The rotation of asymmetric plan structures

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ABSTRACT:
A recent study on the torsional response of ductile structures indicated the need for improvements in current seismic design provisions (Paulay, 1997). This is because most standards deal with the torsion problem using concepts based on elastic response. These provisions may be satisfactory at the serviceability limit state but are generally irrelevant for ductile structures.

The objective of this paper is to promote an understanding of the dynamic response of asymmetric structures. It is illustrated how the mass rotational inertia plays an important role in the response of asymmetric structures. It is shown how strength eccentricity and distribution of mass affect the rotation of the system.

It is suggested that the strength distribution to elements of a system be such as to reduce or eliminate strength eccentricity. This generates a reduction of the system rotation and allows the structure to be modelled as an equivalent single degree of freedom, ESDOF, system. Thus, the system displacement ductility capacity becomes a simple function of the displacement ductility capacity of the critical element.

1 INTRODUCTION

Structural engineers often encounter buildings, which exhibit some degree of plan asymmetry. This asymmetry can be caused by an uneven distribution of strength, stiffness and/or mass. In addition, these structures can be affected by what is called accidental eccentricity. This is considered to compensate for inevitable differences between the assumed and the actual characteristics of the structure. For this purpose most seismic design provisions (IAEE, 1996) recommend the use of a design eccentricity that considers both dynamic and accidental eccentricity. The dynamic eccentricity is defined as the distance from the centre of stiffness at which the base shear V_o has to be applied to develop the torsional moment predicted by a time history analysis (Newmark, 1971). This force-based concept is intended to ensure that the ductility capacity of elements will never be exceeded. This concept contemplates the dynamic torque introduced by the mass rotational inertia by increasing the stiffness eccentricity, e_r, by a predefined value. The validity of this concept has been questioned when ductile behaviour is to be addressed since it was intended for elastic response (Paulay, 1997).

Priestley and Kowalsky (1998) and Paulay (1999, 2000) have recently proposed that, in contrast with the assumptions made in the past of assuming stiffness as being independent of strength element, stiffness is strength dependent. An implication of the element stiffness being strength dependent is that the location of the centre of rigidity, CR, could be quite different from that found with the traditional approach, particularly when element properties based on gross concrete sections are used for analysis.

This paper evaluates the rotation of asymmetric structures and its effect on the displacement ductility demand using a displacement approach based on a realistic element modelling. It is suggested that the strength distribution to elements of a system be such as to reduce or eliminate the strength eccentricity since this result in a reduction of the system rotation. This enables the establishment of a simple relationship between the system displacement ductility demand and the displacement ductility capacity of the critical element.

2 GENERAL DEFINITONS

In the theory of elasticity stiffness, *k,* of a prismatic element is based on the flexural rigidity, *EI*, where *E* is the modulus of elasticity of the material and *I* is the second moment of area of the cross section.

In most reinforced concrete components cracking extends throughout a significant portion of the length. Therefore, stiffness based on the gross section properties can lead to misleading values. Paulay (1999, 2000) showed in detail that the nominal yield displacement for structural walls could be approximately obtained from the curvature at first yield, which is inversely proportional to the length of the elements. Reinforcement ratio and axial load levels, normally encountered in walls, do not significantly influence yield curvature and hence yield displacements. Thus, the element stiffness, k_i, using this concept is,

$$k_i = \frac{V_{ni}}{\Delta_{yi}} \qquad (1)$$

where, V_{ni} is the nominal strength and Δ_{yi} is the nominal yield displacement of the element.

The system strength and stiffness are defined as the sum of strength and stiffness of all the resisting elements, respectively. Thus, the system nominal yield displacement is,

$$\Delta_s = \frac{\sum V_{ni}}{\sum k_i} \qquad (2)$$

The centre of strength, CV, is defined as the location about which the first moment of the element strength is zero. The distance between the centre of mass, CM and the CV is referred to as the strength eccentricity, e_v. This eccentricity is expressed as,

$$e_v = \frac{\sum V_{ni}\, x_i}{\sum V_{ni}} \qquad (3)$$

The radius of gyration of strength, r_v, which is a measure of the distribution of strength in the structure, is defined as,

$$r_v = \sqrt{\frac{\sum V_{in}\, x_i^2}{\sum V_{in}}} \qquad (4)$$

where, x_i is the distance of the element from the CM.

The total mass, *M*, and the polar moment of inertia of the distributed mass, I_m, also affect the dynamic response in asymmetric structures. These two parameters are considered through the radius of gyration of mass, r_m, defined as,

$$r_m = \sqrt{\frac{I_m}{M}} \qquad (5)$$

Note that in equation (4) I_m is defined with respect to the CM. The ratio of the radius of gyration of strength to radius of gyration of mass, r_v/r_m, is an important parameter that accounts for the system rotation.

The CR of elastic systems is defined as the location about which the first moment of stiffness of the elements is zero. Thus is the point through which the application of a static lateral force causes no rotation. The distance between the CM and the CR is referred to as the stiffness eccentricity, e_r, as shown in figure 1. This eccentricity is,

$$e_r = \frac{\sum k_i\, x_i}{\sum k_i} \qquad (6)$$

The system uncoupled translational period, T_y, as expressed by eq. (7), considers the system stiffness as the sum of the stiffness of all the elements, i.e.,

$$T_y = 2\pi\sqrt{\frac{m}{\sum k_i}} \qquad (7)$$

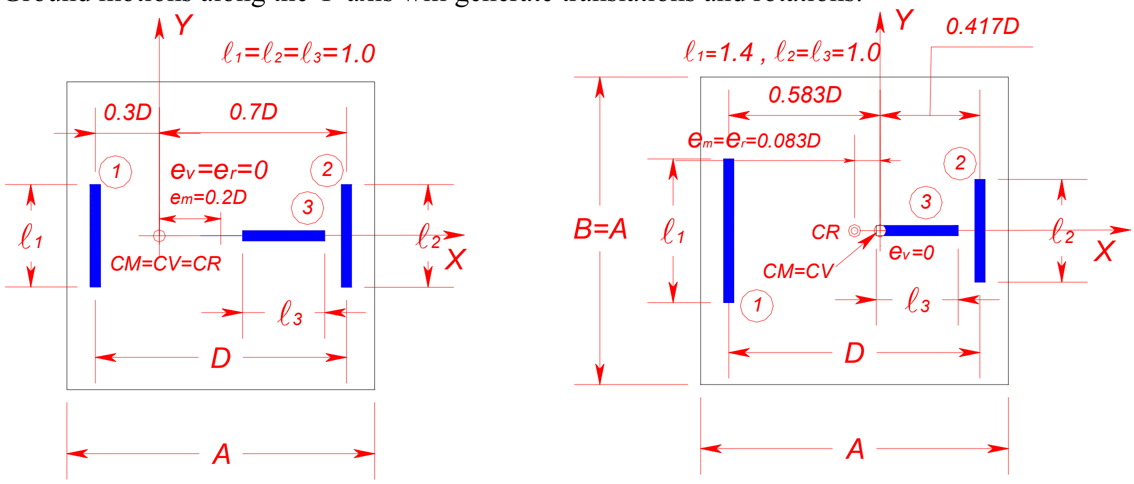
This definition of period implies that the CM and CR are coincident. Thus, no coupling occurs between the translational and the rotational mode shapes.

3 SYSTEM DESCRIPTION

The response of the two torsionally un-restrained simple mass systems shown in figure 1 is investigated to observe the effects caused by mass rotational inertia and strength eccentricity. The translational strength is provided by the two elements parallel to the Y-axis. An additional element is placed along the X-direction to provide transverse strength and stability without adding torsional strength to the structure. The in-plane floor diaphragm is considered infinitely rigid and the cantilever elements are fixed at ground level. Only flexural deformations are considered. The elasto-plastic hysteretic rule is used to represent the non-linear response of the elements.

In System 1 the two lateral force resisting elements parallel to the Y-axis have the same length, that is ℓ_1/ℓ_2=1, as figure 1a shows. The system is symmetric but is mass–eccentric. The mass eccentricity is – 0.20D, where D is the distance between elements 1 and 2. The location of the CM, CR and CV is the same. Thus, the system is traditionally classified as torsionally balanced. Ground motions applied to this system in the Y direction will only generate translation.

System 2, as shown in figure 1b, is asymmetric and is mass-eccentric. It has the same elements as System 1 with the difference that the ratio ℓ_1/ℓ_2 is 1.4. The mass eccentricity is equal to +0.083D. The CM and CV coincide. The difference in length between the elements generates a stiffness eccentricity. Thus, the system is classified as torsionally unbalanced. Ground motions along the Y-axis will generate translations and rotations.



(a) System 1 (b) System 2

Figure 1. Torsionally un-restrained systems.

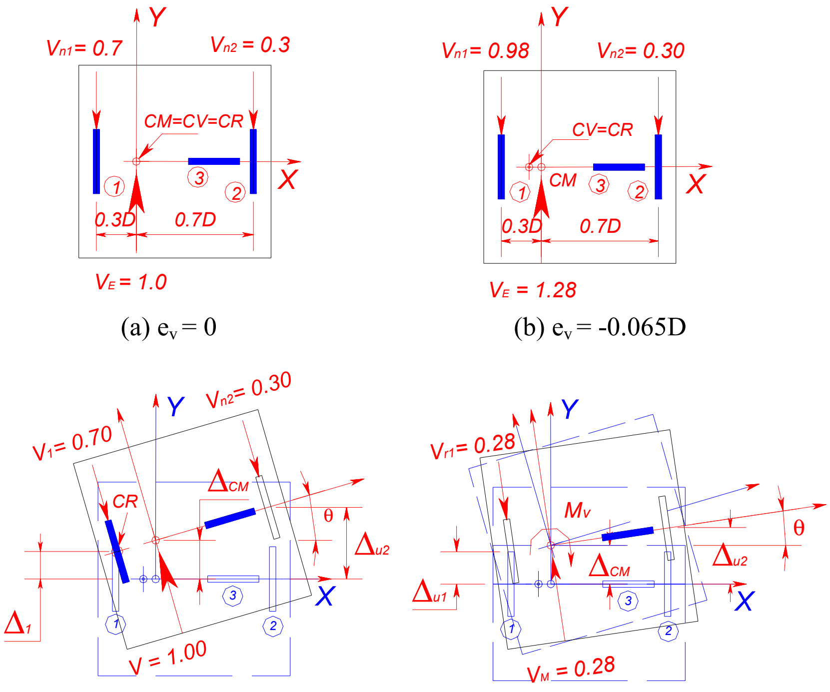
Both systems were subjected to an input ground motion along the Y-axis. The ground motion was generated to match the loading standard (NZS 4203, 1992) response spectra for intermediate soil conditions. The ground motion used was seeded from the N-S component of El Centro earthquake of 1940. The systems were assumed to have a 5% viscous damping on the two modes of vibration.

The systems were provided with a total base shear capacity to meet a displacement ductility demand of 5. Design seismic coefficients were obtained from an inelastic response spectrum for an equivalent 5% damping ratio. Systems 1 and 2 have an uncoupled translational period of 1.3 sec. The corresponding seismic coefficient was 0.10.

4 EFFECT OF THE MASS ROTATIONAL INERTIA

In case of asymmetric systems, the mass inertia and the mass rotational inertia generate both translation and rotation, whereas only translation is generated in symmetric systems. The importance of the mass rotational inertia in the dynamic response of the system is explained by combining statics with a dynamically induced torque (Paulay, 1999) as explained below.

Figure 2a shows the strength distribution to be provided to elements 1 and 2 in System 1 to satisfy static equilibrium. Let us assume that for some reason element 1 ends up with 40% excess strength. That is, the excess strength factor for element 1 is λ_1 =1.4, see Figure 2b. This case, is studied only for research purposes since a symmetric structure should never end up with such a strength eccentricity. The excess strength of element 1 generates a 28% increase of the system's strength and a reduction of the uncoupled translational period. This introduces a strength eccentricity, e_v = –0.065D. Since the length of elements 1 and 2 is the same, the strength and stiffness eccentricities are equal.



(a) e_v = 0 (b) e_v = -0.065D

(c) Static (d) Dynamic

Figure 2. Effect of the translational mass and mass rotational inertia on system response.

Figure 2c shows the case when a force V=1.0 is applied at the CM. The system develops a static mechanism when the nominal strength of element 2 is reached. According to the figure, a change in colour on an element from black to white implies that the element has reached its nominal strength. At this force level Element 1 remains elastic with reserve strength, V_{r1}, of 0.28. Once the static mechanism is attained the centre of rotation shifts to element 1. The centre of strength remains unaltered since its position is not affected by yielding of the elements. The displacement of element 1, $\Delta_{1,}$ is smaller than its nominal yield displacement, Δ_{y1}. In contrast, element 2 develops a displacement, $\Delta_{u2,}$ beyond it nominal yield value, Δ_{y2}.

Figure 2d shows that as element 2 yields it is feasible for element 1 to reach its strength and hence its nominal yield displacement, Δ_{y1}, if a clockwise torque, $M_{v,}$ is introduced. In fact, such torque is generated during the dynamic response due to the mass rotational inertia. This dynamic torque can be introduced because there is additional resistance provided by the pair generated by the reserve strength of element 1 and the resistance provided by the mass inertia force. The torque increases the displacement of element 1 beyond its yield displacement, Δ_{u1}, while reduces the displacement in element 2. The clockwise dynamic torque, M_v, is equal to the reserve strength of element1,V_{r1} times the distance between the element and the centre of mass, 0.30D. This torque is equal to the system's strength, V_E, times the strength eccentricity, 1.28 x 0.065D.

5 DUCTILE RESPONSE OF A TORSIONALLY UNBALANCED SYSTEM

System 2, see figure 1b, is utilised to study the effect of the strength eccentricity and the mass rotational inertia on the dynamic response.

The strength is distributed so that static equilibrium is attained (e_v=0). A negative strength eccentricity develops in System 2 when the strength of element 1 is increased by an excess factor λ_1 while the strength in element 2 is not varied. The opposite occurs if the excess strength factor λ_2 is applied to element 2.

The variation of the normalized eccentricities e_v/D and e_r/D versus λ_1 and λ_2 is plotted in figure 3. It is observed a change in the location of the CV and CR when the strength factor λ varies. The difference between the eccentricities is caused because element 1 is 1.4 times longer than element 2. The difference in the element length is considered through the element length ratio, $\alpha=\ell_1/\ell_2$, where ℓ_1 and ℓ_2 is the length of elements 1 and 2, respectively. In this particular case and according to equation 8 for α=1.4 and λ_1 =1 (e_v=0), a stiffness eccentricity of -0.0833D is obtained.

$$\frac{e_v}{D}=\frac{1}{2.33\lambda_1+1}-0.3 \quad \text{and} \quad \frac{e_r}{D}=0.58\frac{(1-\alpha\lambda_1)}{(1.4+\alpha\lambda_1)} \tag{8}$$

As an excess strength is assigned to either element, a strength eccentricity is introduced. This brings together a variation in the stiffness eccentricity. Note that in this system the distance between CV and CR remains almost constant as λ varies.

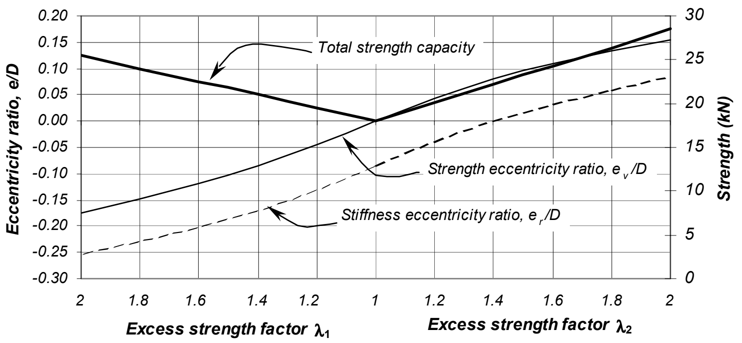
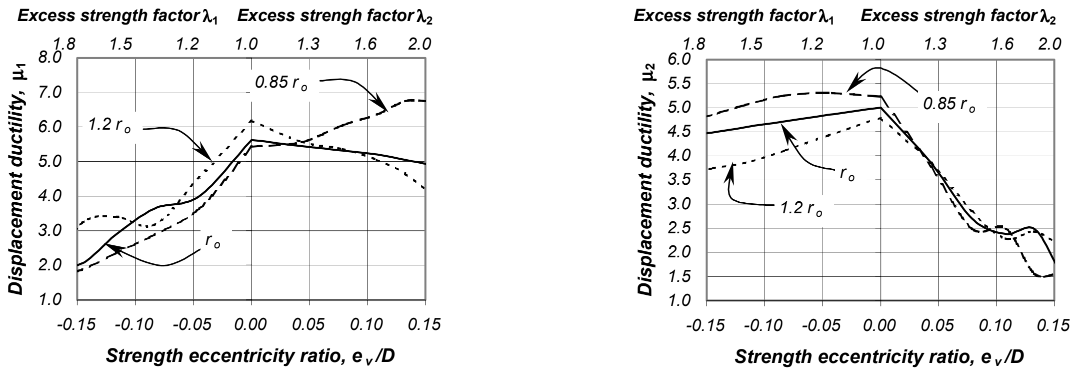


Figure 3. Relationship between strength and stiffness eccentricity ratios



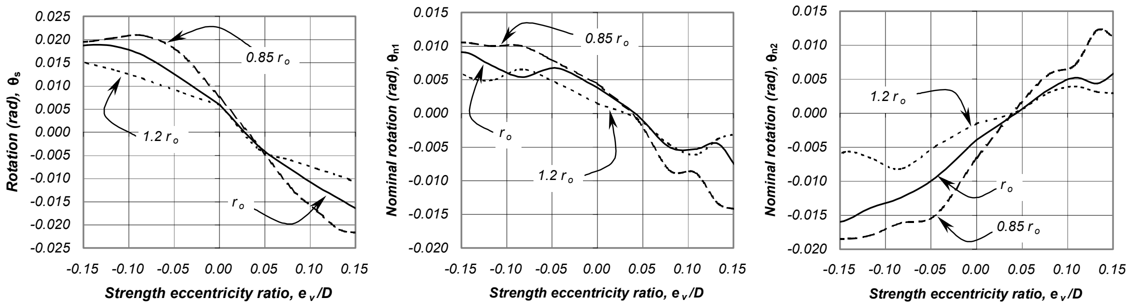
(a) Element 1 (b) Element 2

Figure 4. Element displacement ductility demand for a given strength eccentricity. System 2 with T_y=1.3 sec and α=1.4.

The centre of mass and strength in a system are fixed once the location and length of the elements and the plan configuration is known. System 2 has a square plan that corresponds to a radius of gyration of mass equal to r_m=r_o. In addition, r_m is varied to 0.85 r_o and 1.2 r_o to cover other plan configurations provided the mass remains constant. Once the location and length of the elements is fixed the radius of gyration of strength with respect to the CM, see equation 4, is equal to r_v. The ratio between these two quantities r_v/r_m is a normalised expression that takes into account the effect of the mass rotational inertia on the system rotation. This ratio is directly related to elastic systems since the radius of gyrations of stiffness, r_k, is proportional to r_v. The values of the r_v/r_m ratio associated with r_m values of 0.85 r_o, r_o and 1.2 r_o are 1.16, 0.99 and 0.83, respectively.

Figure 4 shows the variation in the displacement ductility demand, μ_1 and μ_2, on elements 1 and 2 for different values of the strength eccentricity ratio. This figure also plots the response of the elements due to the variation of the radius of gyration of mass, $r_{m,}$ which is associated with the r_v/r_m ratio. It is seen that an increase in the element strength reduces the displacement ductility demand of the respective element. On the other hand, the element where the strength is not varied the displacement ductility demand does not significantly vary from the ductility reached when e_v=0. This results show that the increase in the rotation generated by the strength eccentricity is compensated by the increase in the system strength. Figure 4 also shows that the displacement ductility demand is also influenced by the magnitude of r_m. This reduction is larger as r_m increases. The above results indicates that a strength eccentricity can be introduced in a system as long as the strength assigned to the elements is equal or larger to the strength required to comply with static equilibrium, CV=CM. Thus, an introduction of a strength eccentricity is associated with an increase in the system's strength.

Figure 5a shows the effect of the strength eccentricity ratio on the system rotation. A clockwise rotation is generated when the strength eccentricity is negative. As the strength eccentricity is reduced, the rotation diminishes and reaches zero when e_v=+0.025D. Thereafter, the rotation of the system changes direction and increases once again. This suggests that it is possible to modify the rotation of the structure by introducing a strength eccentricity. Thus, strength eccentricity is synonymous of rotation. However a twist not necessarily creates a harmful situation as long as the ductility capacity of the elements is not exceeded. An exception is observed for r_m=0.85r_o when a positive strength eccentricity increases the ductility demand on element 1. The effect of the mass rotational inertia in the response is also shown in figure 5a. It is seen that the system rotation diminishes as r_m increases.



(a) System (b) Element 1 (c) Element 2

Figure 5. System rotation and the nominal rotation associated with elements 1 and 2 plotted against the strength eccentricity ratio.

The maximum displacements reached by elements 1 and 2 and the CM are not attained at the same time nor these displacements are reached when the maximum system rotation develops. Therefore, it is not possible to directly associate the system maximum displacement and rotation to the maximum response of the elements. Due to this behaviour, the problem is approached through a nominal rotation, θ_n that relates the maximum displacement of elements 1 or 2 to the maximum displacement developed at the CM. This rotation is expressed by,

$$\theta_{ni} = \frac{\Delta_{ucm} - \Delta_{ui}}{x_i} \tag{9}$$

where, Δ_{ucm} is the maximum displacement at the CM, Δ_{ui} is the maximum displacement of element *i* and x_i is the distance between the CM and element *i*.

Figures 5(b) and 5(c) plots the variation of the nominal rotations θ_{n1} and θ_{n2} against the strength eccentricity. It is seen that for strength eccentricity of +0.04D the nominal rotation associated with elements 1 and 2 is equal to zero. This strength eccentricity corresponds to an element strength distribution such that the CM ends up located in between the CV and CR. This behaviour turns out because before yielding occurs in the elements, a clockwise rotation is generated by the elastic torque, $M_r = Ve_r$. As soon as one of the element yields, the mass rotational inertia introduces an opposite dynamic torque and equal to $M_v = Ve_v$. The superposition of both torques induce a minimum in the rotation of the system. This generates the whole system to translate without generating a rotation. Thus, the maximum displacement in the two elements and at the centre of mass results to be equal.

An advantage of the nominal rotation as defined above is that it may allow decoupling the system's rotation from the translation. For this to be applicable it is necessary to assure that the maximum displacement generated at the CM of the system compares with the maximum displacement generated by an ESDOF. The ESDOF has a stiffness $K_s = \Sigma k_i$ and a strength $V_s = \Sigma V_i$. The comparison of the response of System 2 and that of the ESDOF is shown in Figure 6. It is observed a close response up to a strength eccentricity ratio of +- 0.08D. Thereafter, the difference is of the order of 15%. This difference can be neglected since the aim is to distribute strength such that strength eccentricity is reduced or eliminated. The combination of these two effects provides the maximum response to be reached by the elements.

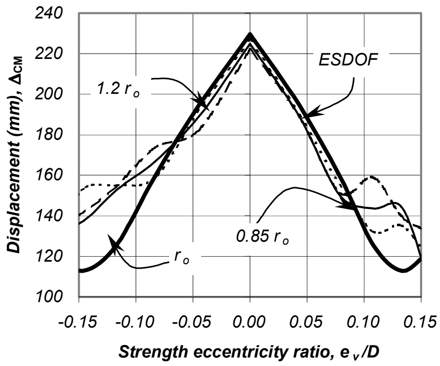


Figure 6. Comparison of the response of System 2 and an ESDOF. T_y=1.3 sec

The objective of reducing or eliminating the strength eccentricity is to minimise the rotation of the system. It is shown in figures 5b and 5c that the rotation is eliminated when the CM ends up located in between the CV and CR. However, instead of having to use cumbersome calculations to find the CR, it is simply enough to aim to design so that CM=CV. Even though, a small rotation is expected in this case, see figure 5, it can be neglected as long as the distance between CV and CR is small. Lets recall that the location of CR is a function of α.

In a system that only translates the critical element is the longest element in the system. The following relationship relates the system displacement ductility capacity and the critical element displacement ductility capacity. This relation is expressed as,

$$\mu_s = \frac{\alpha + 0.58(1-\alpha)}{\alpha}\mu_1 \tag{10}$$

For example, system 2 has an element length ratio of α=1.4 and the displacement ductility capacity of the critical element 1 is equal to μ_1=5. Equation 10 indicates that the system must be designed for a displacement ductility capacity of μ_s = 4.2. This ductility, μ_s, can now be used to compute the strength required by the system.

6 **CONCLUSIONS**

1. The magnitude of the dynamic torque introduced by the mass rotational inertia is proportional to the strength eccentricity introduced to the system. The parameter r_m also affects the response. The variation of this parameter modifies the rotation of the system for a given strength eccentricity. As r_m is increased the torque and the rotation of the system are reduced.
2. It is considered practical to distribute the strength on the elements so that CV=CM. The rotation that generates this strength distribution is considered small, thus it can be neglected as long as the distance between CR and CV is small. A system with this characteristic will exhibit translation only. This enables the determination of a simple relationship that relates the displacement ductility demand of the system with the displacement ductility capacity of the critical element.
3. A strength eccentricity can be introduced to a system as long as the strength provided to the elements is equal or larger to the strength required to comply with static equilibrium, CV=CM.
4. It is suggested that the response an asymmetric system can be considered as the translation generated by an ESDOF plus a nominal rotation. The combination of these two effects provides the maximum response to be reached by the elements.

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7 **RETURN TO INDEX**