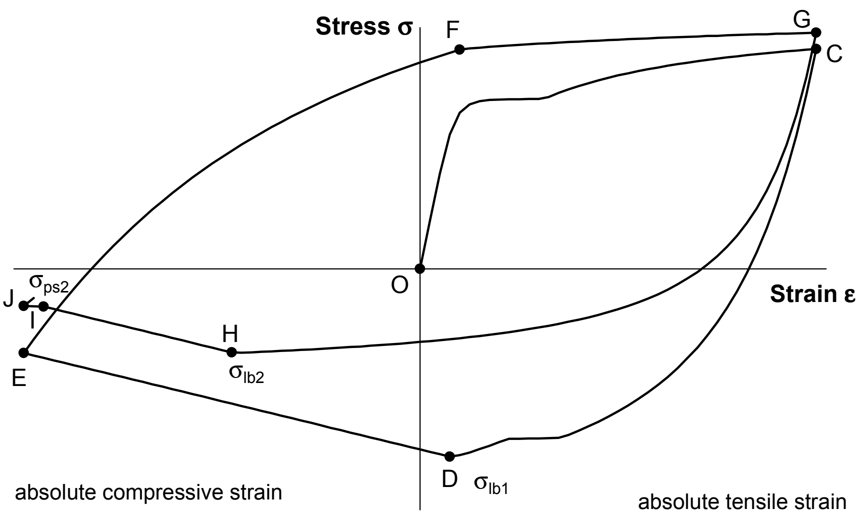


Figure 4 Buckling model for cyclic behaviour of steel reinforcement.

again followed to point G, where another load reversal is experienced. For this load cycle the stresses are lower than the first cycle and the buckling criteria is reached for cycle 2 at point H. After point H the stresses reduce linearly from $\sigma_{lb2}$ to $\sigma_{ps2}$ at point I. Following point I, the post-buckling softening slope is reduced until point J, where there is another load reversal. The model then continues for many cycles depending on the nature of loading.

To evaluate the critical buckling strain, $\varepsilon_{lb}$, relationships such as those developed by Nakatsuka et al. (1999), Papia & Russo (1989) and Kato et al. (1996) can be used. Alternatively the model can be fitted to experimental data if available. In this work the Nakatsuka et al. (1999) relationship given in Equation 1 was adopted due to its simplicity for rectangular transverse reinforcement. This relationship takes into account the effects on buckling strain due to: lateral reinforcement spacing, $s$, to confined core diameter ratios; confining stress; yield strength of lateral reinforcement, $f_{yh}$; shape of reinforcement (circular, rectangular); and the compressive strength of plain concrete.

$$\varepsilon_{\mathrm{bu}} = \varepsilon_{c0} + f_1 f_2 f_3 f_4 f_5 \tag{1}$$

where, $$f_1 = \begin{cases} 3.6 - 4.8(s/d) & 0.1 \le s/d \le 0.75 \\ 0 & s/d > 0.75 \end{cases}$$

$$f_2 = \left(p_s\, f_{yh}\right)^2$$

$f_3 = 1.0$ for bar in circular column; 0.9 for corner bar; 0.18 for intermediate bar

$$f_4 = \begin{cases} 110/{}_c\sigma_b - 1 & 30\text{MPa} \le {}_c\sigma_b \le 110\text{MPa} \\ 0 & {}_c\sigma_b \ge 110\text{MPa} \end{cases}$$

$$f_5 = \left(\frac{600}{f_{yh}} + 0.5\right) \times 10^{-4} \qquad f_{yh} \ge 400\text{MPa}$$

where, $d$ = smallest side length of concrete cross section surrounded by lateral reinforcements, ${}_c\sigma_b$ = cylinder concrete strength, $\varepsilon_{c0}$ = maximum strain of plain concrete and $p_s$ = reinforcement ratio of transverse reinforcement to concrete core.

The post-buckling slope, $\tau_{lb}$ is calculated using Equation 2, a relationship developed by Inoue & Shimizu (1988) for post-buckling behaviour of steel struts. The second post-buckling slope is set at $-0.005E_s$ referring to the report by Yamada et al. (1993) for the buckling of steel plates.

$$\tau_{lb} = 100\varepsilon_{sy}\left(\frac{1}{\sqrt{1+0.005\lambda^2}}-1\right) \tag{2}$$

where, $\varepsilon_{sy}$ = yield strain of longitudinal steel, $\lambda = \alpha s/i_r$ , $\alpha$ = 1.0 for corner bars; 0.5 for intermediate bars and $i_r$ = radius of gyration for bar.

## 4 NUMERICAL STUDY OF FRAME BEHAVIOUR INCLUDING BAR BUCKLING

### 4.1 *Frame and Analysis Details*

A numerical study was undertaken to investigate the dynamic behaviour of a reinforced concrete frame when the buckling of longitudinal reinforcement was considered. A 2-bay, 2-storey reinforced concrete frame designed to the Australian Concrete Structures Standard, AS3600 (1994), was used for the analysis. Details of the frame's geometry and reinforcement details are given in Figure 5. The structure was analysed using a fibre-based, non-linear dynamic analysis program. Constitutive relationships used for the analysis were, Popovics (1973) model for concrete and the modified Ohi & Akiyama model for steel (Kato et al., 1973; Meng et al., 1992). Material properties used in the analysis are given in Table 1.

To determine parameters for the buckling modification the column transverse reinforcement was assumed to be R10 stirrups at 240mm spacings. The ratios of stirrup spacing to bar diameter, $s/d_b$ were 10 and 8.57, respectively for the exterior and interior columns, as permitted by AS3600 for columns. The parameters for the buckling model, given in Table 2 were calculated from Equations 1 and 2 for both the corner and intermediate bars. The critical buckling strain, $\varepsilon_{lb}$, is the same for corner and intermediate bars and post-buckling softening will occur at the same strain. Post-buckling softening is more severe for the intermediate bars. Buckling was not considered for the steel relationships in the beam.

Rayleigh damping of 2% was used in the dynamic analysis for the first two modes of response.

### 4.2 *Discussion of Results*

The structure was analysed using the 1940 El Centro NS earthquake record over a range of peak ground accelerations from 0.05g to 1.2g to determine when collapse occurs for the situation; when buckling is not considered and when it is considered. The frame was considered to collapse when the storey deformations/drifts increased rapidly as shown in Figure 6. For the case where buckling was not considered the peak ground acceleration to cause collapse was between 1.0g and 1.1g. When buckling of longitudinal reinforcement was introduced into the

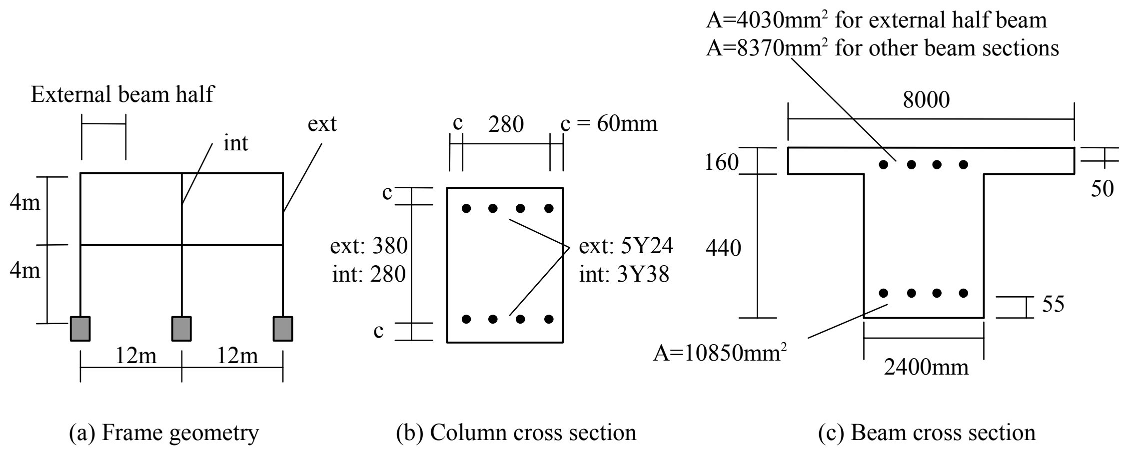


Figure 5 Frame geometry and cross section details for 2-bay, 2-storey frame.

Table 1. Input material properties for analytical frame.

| Material Property | Value |
|---|---|
| Peak concrete compressive strength, $f_c$ | 47 MPa |
| Concrete strain at peak strength, $\varepsilon_c$ | 0.0025 (m/m) |
| Concrete modulus, $E_c$ | 30,000 MPa |
| Steel yield strength, $f_{sy}$ | 460 MPa |
| Steel yield strain, $\varepsilon_{sy}$ | 0.0023 (m/m) |
| Young's modulus for steel, $E_s$ | 200,000 MPa |

Table 2. Values for buckling model parameters used in analysis.

| Bar Type | $\varepsilon_{lb}$ (m/m) | $\tau_{lb}$ |
|---|---|---|
| Exterior Corner | 0.002352 | -0.09721 |
| Exterior Intermediate | 0.002352 | -0.15333 |
| Interior Corner | 0.002352 | -0.08364 |
| Interior Intermediate | 0.002352 | -0.1423 |

analysis, the structure reached collapse more rapidly, occurring between 0.6g and 0.7g. The effect of the longitudinal reinforcement buckling model is to provide a limit on the maximum steel compressive stresses. Figure 7 highlights the influence of longitudinal reinforcement buckling on the deformation response of the structure for two magnitudes of earthquake 0.7 and 1.0g. For the 0.7g magnitude earthquake analyses the difference is quite clear. Without buckling the structure has permanently deformed but not collapsed. When buckling was considered, the structure collapsed. With an increased magnitude of 1.0g, structural collapse occurred with or without buckling considered, however, if it is taken into account the onset of collapse occurred earlier in the time history.

It should be noted that only the buckling of longitudinal reinforcement has been considered in this analysis. When the structure begins to collapse the drift limits are large and not sustainable without major non-structural and structural damage. A solution is still calculated since there is no mathematical limit on the interstorey drifts for the structure. Furthermore, at this stage the steel constitutive relationships have no limit on the maximum tensile strain and therefore the strain in the steel bars can exceed their fracture strain.

The first few cycles of the modified stress-strain behaviour for the exterior column corner and intermediate bars due to buckling of reinforcement are shown in Figure 8. The critical strain for both bars is the same, however, the post-buckling behaviour differs greatly. The intermediate bar has less restraint from outward buckling from the stirrups and hence the post-buckling softening and stress degradation is more severe.

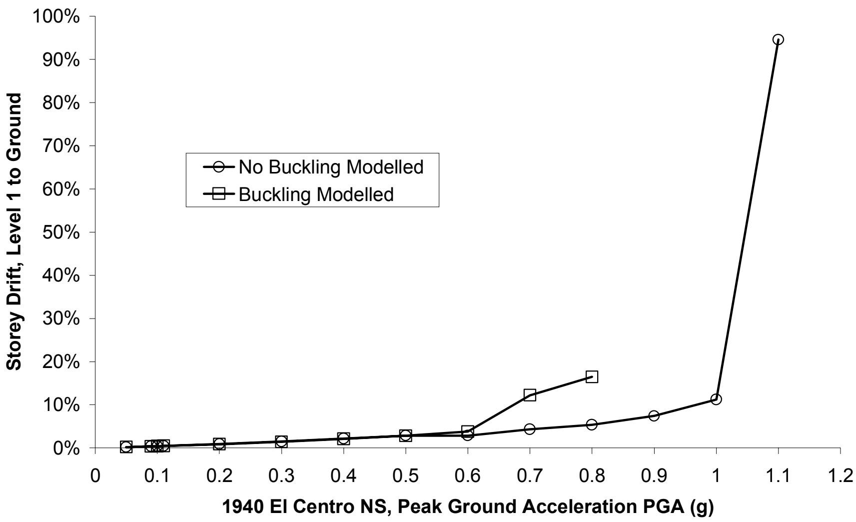


Figure 6 Relationship between interstorey (level 1 to ground) drift and peak ground acceleration.

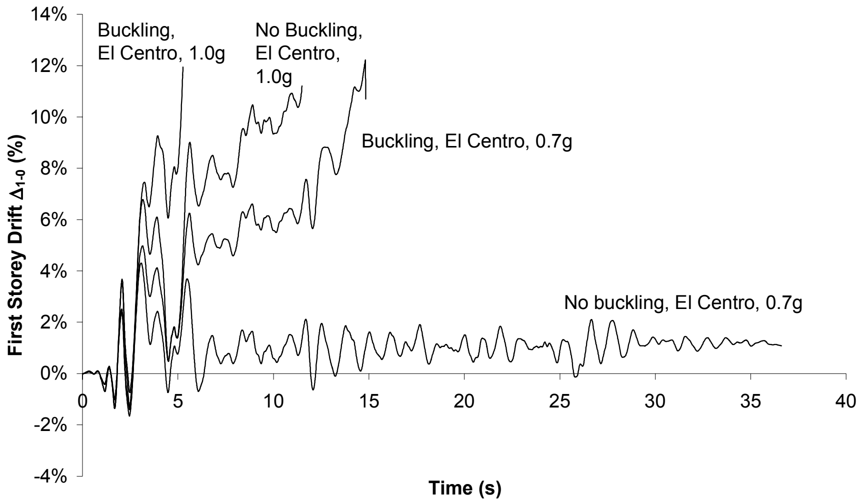


Figure 7 Drift time history for frame analysis.

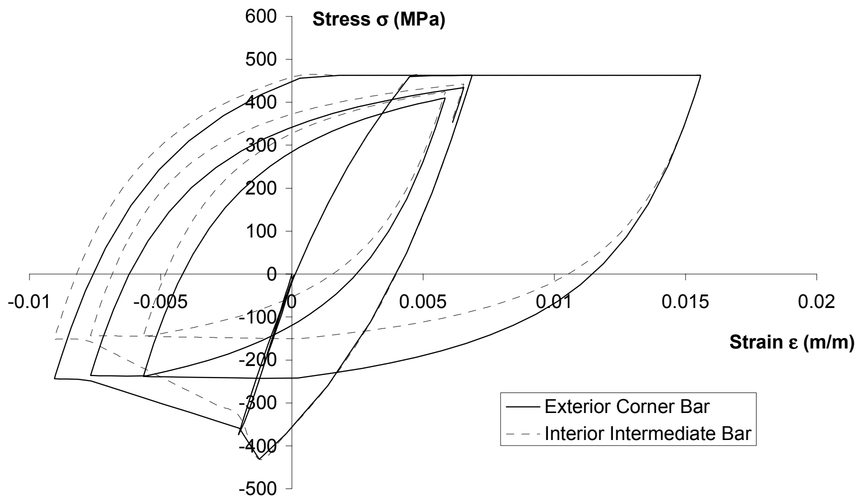


Figure 8 Stress-strain behaviour for exterior column corner and intermediate longitudinal bars.

## 5 SUMMARY

A simplified modification to the steel constitutive relationships has been developed to account for the buckling of longitudinal reinforcement. The modification allows the post-buckling behaviour of reinforcing bars to be modelled so that frames can be analysed up to and beyond collapse. The effects of buckling reduce the analytically determined collapse magnitude earthquake to more realistic levels. Further work is needed to improve the accuracy of these analytical techniques with the inclusion of other secondary effects, such as tensile steel fracture, shear deformations, local shear failure, local joint deformations, and joint failure. Verification of the analytical technique and buckling model with experimental test data needs to be undertaken to ensure model parameters can be estimated accurately to correctly predict analytically the behaviour of reinforced concrete frames when subjected to severe ground motions to cause collapse.

## 6 ACKNOWLEDGEMENTS

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REFERENCES:

Gomes, A. and Appleton, J., (1997), Non-linear cyclic stress-strain relationship of reinforcing bars including buckling, *Engineering Structures*, Vol. 19(10) 822-826.

Inoue, K & Shimizu, N., (1988), Plastic Collapse Load of Steel Braced Frames Subjected to Horizontal Force, *Journal of Structural and Construction Engineering*, AIJ, No. 388, 59-69 (in Japanese).

Kato, B., Akiyama, H. and Yamauchi, Y., (1973), Predictable Properties of Material Under Incremental Cyclic Loading, *IABSE, Report of the Working Commissions, Band 13*, *Symposium on Resistance and Ultimate Deformability of Structures Acted on by Well Defined Repeated Loads, Lisboa*, 119-124.

Kato, D, Kanaya, J, Wakatsuki, K & Honda, Y, (1995), Design method to prevent buckling of main bars in RC members, *Proc. Pacific Conference on Earthquake Engineering,* Vol. 3, 117-126.

Kawano, A., Griffith, M.C., Joshi, H.R. & Warner, R.F., (1998), Analysis of the Behaviour and Collapse of Concrete Frames Subjected to Severe Ground Motion, Department of Civil and Environmental Engineering, The University of Adelaide, Research Report R160, 76p.

Menegotto, M. & Pinto, P.E., (1973), Method of Analysis for Cyclically Loaded R.C. Plane Elements Including Changes in Geometry and Non-Elastic Behaviour of Elements under Combined Normal Force and Bending, *IABSE, Report of the Working Commissions, Band 13*, *Symposium on Resistance and Ultimate Deformability of Structures Acted on by Well Defined Repeated Loads, Lisboa.*

Meng, L., Ohi, K. and Tkanashi, K. (1992), A Simplified Model of Steel Structural Members with Strength Deterioration Used for Earthquake Response Analysis, *Journal of Structural and Construction Engineering*, AIJ, No. 437, 115-124 (in Japanese).

Monti, G. & Nuti, C., (1992), Non-linear Cyclic Behaviour of Reinforcing Bars Including Buckling, *Journal of Structural Engineering*, ASCE, Vol. 118(12), 3269-3284.

Nakatsuka, T., Maekawa, M. & Nakagawa, H., (1999), Equations to Estimate Strains at Buckling of Longitudinal Reinforcement Under Uniaxial and Monotonical Compression, *Journal of Structural and Construction Engineering*, AIJ, No. 516, 145-149 (in Japanese).

Papia, M. and Russo, G., (1989), Compressive Concrete Strain at Buckling of Longitudinal Reinforcement, *Journal of Structural Engineering*, ASCE, Vol. 115(2), 382-397.

Popovics, S., (1973), A Numerical Approach to the Complete Stress-Strain Curve of Concrete, *Cement and Concrete Research*, Permagon Press Inc., Vol. 3(5), 583-599.

Rodriguez, M.E., Botero, J.C. & Villa, J. (1999), Cyclic Stress-Strain Behaviour of Reinforcing Steel Including Effect of Buckling, *Journal of Structural Engineering*, ASCE, Vol. 125(6), 605-612.

Suda, K, Murayama, Y. Ichinomiya, T. & Shimbo, H., (1996), Buckling Behaviour of Longitudinal Reinforcing Bars in Concrete Columns Subjected to Reverse Lateral Loading, *Eleventh World Conference on Earthquake Engineering, 1996.*

Yamada, S., Akiyama, H., & Kuwamura, H., (1993), Post-Buckling and Deteriorating behaviour of Box-Section Steel Members, *Journal of Structural and Construction Engineering*, AIJ, No. 444, 135-143 (in Japanese).

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